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Violation of angular momentum selection rules in quantum gravity

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Abstract

A simple consequence of the angular momentum conservation in quantum field theories is that the interference of s-channel amplitudes exchanging particles with different spin J vanishes after complete angular integration. We show that, while this rule holds in scattering processes mediated by a *massive* graviton in quantum gravity, a *massless* graviton s-channel exchange breaks orthogonality when considering its interference with a scalar-particle s-channel exchange, whenever all the external states are massive. As a consequence, we find that, in the Einstein theory, unitarity implies that angular momentum is not conserved at quantum level in the graviton coupling to massive matter fields. This result can be interpreted as a new anomaly, revealing unknown aspects of the well-known van Dam–Veltman–Zakharov discontinuity.

1. Introduction

It is well known that, when considering a massive spin-2 gravitational field in quantum gravity, the limit of vanishing graviton mass is distinct from the prediction of the massless-graviton Einstein theory. In [1,2], van Dam, Veltman, and Zakharov (vDVZ) stressed this problem considering the leading tree-level approximation to the graviton exchange between matter sources, for a massive graviton coupled to matter as $h^{\mu\nu}T_{\mu\nu}$ (with $T_{\mu\nu}$ the conserved energy–momentum tensor and $h^{\mu\nu}$ the graviton field). The vDVZ discontinuity is shown to arise from the fact that a massive spin-2 tensor field has five polarization degrees of freedom, while a massless spin-2 graviton has simply two. In the massless limit, the massive graviton decomposes into three massless fields with spin-2, spin-1 and spin-0, respectively. The spin-1 vector field has a derivative coupling to the conserved energy–momentum tensor, and its contribution to the one graviton exchange amplitude vanishes. On the other hand, the spin-0 scalar field is coupled to the trace of the energy–momentum tensor and contributes in general to the scattering amplitude. This scalar component does not decouple even in the massless graviton limit. This gives rise to a discontinuity in the predictions of the massive and massless theory in the lowest tree-level approximation. As a consequence, in the
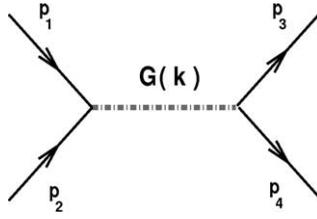
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Fig. 1. Scattering $p_1 p_2 \to p_3 p_4$ in the s-channel with a graviton exchange.

massive theory (even in the limit of small masses) the light bending by the Sun and the precession of the Mercury perihelion differ by numerical factors from the predictions of the Einstein theory.

Many papers have elaborated on the possibility to fix this apparent inconsistency of the massive theory, in different directions [3–8]. For instance, in [3] it is claimed that, if the light bending by the sun is computed by solving the exact space–time metric equation in the presence of a small graviton mass, no discontinuity arises in the limit of small graviton mass. In fact, the discontinuity could be connected to the use of perturbation theory for the metric fluctuations around the flat space–time. More recently, it has been shown that there is not any vDVZ discontinuity in the de Sitter space [5] (or in the anti-de Sitter space [6]), where the massless graviton limit is smooth (see also [7,8] for other solutions).

Here, we present a different class of problems connected to the vDVZ discontinuity. In particular, we stress the fact that there are cases where, while the massive theory is well-behaved, a massless graviton gives rise to inconsistencies. In particular, we show that the massless graviton propagator in the Einstein theory breaks angular momentum selection rules.

Let us consider the tree-level amplitude for the graviton exchange in the s-channel between two on-shell matter fields (Fig. 1). The two on-shell matter fields enter into the amplitude through the conserved (at the zeroth order in $h_{\mu\nu}$) symmetric energy–momentum tensors $T_{\mu\nu}$ and $T'_{\alpha\beta}$, respectively.[1]

For a *massive* spin-2 field of momentum k and mass m_G, one has five independent polarization tensors $\epsilon_{\mu\nu}(k, \sigma)$, where the index σ runs over the polarization states. Summing over all polarizations, one gets [1]

$$\sum_{\sigma=1}^{5} \epsilon_{\mu\nu}(k, \sigma) \epsilon_{\alpha\beta}(k, \sigma) = P^m_{\mu\nu\alpha\beta}(k) \tag{1}$$

with

$$P^m_{\mu\nu\alpha\beta}(k) = \frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta}) - \frac{1}{2m_G^2}(\eta_{\mu\alpha}k_\mu k_\beta + \eta_{\nu\beta}k_\mu k_\alpha + \eta_{\mu\beta}k_\nu k_\beta + \eta_{\nu\alpha}k_\mu k_\beta)$$
$$+ \frac{1}{6}\left(\eta_{\mu\nu} + \frac{2}{m_G^2}k_\mu k_\nu\right)\left(\eta_{\alpha\beta} + \frac{2}{m_G^2}k_\alpha k_\beta\right). \tag{2}$$

The projector $P^m_{\mu\nu\alpha\beta}$ is symmetric and traceless in both (μ, ν) and (α, β) indices, and satisfies the transversality conditions $k^\mu P^m_{\mu\nu\alpha\beta} = k^\alpha P^m_{\mu\nu\alpha\beta} = 0$.

For a massless graviton, one has just two transverse polarization states ($\sigma = 1, 2$), that correspond to the helicity values $\lambda = \pm 2$. The sum over polarizations is then [1]

$$\sum_{\sigma=1}^{2} \epsilon_{\mu\nu}(k, \sigma)\, \epsilon_{\alpha\beta}(k, \sigma) = P_{\mu\nu\alpha\beta}(k) = \frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta}) + \cdots, \tag{3}$$

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[1] In this Letter, indices $(\mu, \nu, \alpha, \beta)$ are contracted according to the Minkowski metric $\eta_{\mu\nu} = \text{Diag}(1, -1, -1, -1)$.

where dots stand for terms containing at least one graviton momentum.

In the unitary gauge, the corresponding massive and massless graviton propagators are proportional to the projectors $P^m_{\mu\nu\alpha\beta}$ and $P_{\mu\nu\alpha\beta}$, respectively [1]. However, terms proportional to the graviton momentum in Eqs. (2) and (3) vanish when contracted with $T_{\mu\nu}$ in the on-shell matrix elements, due to the conservation of the energy–momentum tensor. For this reason, the tree-level diagram with one graviton exchange in Fig. 1 is gauge invariant, and the effective massive and massless graviton propagators become [1]

$$G^m_{\mu\nu\alpha\beta}(k) = i\frac{(1/2)\eta_{\mu\alpha}\eta_{\nu\beta} + (1/2)\eta_{\mu\beta}\eta_{\nu\alpha} - (1/3)\eta_{\mu\nu}\eta_{\alpha\beta}}{k^2 - m_G^2 + i\epsilon}, \tag{4}$$

$$G_{\mu\nu\alpha\beta}(k) = i\frac{(1/2)\eta_{\mu\alpha}\eta_{\nu\beta} + (1/2)\eta_{\mu\beta}\eta_{\nu\alpha} - (1/2)\eta_{\mu\nu}\eta_{\alpha\beta}}{k^2 + i\epsilon}. \tag{5}$$

As shown in [1], unitarity fixes uniquely the coefficients of the Minkowski metric products in Eqs. (4) and (5).

The corresponding on-shell s-channel matrix elements will be then, up to some coupling constant,

$$\mathcal{A}^m \sim T^{\mu\nu} G^m_{\mu\nu\alpha\beta}(k) T'^{\alpha\beta} \tag{6}$$

and

$$\mathcal{A} \sim T^{\mu\nu} G_{\mu\nu\alpha\beta}(k) T'^{\alpha\beta}. \tag{7}$$

In the limit $m_G \to 0$, Eqs. (6) and (7) only differ by the coefficients of the $\eta_{\mu\nu}\eta_{\alpha\beta}$ term in Eqs. (4) and (5). When contracted with the energy–momentum tensors, the latter give terms proportional to the traces T^μ_μ and T'^α_α, that are nonvanishing for massive external fields. From this difference, the vDVZ discontinuity arises [1].

Note that the terms in the amplitudes corresponding to the $\eta_{\mu\nu}\eta_{\alpha\beta}$ terms in the graviton propagators can be interpreted as a scalar field exchange amplitude.[2]

Let us consider now the interference of the s-channel amplitudes exchanging particles of different spin J (Fig. 2).

$$\mathcal{I}(i, j) \sim \mathcal{A}^\star(J = i) \times \mathcal{A}(J = j) + \text{h.c.} \quad (j \neq i). \tag{8}$$

A simple consequence of angular momentum conservation is that, after complete angular integration on the final state, this quantity must vanish, that is

$$\int d\cos\theta \, d\varphi \, \mathcal{I}(i, j) = 0 \quad (j \neq i), \tag{9}$$

where θ is the scattering angle and φ is the azimuthal angle in the center of mass frame. For instance, it is straightforward to verify this in gauge theories, looking at the interference of a vector boson exchange with a scalar (Higgs boson) particle exchange.

One then expects the same is true for the interference of the $J = 2$ and $J = 0$ amplitudes. On the other hand, we have seen above that (in the small-m_G limit) the massive and massless graviton propagator effectively differs by a scalar field exchange, when the external fields are massive. This extra scalar field component, when interfering with a spin-0 exchange amplitude, will give a nonvanishing contribution to $\int d\cos\theta \, d\varphi \, \mathcal{I}(2, 0)$. This implies that the orthogonality condition in Eq. (9) for the interference $\mathcal{I}(2, 0)$ can be verified *either* for the massive graviton exchange *or* for the massless graviton exchange, but *cannot* hold in *both* cases at the same time.

We checked the above statement by an explicit calculation. The result is that the orthogonality condition in Eq. (9) holds for the *massive* graviton exchange, but not in the Einstein theory!

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[2] The different coefficients of the $\eta_{\mu\nu}\eta_{\alpha\beta}$ term in the massive and massless graviton propagators is usually interpreted as an extra spin-0 field, corresponding to one of the five polarization states of a massive graviton contributing to the massive-graviton amplitude in the limit $m_G \to 0$, as discussed above.
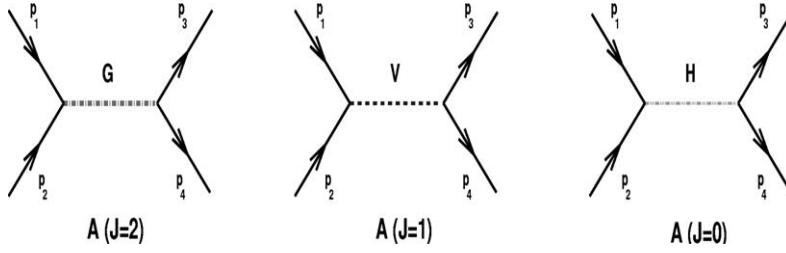
Fig. 2. Scattering $p_1 p_2 \to p_3 p_4$ in the s-channel with different spin-J particle exchange.

For a massless graviton and massive external states, one finds

$$\int d\cos\theta \, d\varphi \, \mathcal{I}(2,0) \neq 0. \tag{10}$$

In the following, we illustrate this result, by giving the explicit expressions of the above discontinuity for the scattering of different external states. We will also extend the discussion to the interferences of the graviton graphs with vector-boson exchange diagrams in the s-channel. As a theoretical framework, we assume the Standard Model minimally coupled to gravity (e.g., as in [9]).

2. The graviton-scalar interference

In the following, we will discuss the interference of the on-shell tree-level scattering amplitudes in the s-channel mediated by a graviton ($J = 2$) with either a scalar particle exchange ($J = 0$) or a vector particle exchange ($J = 1$), as in Fig. 2. We consider initial and final states containing either massive fermions or massive vector bosons. For each s scattering channel,

$$a + \bar{a} \to b + \bar{b}, \tag{11}$$

it is convenient to introduce the dimensionless quantities $\mathcal{I}_{a,b}^m(2, j)$ and $\mathcal{I}_{a,b}(2, j)$ connected to the interferences of the massive and massless graviton amplitudes, $\mathcal{A}_{a,b}^m(J = 2)$ and $\mathcal{A}_{a,b}(J = 2)$, respectively, and the amplitude mediated by a particle of spin j, $\mathcal{A}_{a,b}(J = j)$, with $j = 0, 1$.

The crucial point is that the two amplitudes $\mathcal{A}_{a,b}^m(J = 2)$ and $\mathcal{A}_{a,b}(J = 2)$ depend on the two different (massive or massless) graviton propagators in Eqs. (4) and (5), respectively.

By setting $r_j = m_j^2/s$ (with $m_j = m_0$ (m_1) for the exchange of a scalar (vector) particle of mass m_0 (m_1)) and $r_G = m_G^2/s$, with $\sqrt{s}$ the c.m. scattering energy, we define

$$\mathcal{I}_{a,b}^m(2, j) \equiv \frac{M_P^2}{s}(1 - r_j)(1 - r_G) \sum_{\text{pol}} \mathcal{A}_{a,b}^\star(J = j) \times \mathcal{A}_{a,b}^m(J = 2) + \text{h.c.}, \tag{12}$$

$$\mathcal{I}_{a,b}(2, j) \equiv \frac{M_P^2}{s}(1 - r_j) \sum_{\text{pol}} \mathcal{A}_{a,b}^\star(J = j) \times \mathcal{A}_{a,b}(J = 2) + \text{h.c.}, \tag{13}$$

where M_P is the reduced Planck mass (see Appendix A), and a sum over all the external particles polarization states is performed.

Note that, by definition, the quantities $\mathcal{I}_{a,b}^m(2, j)$ and $\mathcal{I}_{a,b}(2, j)$ depend neither on the masses of particles exchanged in the propagators nor on the Planck mass.

Since we are interested into the discontinuity in the massive and massless graviton interferences, it is useful to define also the quantity $\Delta_{a,b}(2, j)$,

$$\Delta_{a,b}(2, j) \equiv \mathcal{I}_{a,b}(2, j) - \mathcal{I}_{a,b}^m(2, j), \tag{14}$$

that gives the *excess* in the Einstein interference $\mathcal{I}_{a,b}(2, j)$ with respect to the massive graviton interference $\mathcal{I}_{a,b}^m(2, j)$ (when $j = 0$, $\Delta_{a,b}(2, j)$ will be directly connected to the vDVZ discontinuity).

Following the discussion in the previous section, we now concentrate on the graviton interference with a scalar particle, and express all our results in terms of the massive graviton interference $\mathcal{I}_{a,b}^m(2, 0)$ and the discontinuity $\Delta_{a,b}(2, 0)$. In the $J = 0$ propagator, we assume as a scalar particle a Higgs boson, coupled as in the standard model (see Appendix A). The following external states are considered:[3]

(a) the scattering of two electrons into a pair of fermions f, with $f \neq e$;
(b) the scattering of two electrons into a pair of gauge vector bosons W;
(c) the scattering of two W's into a pair of gauge vector bosons W', with $W' \neq W$.

In the following, $r_i = m_i^2/s$, $\beta_i = \sqrt{1 - 4r_i}$ ($i = e, f, W, W'$), and λ_e (λ_f) is the e (f) Yukawa coupling. The angle θ is the scattering angle of a final particle with given electric charge with respect to the initial particle of same charge, in the c.m. system.

Following the Feynman rules in Appendix A, one then gets[4]

- $e^+ e^- \to f\bar{f}$

$$\mathcal{I}_{e,f}^m(2, 0) = -\frac{8}{3}\lambda_e\lambda_f\beta_e^2\beta_f^2\sqrt{r_e r_f}\left(1 - 3\cos^2\theta\right) \tag{15}$$

and

$$\Delta_{e,f}(2, 0) = -\frac{4}{3}\lambda_e\lambda_f\beta_e^2\beta_f^2\sqrt{r_e r_f}, \tag{16}$$

- $e^+ e^- \to W^+ W^-$

$$\mathcal{I}_{e,W}^m(2, 0) = -\frac{1}{3}\lambda_e g_W\sqrt{\frac{r_e}{r_W}}\,\beta_e^2\beta_W^2(1 + 6r_W)\left(1 - 3\cos^2\theta\right) \tag{17}$$

and

$$\Delta_{e,W}(2, 0) = \frac{1}{12}\lambda_e g_W\sqrt{\frac{r_e}{r_W}}\,\beta_e^2\left(3 + \beta_W^2(1 - 12r_W)\right), \tag{18}$$

- $W^+ W^- \to W'^+ W'^-$

$$\mathcal{I}_{W,W'}^m(2, 0) = -\frac{1}{24}\frac{g_W g_{W'}}{\sqrt{r_W r_{W'}}}\beta_W^2\beta_{W'}^2(1 + 6r_W)(1 + 6r_{W'})\left(1 - 3\cos^2\theta\right) \tag{19}$$

and

$$\Delta_{W,W'}(2, 0) = -\frac{1}{12}\frac{g_W g_{W'}}{\sqrt{r_W r_{W'}}}\beta_W^2\beta_{W'}^2\left(\beta_W^2 + 12r_W^2\right)\left(\beta_{W'}^2 + 12r_{W'}^2\right). \tag{20}$$

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[3] We consider only processes that do not receive contributions from t (u) channel exchanges.
[4] Results in Eqs. (15) and (17) were first obtained in [10], although in a different context.

Then, in each of the above channels, we have for the graviton-scalar interference in the Einstein theory

$$\mathcal{I}_{a,b}(2,0) = \mathcal{I}_{a,b}^m(2,0) + \Delta_{a,b}(2,0),\tag{21}$$

with a θ *independent* discontinuity $\Delta_{a,b}(2,0)$.

The angular integration $\int d\cos\theta$ of all the *massive* graviton interferences, $\mathcal{I}_{a,b}^m(2,0)$, has a vanishing results (respecting angular momentum selection rules). On the other hand, the angular integration of the *massless* graviton interference always gives rise to a nonnull results (for massive external states), that is

$$\int\limits_{-1}^{1} d\cos\theta\, \mathcal{I}_{a,b}(2,0) = \int\limits_{-1}^{1} d\cos\theta\, \Delta_{a,b}(2,0) = 2\Delta_{a,b}(2,0) \neq 0,\tag{22}$$

that is connected to the vDVZ discontinuity.

Note that the results above do not depend on the gauge choice. For instance, in a covariant gauge, the gauge dependence affects the graviton propagators only through momentum dependent terms, that vanish after contraction with the energy–momentum tensors.

In Eqs. (15)–(18), the interferences are all vanishing in the massless fermion limit ($r_{e,f} \to 0$), due to fermion chirality. The $J = 2$ amplitude conserves the chirality, while the opposite is true for the $J = 0$ scalar channel. Then, in order to get a nonvanishing result for the interference, a chirality flip is needed in the initial/final fermion states, giving rise to the fermion mass factor. In Eqs. (17)–(20), the singularity in the external gauge-boson mass ($1/\sqrt{r_W}$ and $1/\sqrt{r'_W}$ terms) arises from the sum over the gauge bosons polarizations, since longitudinal modes do not decouple in the massless gauge-boson limit.[5]

From the results above, assuming angular momentum conservation at each interaction vertex, one could conclude that the Einstein graviton propagator behaves as if it was propagating a further scalar degree of freedom that is coupled to the masses of external states. However, this would be in contrast with unitarity and the conservation of the energy momentum tensor. Indeed, only the spin-2 transverse polarizations $\epsilon_{\mu\nu}(k,\sigma)$ with helicities $\lambda = \pm 2$ are effectively exchanged in the massless graviton propagator (see [1] for details). Then, in the Einstein theory, unitarity implies that angular momentum is not conserved at quantum level in the graviton coupling to massive matter fields, even if the total angular momentum is conserved in the scattering process.

We checked the results relative to the fermion–fermion scattering by computing the expansion in terms of spherical harmonics (i.e., the angular momentum eigenstates, $\mathbf{Y}_l^m(\theta,\varphi)$, defined in the Appendix A) of the scattering amplitudes, for the four-fermion processes

$$e^+(p_1,\nu_1) + e^-(p_2,\nu_2) \to (J = 0,1,2) \to \bar{f}(p_3,\nu_3) + f(p_4,\nu_4)\tag{23}$$

where a virtual particle of spin $J = 0,1,2$ is exchanged in the s-channel, and p_i and ν_i ($i = 1,2,3,4$) stands for the external particles momenta and helicities, respectively. We will work in the c.m. frame, where the momenta p_i can be cast in the following form

$$p_1 = \frac{\sqrt{s}}{2}(1,0,0,\beta_e), \qquad p_2 = \frac{\sqrt{s}}{2}(1,0,0,-\beta_e),$$

$$p_3 = \frac{\sqrt{s}}{2}(1,\beta_f\sin\theta\cos\varphi,\beta_f\sin\theta\sin\varphi,\beta_f\cos\theta),$$

$$p_4 = \frac{\sqrt{s}}{2}(1,-\beta_f\sin\theta\cos\varphi,-\beta_f\sin\theta\sin\varphi,-\beta_f\cos\theta),\tag{24}$$

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[5] Note that the s-channel diagram mediated by a scalar particle with external gauge bosons does not exist in the gauge symmetric phase, but only after spontaneous symmetry breaking.

with φ being the azimuthal angle.

In order to express the $J = 0, 1, 2$ helicity amplitudes as a linear combination of the spherical harmonics $\mathbf{Y}_l^m(\theta, \varphi)$, it is convenient to use the solution of the Dirac equation for the particle (U) and antiparticle (V) bispinors in the momentum space [11]

$$U(p, \nu) = \begin{pmatrix} \sqrt{\epsilon + m}\, \omega_\nu(\underline{n}) \\ \sqrt{\epsilon - m}\, (\underline{\sigma} \cdot \underline{n})\omega_\nu(\underline{n}) \end{pmatrix}, \qquad V(p, -\nu) = \begin{pmatrix} \sqrt{\epsilon - m}\, (\underline{\sigma} \cdot \underline{n})\omega_\nu(\underline{n}) \\ \sqrt{\epsilon + m}\, \omega_\nu(\underline{n}) \end{pmatrix}, \tag{25}$$

where the 2-component spinors $\omega_\nu(\underline{n})$ (with $\nu = \pm 1$) are the eigenstates of the helicity operator $(\underline{\sigma} \cdot \underline{n})\omega_\nu(\underline{n}) = \nu\omega_\nu(\underline{n})$, and σ_i are the Pauli matrices. Here, $\underline{n} = \underline{p}/|\underline{p}|$, where $\underline{p}$ is the 3-momentum $\underline{p} = |\underline{p}|(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$, and ϵ is the corresponding energy. In polar coordinates, $\omega_\nu(\underline{n})$ can be expressed as

$$\omega_{+1}(\underline{n}) = \begin{pmatrix} e^{-i\frac{\varphi}{2}}\cos\frac{\theta}{2} \\ e^{i\frac{\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}, \qquad \omega_{-1}(\underline{n}) = \begin{pmatrix} -e^{-i\frac{\varphi}{2}}\sin\frac{\theta}{2} \\ e^{i\frac{\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}. \tag{26}$$

After some straightforward algebra, the $\mathcal{A}(J = 0, 1, 2)$ helicity amplitudes for the channels in Eq. (23) can be cast in the following form, as a function of the initial and final helicities ($\nu_i = \pm 1$),[6]

$$\mathcal{A}(J = 0) = R_0\big\{\delta_{\nu_1,\nu_2}\delta_{\nu_3,\nu_4}\nu_1\nu_3\,\mathbf{Y}_0^0\big\}, \tag{27}$$

$$\mathcal{A}(J = 1) = R_1\big\{\delta_{\nu_1,-\nu_2}\delta_{\nu_3,-\nu_4}\big(\mathbf{Y}_1^0 + \sqrt{3}\,\nu_1\nu_3\mathbf{Y}_0^0\big) - \delta_{\nu_1,-\nu_2}\delta_{\nu_3,\nu_4}\big(2\sqrt{2r_f}\,\nu_1\nu_3\mathbf{Y}_1^{\nu_1}\big)$$
$$- \delta_{\nu_1,\nu_2}\delta_{\nu_3,-\nu_4}\big(2\sqrt{2r_e}\,\mathbf{Y}_1^{-\nu_1}\big) + \delta_{\nu_1,\nu_2}\delta_{\nu_3,\nu_4}\big(4\nu_1\nu_3\sqrt{r_e r_f}\,\mathbf{Y}_1^0\big)\big\}, \tag{28}$$

$$\mathcal{A}^\xi(J = 2) = R_2\big\{\delta_{\nu_1,-\nu_2}\delta_{\nu_3,-\nu_4}\big(4\mathbf{Y}_2^0 + \sqrt{5}\big(\mathbf{Y}_0^0 - \sqrt{3}\,\nu_1\nu_3\mathbf{Y}_1^0\big)\big) + \delta_{\nu_1,-\nu_2}\delta_{\nu_3,\nu_4}\big(4\sqrt{6r_f}\,\nu_1\nu_3\mathbf{Y}_2^{\nu_1}\big)$$
$$+ \delta_{\nu_1,\nu_2}\delta_{\nu_3,-\nu_4}\big(4\sqrt{6r_e}\,\mathbf{Y}_2^{-\nu_1}\big) + \delta_{\nu_1,\nu_2}\delta_{\nu_3,\nu_4}\big(8\sqrt{r_e r_f}\,\nu_1\nu_3\big(2\mathbf{Y}_2^0 - \sqrt{5}(1 - 3\xi)\mathbf{Y}_0^0\big)\big)\big\}, \tag{29}$$

where $\delta_{\nu_i,\nu_j} = 1$ if $\nu_i = \nu_j$ and zero otherwise,

$$R_0 = \sqrt{4\pi}\,\frac{\lambda_e \lambda_f}{1 - r_0}\beta_e\beta_f, \qquad R_1 = \sqrt{\frac{\pi}{12}}\,\frac{g_V^e g_V^f}{1 - r_1}, \qquad R_2 = -\frac{1}{12}\sqrt{\frac{\pi}{5}}\left(\frac{s}{M_P^2}\right)\frac{\beta_e\beta_f}{1 - r_G}.$$

In the $\mathcal{A}^\xi(J = 2)$ graviton amplitude, the quantity ξ parametrizes the vDVZ discontinuity, with $\xi = 1/3$ and $\xi = 1/2$ for the massive and massless graviton propagator, respectively. The functions $\mathbf{Y}_l^m(\theta, \varphi)$ (note that the relevant ones are reported in Appendix A) satisfy the following normalization condition

$$\int\limits_{-1}^{1} d\cos\theta \int\limits_{0}^{2\pi} d\varphi \,\big(\mathbf{Y}_l^m(\theta, \varphi)\big)^\star \mathbf{Y}_{l'}^{m'}(\theta, \varphi) = \delta_{l,l'}\delta_{m,m'}. \tag{30}$$

When considering the interference of $\mathcal{A}^\xi(J = 2)$ with the scalar exchange amplitude $\mathcal{A}(J = 0)$, only the last component in $\mathbf{Y}_0^0(\theta, \varphi)$ of the graviton amplitude (that is proportional to $(1 - 3\xi)$) survives after angular integration, for equal initial and equal final helicities. Then, the coefficient of this residual component vanishes only in the case of a massive graviton propagator, for which $\xi = 1/3$. In the Einstein theory ($\xi = 1/2$), the coefficient does not vanish, and it is responsible for the nonorthogonality of the graviton and scalar amplitudes.

By summing the graviton-scalar interference obtained starting from the amplitudes in Eqs. (27) and (29) over the external particles helicities, one easily recovers the results in Eqs. (15) and (16) obtained by summing the interference over the external polarizations.

On the basis of Eqs. (28) and (29), it is now straightforward to verify that there are not problems with angular momentum selection rules, as far as the interference of the graviton amplitudes and the vector-boson ($J = 1$)

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[6] We do not include the axial coupling in the $\mathcal{A}(J = 1)$ amplitude, since the latter does not affect the discontinuity.

exchange amplitudes are concerned. For the sake of completeness, we present in Appendix B the corresponding results for $\mathcal{I}^m_{a,b}(2, 1)$ and $\Delta_{a,b}(2, 1)$, for all the external fermion and vector-boson states considered for the graviton-scalar interferences.

3. Conclusions

Selection rules for angular momentum conservation have been considered in the framework of quantum gravity. As required by angular momentum conservation, the interferences of s-channel amplitudes mediated by particles with different spins $J = 0, 1, 2$ must vanish after complete angular integration on the final state. We find that, in the case of a propagating *massive* graviton, these selection rules are satisfied for any graviton mass. On the contrary, as a consequence of the vDVZ discontinuity (for which the massless limit of massive gravity is different from the Einstein theory), the interferences of $J = 0$ and $J = 2$ amplitudes do not vanish in the *massless* gravity, whenever all the external states are massive. We checked this property in the s-channel $p_1 p_2 \rightarrow p_3 p_4$ scatterings, where initial and final states are either fermions or gauge bosons. We conclude that angular momentum selection rules in the quantum gravity of the Einstein theory are broken.

This result could be interpreted in the following way. Assuming angular momentum conservation at each interaction vertex, a massless graviton propagator behaves as if it was carrying a further scalar degree of freedom coupled to the masses of matter fields with gravitational strength. This extra scalar field would not decouple in physical processes, leading to the breaking of angular momentum selection rules.

The latter interpretation would anyhow be in contrast with unitarity and the energy–momentum tensor conservation, since, in the processes considered, only the spin-2 transverse polarizations (with helicities $\lambda = \pm 2$) are exchanged in the massless graviton propagator.

Then, we conclude that, in the Einstein theory, angular momentum is not conserved at quantum level in the graviton coupling to massive matter fields, even if the total angular momentum is conserved in the scattering process. This effect could be interpreted as a new kind of quantum anomaly. In this regard, the *massive* quantum gravity, or even its massless limit, is a better-behaved theory, being *anomaly free*.

The present results could be due to the use of perturbation theory around the flat metric. Then, the breaking of angular momentum selection rules could simply suggest that the standard approach to perturbation theory in quantum gravity is not completely consistent.

On the other hand, assuming that quantum gravity based on the Einstein theory correctly describes the gravitational interactions, the present breaking of angular momentum selection rules seems to be connected to a new quantum effect that should show up in some physical process. In particular, it could *in principle* be measured by some experiment (although unrealistically at the moment), if the Higgs boson will be discovered.

Acknowledgements

Appendix A

Feynman rules

The Feynman rules used in this Letter are the following [9]

$$H - \bar{f} - f = -i\,\lambda_f,$$

$$H - W_\alpha^+ - W_\beta^- = ig_W m_W g_{\alpha\beta},$$

$$V_\mu - \bar{f} - f = \frac{i}{2}\left(g_V^f \gamma_\mu - g_A^f \gamma_\mu \gamma_5\right),$$

$$V_\mu(q) - W_\alpha^+(p^+) - W_\beta^-(p^-) = ig_W\left\{g_{\mu\alpha}(q_\beta - p_\beta^+) + g_{\mu\beta}(p_\alpha^- - q_\alpha) + g_{\alpha\beta}(p_\mu^+ - p_\mu^-)\right\},$$

$$G_{\mu\nu} - \bar{f}(k_2) - f(k_1) = -\frac{i}{4M_P}\left\{W_{\mu\nu}^{(f)}(k_1, k_2) + W_{\nu\mu}^{(f)}(k_1, k_2)\right\},$$

$$G_{\mu\nu} - W_\alpha^+(k_1) - W_\beta^-(k_2) = -\frac{i}{M_P}\left\{W_{\mu\nu\alpha\beta}^{(W)}(k_1, k_2) + W_{\nu\mu\alpha\beta}^{(W)}(k_1, k_2)\right\},$$

where

$$W_{\mu\nu}^{(f)}(k_1, k_2) = \gamma_\mu(k_{1\nu} + k_{2\nu}) - \eta_{\mu\nu}(\slashed{k}_1 + \slashed{k}_2 - 2m_f),$$

$$W_{\mu\nu\alpha\beta}^{(W)}(k_1, k_2) = \frac{1}{2}\eta_{\mu\nu}(k_{2\alpha}k_{1\beta} - \eta_{\alpha\beta}\,k_1 \cdot k_2) + \eta_{\alpha\beta}k_{1\mu}k_{2\nu} - \eta_{\mu\beta}k_{1\nu}k_{2\alpha} + \eta_{\mu\alpha}(\eta_{\nu\beta}\,k_1 \cdot k_2 - k_{2\nu}k_{1\beta})$$

$$+ m_W^2\left(\eta_{\mu\alpha}\eta_{\nu\beta} - \frac{1}{2}\eta_{\mu\nu}\eta_{\alpha\beta}\right).$$

Above, $\slashed{p} = \gamma^\alpha p_\alpha$, M_P is the reduced Planck mass, defined as $M_P^2 = (8\pi G_N)^{-1}$ (where G_N is the Newton constant), and m_f, m_W are the fermion, vector-boson masses, respectively. V_μ, H, and $G_{\mu\nu}$ are a neutral vector gauge boson, Higgs boson and graviton fields, respectively. The momenta in the G–W–W Feynman rule are entering into the vertex, while in G–$\bar{f}(k_2)$–$f(k_1)$, $f(k_1)/\bar{f}(k_2)$ stands for an incoming/outgoing fermion f of momenta k_1/k_2, respectively.

The corresponding vertices for the W' vector boson, are obtained just changing $g_W \to g_{W'}$ and $m_W \to m_{W'}$.

Spherical harmonics

The spherical harmonics $\mathbf{Y}_l^m(\theta, \varphi)$ are eigenstates of the angular momentum operator $\hat{\mathbf{L}}^2$ and its projection on the z axis $\hat{\mathbf{L}}_z$, satisfying $\hat{\mathbf{L}}^2\mathbf{Y}_l^m = l(l+1)\mathbf{Y}_l^m$ and $\hat{\mathbf{L}}_z\mathbf{Y}_l^m = m\mathbf{Y}_l^m$. Below, we report explicitly the spherical harmonics entering into Eqs. (27)–(29)

$$\mathbf{Y}_0^0(\theta, \varphi) = \frac{1}{\sqrt{4\pi}}, \qquad \mathbf{Y}_1^0(\theta, \varphi) = \sqrt{\frac{3}{4\pi}}\cos\theta, \qquad \mathbf{Y}_1^{\pm 1}(\theta, \varphi) = \pm\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{\pm i\varphi},$$

$$\mathbf{Y}_2^0(\theta, \varphi) = \sqrt{\frac{5}{16\pi}}\left(1 - 3\cos^2\theta\right), \qquad \mathbf{Y}_2^{\pm 1}(\theta, \varphi) = \pm\sqrt{\frac{15}{8\pi}}\cos\theta\sin\theta\, e^{\pm i\varphi}.$$

Appendix B

In this appendix, we consider the interferences of the $J = 2$ and $J = 1$ amplitudes, assuming the definitions in Eqs. (12)–(14). Terms arising from the axial-vector coupling of fermions are included, too, although they do not give rise to any discontinuity.

- $e^+e^- \to f\bar{f}$

$$\mathcal{I}_{e,f}^m(2,1) = 2g_V^e g_V^f\left\{\beta_e\beta_f\left(r_f + r_e\left(1 - \frac{4}{3}r_f\right)\right)\cos\theta + \frac{1}{4}\beta_e^3\beta_f^3\cos^3\theta\right\} - \frac{g_A^e g_A^f}{4}\beta_e^2\beta_f^2\left(1 - 3\cos^2\theta\right),$$

$$(B.1)$$

and

$$\Delta_{e,f}(2,1) = -\frac{4}{3} g_V^e g_V^f \beta_e \beta_f r_e r_f \cos\theta. \tag{B.2}$$

In this case, the discontinuity vanishes after total angular integration, and is proportional to $r_e r_f \sim m_e^2 m_f^2$, since it is connected to the traces of the energy–momentum tensors of the initial and final states. In the limit of massless fermions, the interference does not vanish. Indeed, contrary to the $(J=0)$-channel, the $(J=1)$-channel has the same chirality structure as the $(J=2)$-channel, and the $(J=1)$–$(J=2)$ interference survives also in the massless fermion limit.

The orthogonality in Eq. (B.1) was first noticed in [12], although the corresponding results were obtained in a different context and in the massless fermion limit.

- $e^+ e^- \to W^+ W^-$

$$\mathcal{I}_{e,W}^m(2,1) = -\frac{g_W g_V^e}{r_W}\left\{\beta_e \beta_W\left(\frac{1}{4} - \frac{1}{3}r_e + \frac{3}{2}r_W + 14 r_e r_W + 6 r_W^2 - 8 r_e r_W^2\right)\cos\theta \right.$$
$$\left. + \beta_e^3 \beta_W^3 \left(-\frac{1}{4} + \frac{3}{2}r_W\right)\cos^3\theta\right\} \tag{B.3}$$

and

$$\Delta_{e,W}(2,1) = -\frac{g_W g_V^e}{3 r_W}\beta_e \beta_W r_e\left(1 - 12 r_W^2\right)\cos\theta. \tag{B.4}$$

In this case the contribution of the fermion axial coupling exactly vanishes. The r_e/r_W dependence in the discontinuity arises from terms proportional to $1/m_W^4$ in the sum over polarizations of the two final W's, combined with the terms $r_e r_W$ emerging from the vDVZ discontinuity.

- $W^+ W^- \to W'^+ W'^-$

$$\mathcal{I}_{W,W'}^m(2,1) = -\frac{g_W g_{W'}}{r_W r_{W'}}\beta_W \beta_{W'}\left\{\left(-\frac{1}{48} + \frac{7}{4}r_W - r_W^2 + \frac{45}{4}r_W r_{W'} + 42 r_W^2 r_{W'} - 12 r_W^2 r_{W'}^2\right)\cos\theta\right.$$
$$\left. + \beta_W^2 \beta_{W'}^2\left(\frac{1}{16} + \frac{9}{4}r_W r_{W'} - \frac{3}{4}r_W\right)\cos^3\theta\right\} + (r_W \leftrightarrow r_{W'}) \tag{B.5}$$

and

$$\Delta_{W,W'}(2,1) = \frac{g_W g_{W'}}{12 r_W r_{W'}}\beta_W \beta_{W'}\left(1 - 12 r_W^2\right)\left(1 - 12 r_{W'}^2\right)\cos\theta. \tag{B.6}$$

In the above equations, g_V^e and g_A^e are the vectorial and axial coupling of fermions to the neutral gauge boson V, and g_W and $g_{W'}$ are the couplings of the gauge bosons $W^\pm$ and $W'^\pm$ to V, respectively (cf. Appendix A).

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