Virtual graviton exchanges at the Z pole in large extra dimensions

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Abstract

In the framework of quantum gravity propagating in large extra dimensions, the effects of virtual Kaluza-Klein gravitons on the imaginary part of the amplitude of the process $e^+e^- \to f\bar{f}$ are analyzed at the Z pole. Notably, the interference of the almost-continuous spectrum of the KK gravitons with the standard model resonant amplitude is finite and predictable in terms of the fundamental D-dimensional Plank scale M_D. We find that, while the virtual-graviton effect on total cross sections vanishes at tree-level, both angular and polarization asymmetries are modified by terms whose *relative* effect is at most of order 10^{-4} for $M_D > 1$ TeV. Possible shifts in M_Z, due to the changes in Breit-Wigner line shape are also discussed.

1   Introduction

As shown by Arkani-Hamed, Dimopoulos, and Dvali (ADD) a few years ago [1], the existence of large extra dimensions with only gravity propagating in the bulk, provides an interesting alternative solution to the hierarchy problem. A dramatic consequence of this scenario is that gravity might become strong at the TeV scale, hence making plausible the possibility of detecting quantum gravity effects at future collider experiments [2, 3, 4].

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In the ADD scenario, the Newton's constant G_N in the 3+1 dimensional space is related to the corresponding Planck scale M_D in the $D = 4 + \delta$ dimensional space by [1]

$$G_N^{-1} = 8\pi R^\delta M_D^{2+\delta} \tag{1}$$

where R is the radius of the compact manifold assumed to be on a torus. Therefore, the weakness of gravity might be due to the large size of the compactified extra dimensional space. In particular, if $M_D \sim$ TeV then deviations from the Newton's law are expected at distances of order $R < 10^{32/\delta-19}$ meters [5]. This is compatible with the present experimental sensitivity in gravity tests, which is above the millimeter scale. Then, assuming $M_D \sim$ TeV, Eq.(1) implies $\delta \geq 2$.

The effective Einstein theory in 3+1 dimension is then obtained by integrating out compact extra dimensions. It describes massive Kaluza-Klein (KK) excitations of the standard graviton field.* The KK excitations are very narrowly spaced in comparison to the M_D scale, and can be often treated as a continuum. Remarkably, in the inclusive production (or virtual exchange) of KK gravitons, the sum over the allowed tower of KK states replaces the suppression factor $(E/M_P)^2$ associated to a single graviton production by the quantity $(E/M_D)^{2+\delta}$, where E is the typical energy of the process, and M_P is the Plank mass. Therefore, if M_D is of order of TeV, quantum gravity effects may become accessible at future collider experiments.

In the framework of the ADD scenario, two classes of processes have been analyzed in e^+e^- and hadron collisions [2, 3, 4, 6] : the direct production of KK gravitons, detected as missing energy in the final states, and the virtual KK graviton exchange. In the latter case, the graviton exchange induces local dimension-eight operators (associated with the square of the energy momentum-tensor) that will affect, for instance, the standard four fermion processes. The real part of this kind of contribution is divergent [3]. The divergences can be regularized by introducing an unknown ultraviolet cutoff, therefore loosing the predictivity in terms of the fundamental Plank scale M_D. Indeed, in the effective Einstein theory in 3+1 dimension, arising by integrating out extra dimensions, divergences can appear even at tree-level after summing over the KK states of the graviton propagator. In the case of virtual graviton exchange, in many analyses the assumption is made that the *divergent* real part of the amplitude $\mathcal{A}$ is dominated by the lowest dimensional operator (of dimension eight in the case considered)

$$Re\,[\mathcal{A}] = \frac{4\pi}{\Lambda_T^4} \left(T_{\mu\nu}T^{\mu\nu} - \frac{1}{\delta+2}T_\mu^\mu T_\nu^\nu \right) \tag{2}$$

where $T_{\mu\nu}$ is the energy-momentum tensor. The new scale Λ_T is an effective cutoff,

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* For a detailed discussion about the effective four-dimensional theory, see [3].

assumed to be of order of M_D, introduced to parametrize unknown new physics contributions in the ultraviolet region.

On the other hand, the imaginary part $Im[\mathcal{A}]$ is finite, being connected to the branch-cut singularity of real graviton emission [3].

Up to now, phenomenological analyses have considered virtual KK graviton contributions to the standard model (SM) processes in kinematical regions far from the resonances of the SM spectrum. In this case, the leading contribution to the physical process is due to the interference of the amplitude $Re[\mathcal{A}]$ with the real part of the corresponding SM amplitude, and is suppressed by $(E/\Lambda_T)^4$, where E (assumed to be $E < \Lambda_T$) is the typical energy of the process.

On the other hand, when considering SM resonant processes interfering with virtual KK graviton graphs, the corresponding interference is dominated by $Im[\mathcal{A}]$. Hence, it is finite, and predictable in terms of the fundamental Plank scale M_D.

In this paper we study an example of the latter class of processes, that is the two-fermion production in e^+e^- collisions, $e^+e^- \to f\bar{f}$, in the resonant region of the Z pole. We compute the *finite* interference of the SM amplitude with the $e^+e^- \to G \to f\bar{f}$ amplitude (where G is a KK graviton), and provide a detailed analysis of this new-physics effect in the light of our present knowledge on four-fermion processes at LEP and SLD [7].

In the next section, we present the analytical and numerical results for the angular distributions and total cross sections for the process $e^+e^- \to f\bar{f}$ at the Z pole, for both unpolarized and polarized initial beam and final states. We also discuss possible effects on the model-independent determination of the Z mass through the Z line shape. Section 3 is devoted to our conclusions.

2 Virtual-graviton effects on physical observables at the Z pole

We consider the two-fermion production in electron-positron collisions on the Z pole

$$e^-(k_1)e^+(k_2) \to f(k_3)\bar{f}(k_4) \tag{3}$$

where k_i are the particle momenta.

We want to compute the physical effects of the interference of the resonant Z diagram with the amplitude where a virtual KK graviton is exchanged in the s-channel. We define the Mandelstam variables as $s = (k_1+k_2)^2 \ [\sim M_Z^2]$, $t = (k_1-k_3)^2$, $u = (k_1 - k_4)^2$, and neglect all the fermion masses.

The scattering amplitude in the momentum space of the graviton-mediated process is given by a sum over all KK modes [3]

$$\mathcal{A} = \frac{1}{\bar{M}_P^2} \sum_n \left\{ T_{\mu\nu} \frac{P^{\mu\nu\alpha\beta}}{s - m_{G_n}^2} T_{\alpha\beta} + \frac{1}{3} \left(\frac{\delta - 1}{\delta + 2} \right) \frac{T_\mu^\mu T_\nu^\nu}{s - m_{S_n}^2} \right\}, \tag{4}$$

where $\bar{M}_P$ is the reduced Plank mass ($\bar{M}_P = M_P/\sqrt{8\pi}$). The first term in Eq.(4) corresponds to graviton exchange, the second to scalars exchange, with masses m_{G_n} and m_{S_n}, respectively. The $P^{\mu\nu\alpha\beta}$ is the projector of the graviton propagator, that in the unitary gauge is given by

$$P^{\mu\nu\alpha\beta} = \frac{1}{2} \left(\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} \right) - \frac{1}{3}\eta^{\mu\nu}\eta^{\alpha\beta} + \ldots \tag{5}$$

where $\eta^{\mu\nu}$ is the Minkowski metric and dots represent the terms proportional to the momentum of the graviton propagator. Since we are using the massless approximation for initial and final states, terms proportional to the trace of $T_{\mu\nu}$ vanish. Terms in the graviton propagator that are proportional to the graviton momentum q_μ vanish, too, being $q^\mu T_{\mu\nu} = 0$. Hence, Eq.(4) becomes

$$\mathcal{A} = \mathcal{S}(s) T_{\mu\nu} T^{\mu\nu}, \quad \mathcal{S}(s) = \frac{1}{\bar{M}_P^2} \sum_n \frac{1}{s - m_{G_n}^2} \tag{6}$$

The sum above can be easily computed in the continuum approximation for the KK graviton spectrum [3]

$$\mathcal{S}(s) = \frac{1}{M_D^{2+\delta}} \int d^\delta q_T \frac{1}{s - q_T^2} = \frac{\pi^{\frac{\delta}{2}}}{M_D^4} \Gamma(1 - \frac{\delta}{2}) \left(-\frac{s}{M_D^2} \right)^{\frac{\delta}{2}-1} \tag{7}$$

where we used $m_{G_n}^2 = q_T^2$, with q_T the graviton momentum orthogonal to the brane. In the interference with the resonant channel at the Z pole, only $Im[\mathcal{S}(s)]$ will contribute, with [3]

$$Im[\mathcal{S}(s)] = -\frac{\pi}{M_D^{2+\delta}} \frac{S_{\delta-1}}{2} s^{\frac{\delta-2}{2}} \tag{8}$$

where, for $\delta = 2n$, $S_{\delta-1} = 2\pi^n/(n-1)!$ and, for $\delta = 2n+1$, $S_{\delta-1} = 2\pi^n/\prod_{k=0}^{n-1}(k + \frac{1}{2})$, with n integer. While the real part of $\mathcal{S}$ (that is not relevant in our case !) is divergent and must be regularized by introducing an external cut off that parametrizes unknown new physics contributions in the ultraviolet region, the imaginary part is finite and predictable, only depending on the D-dimensional Plank scale M_D and on the number of extra dimensions δ. Indeed, the imaginary part is finite because it corresponds to the branch-cut singularity of real graviton emission.

We now present how the SM cross sections and distributions for the resonant process $e^+e^- \to f\bar{f}$ (assuming f different from both e^- and ν_e) are modified after the inclusion of the interference with the virtual-graviton exchange. We will make a tree-level analysis, i.e. we will neglect non-leading effects in the SM amplitude, such as the s-channel photon exchange, initial-state radiation, and electroweak and strong corrections. Although such corrections will turn out to be in general larger than the virtual-graviton ones, we will assume that the main effect of the virtual-graviton amplitude arises from its interference with the tree-level SM amplitude. We will also neglect terms corresponding to the squared amplitude of the graviton exchange.

One can find how the SM differential distribution $d\sigma_f/dcos\theta$ for the process $e^+e^- \to f\bar{f}$ is modified by interference terms with the graviton exchange, by means of Eq.(6) and Eq.(8). At the Z pole (i.e., for $s = M_Z^2$), one gets the new contribution

$$\frac{d\sigma_f^{\text{New}}}{d\cos\theta} = -\frac{N_c G_F S_{\delta-1}}{128\sqrt{2}} \left(\frac{M_Z}{M_D}\right)^{2+\delta} \left(\frac{M_Z}{\Gamma_Z}\right) \left\{ g_A^e g_A^f \left(1 - 3\cos^2\theta\right) - 2g_V^e g_V^f \cos^3\theta \right\} \quad (9)$$

with θ defined by $t = -s(1 - \cos\theta)/2$. G_F is the Fermi constant, Γ_Z is the Z total width, $g_V^f = T_3^f - 2Q_f \sin^2\theta_W$ and $g_A^f = T_3^f$ are the standard vectorial and axial Z couplings (with $T_3^e = -1/2$), and $N_c = 3$ for quarks and 1 for leptons.

Correspondingly, the two-fermion cross section at the Z pole, is modified as follows

$$\frac{d\sigma_f}{d\cos\theta} = \frac{9\pi}{2M_Z^2} \frac{\Gamma_{e^+e^-}\Gamma_{f\bar{f}}}{\Gamma_Z^2} S_f(\cos\theta) \quad (10)$$

where

$$S_f(x) = 1 + x^2 - \Delta_1 \left(1 - 3x^2\right) + 2A_e A_f \left(x + \Delta_2 x^3\right), \quad (11)$$

and $\Gamma_{f\bar{f}}$ is the Z partial width into $f\bar{f}$, $\Gamma_{f\bar{f}} = \frac{N_c G_F M_Z^3}{6\pi\sqrt{2}} \left((g_V^f)^2 + (g_A^f)^2\right)$, and $A_f = 2g_V^f g_A^f / \left((g_V^f)^2 + (g_A^f)^2\right)$. The graviton-interference effects are confined in the coefficients Δ_1 and Δ_2 that are f-flavour dependent [†], with

$$\Delta_1 = R_\delta \frac{A_e A_f}{g_V^e g_V^f}, \qquad \Delta_2 = \frac{R_\delta}{g_A^e g_A^f}, \quad (12)$$

and

$$R_\delta = \frac{\pi S_{\delta-1}}{32\sqrt{2}G_F M_Z^2} \left(\frac{\Gamma_Z}{M_Z}\right) \left(\frac{M_Z}{M_D}\right)^{2+\delta}. \quad (13)$$

In table 1, values of R_δ are shown for $\delta = 2, 3, 4, 5, 6$, and M_D in the range $(0.5, 3.0)$ TeV. The corresponding values of Δ_1 and Δ_2 for number of extra dimensions $\delta = 2$, in the three cases of final charged leptons, up-type quarks and down-type quarks, are given in table 2.

[†]A superscript Δ^f is understood wherever in the paper.

M_D	R_2	R_3	R_4	R_5	R_6
0.5	1.4×10^{-4}	5.0×10^{-5}	1.4×10^{-5}	3.5×10^{-6}	7.4×10^{-7}
1	8.5×10^{-6}	1.6×10^{-6}	2.2×10^{-7}	2.7×10^{-8}	2.9×10^{-9}
2	5.3×10^{-7}	4.8×10^{-8}	3.5×10^{-9}	2.1×10^{-10}	1.1×10^{-11}
3	1.1×10^{-7}	6.4×10^{-9}	3.0×10^{-10}	1.2×10^{-11}	4.4×10^{-13}

Table 1: Numerical values of R_δ for $\delta = 2, 3, 4, 5, 6$, and for a few representative values of M_D expressed in TeV.

Note that the whole dependence on the number of extra dimensions δ and on the D-dimensional Plank scale M_D is contained in R_δ. Values of Δ_1 and Δ_2 for a larger number of extra dimensions $\delta = 3, 4, 5, 6$ can be simply computed by multiplying the values in table 2 by the appropriate factors $R_{3,4,5,6}/R_2$ obtained from table 1.

A first remarkable result in Eq.(10) is that the contribution of the Z-graviton interference in the total cross section at the Z pole exactly vanishes, after integrating the distributions $(1 - 3\cos^2\theta)$ and $(\cos^3\theta)$ in Eq.(11) over the full range $(-1, 1)$ of $\cos\theta$. Indeed, this result holds in a larger class of processes. We found that the contribution to the *total* cross section of the interference of the (non-necessarily resonant) amplitude for the scattering $e^+e^- \to f\bar{f}$ mediated by spin-1 fields in the s-channel (either axially or vectorially coupled) with the amplitude mediated by a spin-2 field in the s-channel always vanishes. [‡] Therefore, in the total cross section for the scattering $e^+e^- \to f\bar{f}$, with $f \neq e, \nu_e$, virtual graviton effects are suppressed, being due, at leading order, to the squared amplitude of the graviton-exchange channel.

On the other hand, the non-trivial effect of the graviton interference on angular distributions gives rise to cross-section variations with respect to the SM values, whenever non-uniform angular conditions are applied. For instance, a forward-backward symmetric angular cut along the beams gives rise to a Δ_1 dependence of the cross section on the Z resonance. An asymmetric cut produces a Δ_2 dependence, too.

Accordingly, we present now how the different asymmetries determined at LEP and SLD [7] are modified by the virtual-graviton interference at the Z pole. In particular, we first show how the SM predictions are modified for the forward-backward asymmetries A^f_{FB}. A non-vanishing effect is also found and shown in the cases of the left-right polarization asymmetries A^f_{LR}, the left-right forward-backward asymmetries $A^{FB}_{LR}(f)$, and the τ polarization asymmetries $P_\tau(\cos\theta)$.

[‡]We checked that the results on fermion scattering mediated by virtual graviton exchanges in [3] satisfy this property.

M_D	Δ_1	Δ_2	Δ_1'	Δ_2'
	charged leptons			
0.5	2.2×10^{-3}	5.4×10^{-4}	1.1×10^{-3}	1.1×10^{-3}
1	1.3×10^{-4}	3.4×10^{-5}	6.8×10^{-5}	6.8×10^{-5}
2	8.4×10^{-6}	2.1×10^{-6}	4.2×10^{-6}	4.2×10^{-6}
3	1.7×10^{-6}	4.2×10^{-7}	8.4×10^{-7}	8.4×10^{-7}
	u − type quarks			
0.5	-1.9×10^{-3}	-5.4×10^{-4}	-9.5×10^{-4}	-1.1×10^{-3}
1	-1.2×10^{-4}	-3.4×10^{-5}	-5.9×10^{-5}	-6.8×10^{-5}
2	-7.4×10^{-6}	-2.1×10^{-6}	-3.7×10^{-6}	-4.2×10^{-6}
3	-1.5×10^{-6}	-4.2×10^{-7}	-7.3×10^{-7}	-8.4×10^{-7}
	d − type quarks			
0.5	1.5×10^{-3}	5.4×10^{-4}	7.4×10^{-4}	1.1×10^{-3}
1	9.1×10^{-5}	3.4×10^{-5}	4.6×10^{-5}	6.8×10^{-5}
2	5.7×10^{-6}	2.1×10^{-6}	2.9×10^{-6}	4.2×10^{-6}
3	1.1×10^{-6}	4.2×10^{-7}	5.7×10^{-7}	8.4×10^{-7}

Table 2: Numerical values of the coefficients $\Delta_{1,2}$, and $\Delta_{1,2}'$, defined in eqs.(12) and (23), respectively, for final states of different flavors and $\delta = 2$. Some representative values of M_D (expressed in TeV) are assumed.

For the forward-backward asymmetry A_{FB}^f, defined as

$$A_{FB}^f = \frac{1}{\sigma_f} \left[\int_0^1 \left(\frac{d\sigma_f}{d\cos\theta} \right) d\cos\theta - \int_{-1}^0 \left(\frac{d\sigma_f}{d\cos\theta} \right) d\cos\theta \right] \quad , \tag{14}$$

we obtain by Eq.(10)

$$A_{FB}^f = \frac{3}{4} A_e A_f \left(1 + \frac{\Delta_2}{2} \right) . \tag{15}$$

Since Δ_1 in Eq.(11) multiplies a $\cos\theta$-even term, the Δ_1 dependence drops in the A_{FB}^f definition. The numerical deviation from the A_{FB}^f standard-model value can be obtained by Eq.(15) from the Δ_2 values shown in table 2. For $\delta \geq 2$ and $M_D > 1$ TeV, one gets $|\Delta_2| < 10^{-4}$ for any flavour f. Hence, the virtual-graviton interference gives rise to a deviation in A_{FB}^f that is much smaller than its present experimental error (about 1% [7]) from the LEP experiments.

One could think about a new kind of asymmetry that maximizes the sensitivity to the graviton-Z interference. In Eq.(11), the SM distribution in $\cos\theta$ has an even part $[(1 + \cos^2\theta)]$ and an odd part $[\cos\theta]$ that behave differently from the graviton-interference even part $[1 - 3\cos^2\theta]$ and odd part $[\cos^3\theta]$, respectively. Then, it is possible to define a new angular asymmetry A_G^f which vanishes at the tree-level in SM,

$$\begin{aligned} A_G^f &= \frac{1}{\sigma_f} \left[\int_{-c^\star}^0 \left(\frac{d\sigma_f}{d\cos\theta} \right) d\cos\theta - \int_{-1}^{-c^\star} \left(\frac{d\sigma_f}{d\cos\theta} \right) d\cos\theta \right] \\ &+ \frac{1}{\sigma_f} \left[\int_0^{c^\star} \left(\frac{d\sigma_f}{d\cos\theta} \right) d\cos\theta - \int_{c^\star}^1 \left(\frac{d\sigma_f}{d\cos\theta} \right) d\cos\theta \right] \quad , \end{aligned} \tag{16}$$

where $c^\star = -(1 + \sqrt{2})^{-1/3} + (1 + \sqrt{2})^{1/3}$ is the (unique) real solution of the equation

$$\int_0^{c^\star} (1 + x^2) dx - \int_{c^\star}^1 (1 + x^2) dx = 0, \tag{17}$$

corresponding to $\theta^\star = \arccos c^\star \simeq 53^0$. From Eq.(10), one then gets for A_G^f a maximal sensitivity to graviton effects

$$A_G^f \simeq -0.5764 \, \Delta_1 . \tag{18}$$

Note that, although the A_G^f angular asymmetry shows maximal sensitivity to the graviton interference at tree level, the inclusion of angular-dependent electroweak radiative corrections can (at least partly) spoil the goodness of the definition in Eq.(16). We do not go further into this point, and leave it as a suggestion for new analysis of experimental data.

We are now going to consider the cross-section at the Z pole, for a longitudinally polarized electron beam. Eq.(10) can be easily generalized to the case of an electron beam with polarization $\psi_p = P_e \psi$ (with the projector $P_e = (1 + p_e \gamma_5)/2$) as follows

$$
\begin{aligned}
\frac{d\sigma_f(p_e)}{d\cos\theta} &= \frac{9\pi}{2M_Z^2}\frac{\Gamma_{e^+e^-}\Gamma_{f\bar{f}}}{\Gamma_Z^2}\hat{S}_f(\cos\theta, p_e) \\
\hat{S}_f(x, p_e) &= (1 - p_e A_e)\left(1 + x^2 - \hat{\Delta}_1(p_e)\left(1 - 3x^2\right)\right) \\
&+ 2A_f\left(A_e - p_e\right)\left(x + \hat{\Delta}_2(p_e)x^3\right)
\end{aligned}
\tag{19}
$$

where the coefficients $\hat{\Delta}_1(p_e)$, and $\hat{\Delta}_2(p_e)$, are given by

$$
\begin{aligned}
\hat{\Delta}_1(p_e) &= R_\delta \frac{A_e A_f}{(1 - p_e A_e)}\left(\frac{1}{g_V^e g_V^f} - \frac{p_e}{g_A^e g_V^f}\right), \\
\hat{\Delta}_2(p_e) &= R_\delta \frac{A_e}{(A_e - p_e)}\left(\frac{1}{g_A^e g_A^f} - \frac{p_e}{g_V^e g_A^f}\right).
\end{aligned}
\tag{20}
$$

Note that $\hat{\Delta}_{1,2}(p_e = 0) = \Delta_{1,2}$, and $\hat{S}_f(x, 0) = S_f(x)$. For instance, for $M_D = 1$ TeV, $\delta = 2$, and charged-lepton final states, one has, for $p_e = 1$, $\hat{\Delta}_1 = 1.5 \times 10^{-4}$ and $\hat{\Delta}_2 = 7.4 \times 10^{-5}$, while, for $p_e = -1$, $\hat{\Delta}_1 = 1.2 \times 10^{-4}$ and $\hat{\Delta}_2 = 6.3 \times 10^{-5}$.

From Eq.(19), using the polarized *total* cross section $\sigma_f(p_e)$ (that, as in the σ_f case, is graviton-interference independent), one can determine the differential left-right asymmetry, defined as

$$
\frac{dA_{LR}^f(p_e)}{d\cos\theta} = \frac{1}{\sigma_f(p_e) + \sigma_f(-p_e)}\left(\frac{d\sigma_f(-p_e)}{d\cos\theta} - \frac{d\sigma_f(p_e)}{d\cos\theta}\right).
\tag{21}
$$

The result is

$$
\frac{dA_{LR}^f(p_e)}{d\cos\theta} = \frac{3}{8}p_e\left\{A_e\left(1 + \cos^2\theta - \Delta_1'\left(1 - 3\cos^2\theta\right)\right) + 2A_f\left(\cos\theta + \Delta_2'\cos^3\theta\right)\right\},
\tag{22}
$$

where Δ_1' and Δ_2' are given by

$$
\Delta_1' = R_\delta\left(\frac{A_f}{g_A^e g_V^f}\right) = \left(\frac{g_V^e}{g_A^e A_e}\right)\Delta_1, \qquad \Delta_2' = R_\delta\left(\frac{A_e}{g_V^e g_A^f}\right) = \left(\frac{g_A^e A_e}{g_V^e}\right)\Delta_2.
\tag{23}
$$

Numerical values of Δ_1' and Δ_2' for $\delta = 2$, and different final states, are reported in table 2. One gets, for $\delta \geq 2$ and $M_D > 1$ TeV, $|\Delta_{1,2}'| < 10^{-4}$ for any flavour f. In the total left-right asymmetry, the Z-graviton contribution vanishes, and one recovers the SM tree-level value $A_{LR}^f(p_e) = p_e A_e$. As discussed previously, the pattern of electroweak radiative corrections could alter these leading-order results in a non-trivial way.

The left-right forward-backward asymmetry, defined as

$$A_{LR}^{FB}(f) = \int_0^1 \left(\frac{dA_{LR}^f(p_e)}{d\cos\theta} \right) d\cos\theta - \int_{-1}^0 \left(\frac{dA_{LR}^f(p_e)}{d\cos\theta} \right) d\cos\theta \,, \tag{24}$$

has been also measured at SLC. For this observable, we obtain the following modification of the SM value

$$A_{LR}^{FB}(f) = \frac{3}{4} p_e A_f \left(1 + \frac{\Delta_2'}{2} \right). \tag{25}$$

Also in this case, the graviton-interference effects are much below the experimental errors [7]. Here, too, one could try to introduce "most sensitive" variables of the kind of A_G^f suggested in Eq.(16), hence getting a new asymmetry proportional to Δ_1'.

We complete now the list of the asymmetries measured at LEP/SLC, by considering the τ polarization asymmetry, that is defined as

$$P_\tau(\cos\theta) = \frac{d\sigma_\tau^R(\cos\theta) - d\sigma_\tau^L(\cos\theta)}{d\sigma_\tau^R(\cos\theta) + d\sigma_\tau^L(\cos\theta)} \tag{26}$$

where $d\sigma_\tau^R(\cos\theta)$ and $d\sigma_\tau^L(\cos\theta)$ stand for the differential cross sections $\frac{d\sigma_\tau}{d\cos\theta}(p_\tau)$ for the production of a right-handed ($p_\tau = 1$) and a left-handed ($p_\tau = -1$) τ-lepton, respectively. The effect of the graviton interference at the Z pole gives in this case

$$P_\tau(\cos\theta) = -\frac{A_\tau \left(1 + \cos^2\theta - \Delta_1' \left(1 - 3\cos^2\theta\right)\right) + 2A_e \left(\cos\theta + \Delta_2' \cos^3\theta\right)}{1 + \cos^2\theta - \Delta_1 \left(1 - 3\cos^2\theta\right) + 2A_e A_\tau \left(\cos\theta + \Delta_2 \cos^3\theta\right)} \tag{27}$$

Here, too, the virtual graviton effect is far from being detectable with the present experimental precision [7].

Finally, we want to discuss the possibility how the Z-graviton interference affects the model-independent determination of M_Z from the Z line shape [8]. We showed that the *total* cross section on the Z resonance is not altered at tree level by Z-graviton interference effects. On the other hand, any cross section measurement is affected by angular cuts that will give rise to finite effects.

The numerical value of the Z mass is closely related to the peak position of the Z line shape. In the resonant region around the Z pole, the differential cross section is given by

$$\begin{aligned}
\frac{d\sigma_f}{d\cos\theta} = & \left(\frac{9\pi}{2M_Z^2} \frac{\Gamma_{e^+e^-}\Gamma_{f\bar{f}}}{\Gamma_Z^2} \right) F(s) \left\{ 1 + \cos^2\theta + 2A_e A_f \cos\theta \right. \\
& \left. - G(s) \left[\Delta_1 \left(1 - 3\cos^2\theta^2 \right) - 2A_e A_f \Delta_2 \cos^3\theta \right] \right\},
\end{aligned} \tag{28}$$

10

with Δ_1 and Δ_2 defined in Eq.(12). The function $F(s)$ is the usual Breit-Wigner resonance shape that is used at LEP to fit the Z mass value [9], while $G(s)$ arises from the Z-graviton interference, and can modify the Breit-Wigner shape

$$F(s) = \frac{s\Gamma_Z^2}{\left(s - M_Z^2\right)^2 + M_Z^2\Gamma_Z^2}, \quad G(s) = \left(\frac{s}{M_Z^2}\right)^{\frac{\delta}{2}}. \tag{29}$$

We are neglecting the contribution of the real part of the graviton mediated amplitude, that is suppressed with respect to its imaginary part by terms of order $(s - M_Z^2)/M_Z^2$. After integrating over the polar angle the differential cross section in Eq.(28) over the interval $(-c', c')$ [with $0 < c' < 1$], one gets

$$
\begin{aligned}
\sigma_f^{c'}(s) &= \left(\frac{9\pi}{2M_Z^2}\frac{\Gamma_{e^+e^-}\Gamma_{f\bar{f}}}{\Gamma_Z^2}\right) F(s)\left\{I_1 - I_2\Delta_1 G(s)\right\} \\
I_1 &= \int_{-c'}^{c'} dx \left(1 + x^2\right), \quad I_2 = \int_{-c'}^{c'} dx \left(1 - 3x^2\right).
\end{aligned}
\tag{30}
$$

The effect of the Z-graviton interference arises when considering angular cuts $c' < 1$, while, for $c' = 1$, one has $I_2 = 0$, and no effect on the Breit-Wigner shape is observed.

Then, in order to estimate the shift of the value of fitted Z mass due to graviton-interference, one has to determine how much the position of the maximum of $\sigma_f^{c'}(s)$ is modified by the function $G(s)$. To do that, one has to solve the equation $\frac{d}{ds}\sigma_f^{c'}(s) = 0$. By Eq.(30), this is equivalent to solve the equation

$$F'(s)\left(1 - \frac{\Delta_1 I_2}{I_1}G(s)\right) - F(s)G'(s)\frac{\Delta_1 I_2}{I_1} = 0, \tag{31}$$

where the primed indices indicate the first derivative with respect to s. To find the solution of Eq.(31), it is convenient to use perturbation theory in the parameter Δ_1. At the first order in Δ_1, we make the ansatz that the maximum of the graviton-modified distribution is at the Z mass value

$$\bar{M}_Z^2 = \hat{M}_Z^2 + \delta_{M_Z^2}, \tag{32}$$

where $\hat{M}_Z^2$ is the peak position of the Breit-Wigner distribution $F(s)$ [that is $F'(\hat{M}_Z^2) = 0$ and $\hat{M}_Z^2 = M_Z^2\sqrt{1 + \Gamma_Z^2/M_Z^2}$], and $\delta_{M_Z^2}$ is of order of $\mathcal{O}(\Delta_1)$. By substituting $s = \bar{M}_Z^2$ in Eq.(31), making an expansion in $\delta_{M_Z^2}$, and retaining only the first-order terms, one has,

$$\delta_{M_Z^2} = -\frac{\Gamma_Z^2\delta\Delta_1}{4}\left(\frac{I_2}{I_1}\right)\left(1 + \mathcal{O}(\frac{\Gamma_Z^2}{M_Z^2})\right). \tag{33}$$

The shift in the Z peak turns out to be quite small, since, it is not proportional to the squared Z mass, but to the squared Z width. Keeping only the leading term in

the Γ_Z^2/M_Z^2 expansion, we find that M_Z is shifted by the quantity

$$\Delta M_Z = -\frac{\Gamma_Z^2 \Delta_1 \delta}{8 M_Z}\left(\frac{I_2}{I_1}\right). \tag{34}$$

Of course the shift is critically dependent on the angular cuts entering into the integrals $I_{1,2}$. For $c' = 1/\sqrt{3}$, $\delta = 2$, $M_D = 1$ TeV, and charged-lepton final states, the Z-graviton interference shifts the Z resonance peak by about –1.4 keV. For up-type and down-type quarks, the shift is about 1.2 keV and –1.0 keV, respectively. The different sign for different final states would partly cancel the effect, that is anyhow much smaller than the present M_Z experimental error (2.1 MeV [9]).

3   Conclusions

In scenarios with quantum gravity propagating in large extra dimensions, we computed the effects of the interference of the virtual KK graviton-exchange amplitude with the resonant amplitude on the Z pole, on $e^+e^- \to f\bar{f}$ physical observables. Remarkably, results are finite and predictable in terms of the fundamental D-dimensional Plank scale M_D. The corresponding effect on total cross sections at tree-level vanishes, and this result remains valid for $e^+e^- \to f\bar{f}$ (with $f \neq e, \nu_e$) far from the Z resonance. On the other hand, non-trivial modifications of the angular distributions give rise to finite effects whenever angular cuts are applied, and Z-graviton interferences contribute to the forward-backward asymmetries. We also computed initial and final polarization asymmetries. In general, the *relative* magnitude of the Z-graviton interference effects on all these observables is of the order 10^{-4} for number of extra dimensions equal to 2 and $M_D \sim 1$ TeV, that is 2 orders of magnitude smaller than the present experimental errors. Even a future linear collider operating in the *GigaZ* mode [10] (that is expected to improve the errors on many Z-pole precision measurements by about an order of magnitude) seems not to be sensitive enough to detect such effects.

Finally, we discussed the shift in M_Z from the model-independent determination due to the modification of the Breit-Wigner resonant shape. Shifts in M_Z are found at most of the order 1 keV. In conclusion, although appealing on theoretical grounds, virtual graviton exchanges do not presently affect Z-resonance observables at a detectable level.

Acknowledgments

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