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Transmission characteristics of the birefringent filter system, where a retarder-rotator combination is used at each stage

Biswajit Chakraborty
Department of Applied Physics, Optics and Optoelectronics Section, University of Calcutta, 92, A.P.C. Road, Calcutta 700 009, India

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Polarizers and retarders have long been known as the basic building blocks of Lyot and Solc filters. Recently the use of an optically active medium along with polarizers and retarders has been found to be useful for modifying the transmission characteristics of the birefringent filter system. In this paper, the effect of using an optically active medium as the interstage element of a Solc filter has been studied. The idea of a birefringent band suppression filter has been found and the tuning procedures of these fixed-type filters have also been discussed.

Solc and Lyot filters are well-known examples of linear birefringent networks. The state of polarization and the spectral distribution of intensity are the two properties of the wave that are simultaneously operated upon by such polarizing optical systems. These types of filters, which depend for their action on the interference of polarized light, can be designed for narrow-passband (about a fraction of an angstrom) spectral intensity transmittance. A useful discussion on Lyot and Solc filters and a review of the elementary theory can be found in the works of Evans.[1,2] The comparison between Lyot and Solc filters can be found in Refs. 2 and 3. In both fan- and folded-type Solc filters, the azimuths of different retarders are obtained by simple mathematical relations. The intensity transmission at the principal pass gets sharper as the number of stages increases, but this will enhance the neighboring maximas. An interesting development in the birefringent filter system was described by Harris, Amman, and others.[3] They were able to design for any arbitrarily prescribed amplitude transmittance, by impulse response technique, subject to the conditions that the amplitude transmittance must be periodic with frequency and the corresponding impulse response must be real and causal. Solc and Lyot filters may be considered as the special cases of this generalized filter system. Recently Chakraborty[4] and Ghosh[5] have shown the possibility of using an optically active medium, i.e., rotators, in a birefringent filter system. The frequency-dependent rotation introduced by each rotator can be usefully applied for modifying the spectral intensity transmittance of the birefringent filter system.

Tunability of the birefringent filter is an important criterion from an application point of view. Billings[6] showed a series of methods by which a Lyot–Öhman polarization filter could be tuned over a range of wavelengths. In the case of generalized birefringent networks,[3] the use of an electro-optic cell in place of a quartz or calcite retarder is of great advantage where variable tunability is required.[3,7]

In this communication, we will first derive an expression for intensity transmission of a fan-type Solc filter, where a rotator is placed after each retarder (Fig. 1). Next we will derive an expression for the intensity transmission of the above-mentioned filter system, where the slow axis of each retarder is parallel to the transmission axes of the polarizers. These fixed-type filters can be tuned at any wavelength in the visible region by changing the thickness of the retarders and rotators. The tuning procedures of these filter systems will also be discussed.

Let the Jones vector of the input beam be represented by

$$\epsilon_i = \begin{vmatrix} \epsilon_x \\ \epsilon_y \end{vmatrix} . \quad (1)$$

With the help of Fig. 1, the Jones vector of the output beam can be written as

$$\epsilon_0 = P_{\text{out}}(0)R(\theta/2)R(n\theta)[R(\alpha-\theta)C(\delta)]^n \times R(-\theta/2)P_{\text{in}}(0)\epsilon_i , \quad (2)$$

where n is the number of retardation plates which are equal to the number of rotators. Here $C(\delta)$ and $R(\alpha)$ represent the Jones matrices of the retarder and rotator respectively, given by

$$C(\delta) = \begin{vmatrix} e^{i\delta/2} & 0 \\ 0 & e^{-i\delta/2} \end{vmatrix} , \quad (3)$$

$$R(\alpha) = \begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix} ,$$

$P_{\text{in}}(0)$ and $P_{\text{out}}(0)$ represent, respectively, the Jones matrices of the input and output linear polarizers; both have transmission axes parallel to the X axis of the reference coordinate system and are given by

$$P_{\text{in}}(0) = P_{\text{out}}(0) = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} . \quad (3a)$$

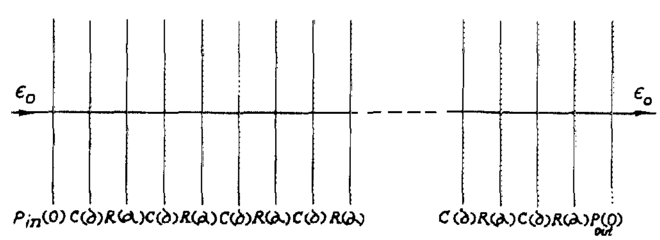

FIG. 1. The diagram shows a fan-type Solc filter, where a rotator $R(\alpha)$ is placed after each retarder $C(\delta)$. $P_{\text{in}}(0)$ and $P_{\text{out}}(0)$ represent linear polarizers, with transmission axes parallel to the X direction of the reference coordinate system.

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In the fan-type Solc filter the azimuth of the jth retarder is given by

$$\theta_j = (\theta/2) + (j-1)\theta, \tag{4}$$

where θ can be obtained by the condition $n\theta \simeq \pi/2$.

Now we will use the well-known property of an unimodular matrix[4,5,8] in order to obtain the expression for intensity transmission of the filter system. The unimodular matrix m_0 is given by

$$m_0 = R(\alpha-\theta)C(\delta) = \begin{vmatrix} e^{i\delta/2}\cos(\alpha-\theta) & -\sin(\alpha-\theta)e^{-i\delta/2} \\ e^{i\delta/2}\sin(\alpha-\theta) & \cos(\alpha-\theta)e^{-i\delta/2} \end{vmatrix} = \begin{vmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{vmatrix}. \tag{5}$$

The nth power of an unimodular matrix is

$$m_0^n = \begin{vmatrix} m_{11}P_{n-1}(x) - P_{n-2}(x) & m_{12}P_{n-1}(x) \\ m_{21}P_{n-1}(x) & m_{22}P_{n-1}(x) - P_{n-2}(x) \end{vmatrix}, \tag{6}$$

where

$$x = \tfrac{1}{2}(m_{11} + m_{22}) \tag{7}$$

and $P_n(x)$ is the Chebyshev polynomial of the second kind, given by

$$P_n(x) = \sin[(n+1)\cos^{-1}x]/\sqrt{1-x^2}. \tag{8}$$

Using the above result for m_0^n in expression (2), we have

$$\epsilon_0 = \begin{vmatrix} p \\ 0 \end{vmatrix} \epsilon_x, \tag{9}$$

where

$$p = \frac{\sin(n\chi)}{\sin\chi}\cos\left(\frac{\theta}{2}\right)\cos\left(\alpha + n\theta - \frac{\theta}{2}\right)e^{i\delta/2} - \frac{\sin[(n-1)\chi]}{\sin\chi}\cos(n\theta) + \frac{\sin(n\chi)}{\sin\chi}\sin\left(\frac{\theta}{2}\right)\sin\left(\alpha + n\theta - \frac{\theta}{2}\right)e^{-i\delta/2}. \tag{10}$$

Now making use of the condition $n\theta \simeq \pi/2$ in the above expression, the intensity transmittance of the filter, apart from a constant photometric factor is given by

$$T = \left(\frac{\sin(n\chi)}{\sin\chi}\right)^2\left[\sin^2(\alpha-\theta)\cos^2\left(\frac{\delta}{2}\right) + \sin^2\alpha\sin^2\left(\frac{\delta}{2}\right)\right], \tag{11}$$

where

$$\chi = \cos^{-1}[\cos(\delta/2)\cos(\alpha-\theta)]. \tag{12}$$

Putting $\alpha = 0$ in expression (11), we get the intensity transmittance of an original Solc filter,[2] given by

$$T = \left(\frac{\sin(n\chi)}{\sin\chi}\cos\chi\tan\theta\right)^2, \tag{13}$$

where

$$\chi = \cos^{-1}[\cos(\delta/2)\cos\theta]. \tag{14}$$

Now we will derive an expression for the intensity transmittance of the filter, where the slow axis of each retarder is parallel to the transmission axes of the polarizers. In that case the number of retarders is equal to one less than the number of rotators.

Substituting $\theta = 0$ in expression (10), the intensity transmittance of the filter is found as

$$T = 1 - \left(\frac{\sin(n\chi)}{\sin\chi}\sin\alpha\right)^2, \tag{15}$$

where

$$\chi = \cos^{-1}[\cos\alpha\cos(\delta/2)]. \tag{16}$$

Here n is the number of rotator in the filter system. The retarder $C(\delta)$, placed just after $P_{in}(0)$ (Fig. 1), can be dropped. Expression (15) gives the transmission characteristics of a birefringent band suppression filter (BBSF).

For the purpose of calculation, the rotation α produced by a quartz plate is given by the well-known Biot's law

$$\alpha = A + B\nu^2,$$

where $A = -1.845$ deg/mm and $B = 9.00 \times 10^{-29}$ deg S^2/mm.[9] Since the variation of birefringence $(n_e - n_0)$ is only 3.8% in the case of quartz and 4.9% in the case of calcites around their mean values, within the entire visible range,[10] the phase difference δ introduced between two orthogonal components of the light vibration of wavelength λ by a retardation plate of thickness d, is given by

$$\delta = (2\pi/\lambda)(n_e - n_0)d = C_1/\lambda,$$

where C_1 is a constant.

In the case of narrow-passband filter, the retarder and rotator thickness is chosen in such a way that at principal pass wavelength λ_{0P} the phase difference δ introduced by each retarder is 360 deg and the linearly polarized component λ_{0P} of the light vibration comes out of the filter system, being unobstructed by the output polarizer $P_{out}(0)$. On the other hand, in the case of BBSF, the linearly polarized component λ_{0R} of the light vibration is completely cut off by the output polarizer $P_{out}(0)$. At λ_{0R}, the phase difference δ introduced by each retarder is also 360 deg and we may call λ_{0R} the principal rejection wavelength.

Plots (a), (b), and (c) of Fig. 2 shows the variation of intensity transmittance T with wavelength λ of a 10-stage filter. Each stage consists of a retarder and a rotator as shown in Fig. 1. For plots (a), (b), and (c), the principal pass wavelengths are 509, 453, and 640 nm and the rotator thicknesses are 0.60, 0.48, and 1 mm, respectively. The azimuth of each retarder is given by expression (4). The rotator thicknesses are obtained with the help of the relation $\alpha_{0P} = m\pi/2n$, where α_{0P} is the amount of rotation in degrees introduced by each rotator for the principal pass wavelength λ_{0P}, n is the number of the stage, and $m = 1,4,6\ldots$, etc. In our calculations we have taken $m = 2$. Plots (a), (b), and (c) show that the effect of secondary maximas are more pronounced on the left-hand side of the principal maxima than on the right-hand side. The amplitudes of the secondary maximas on both sides of the principal maxima increase, as we take higher values of m (4,6,8,etc.). Plot (d) shows the intensity transmittance of a 10-stage Solc filter tuned at $\lambda_{0P} = 509$ nm. A comparison of plot (a) with plot (d) shows that in plot (a) the amplitude of the secondary maxima on the right-hand side of the principal maxima is lower, whereas the secondary maxima on the left-hand side of the principal maxima is much higher than that of the Solc filter [plot (d)]. The widths of the principal maximas are more or less the same in both the cases.

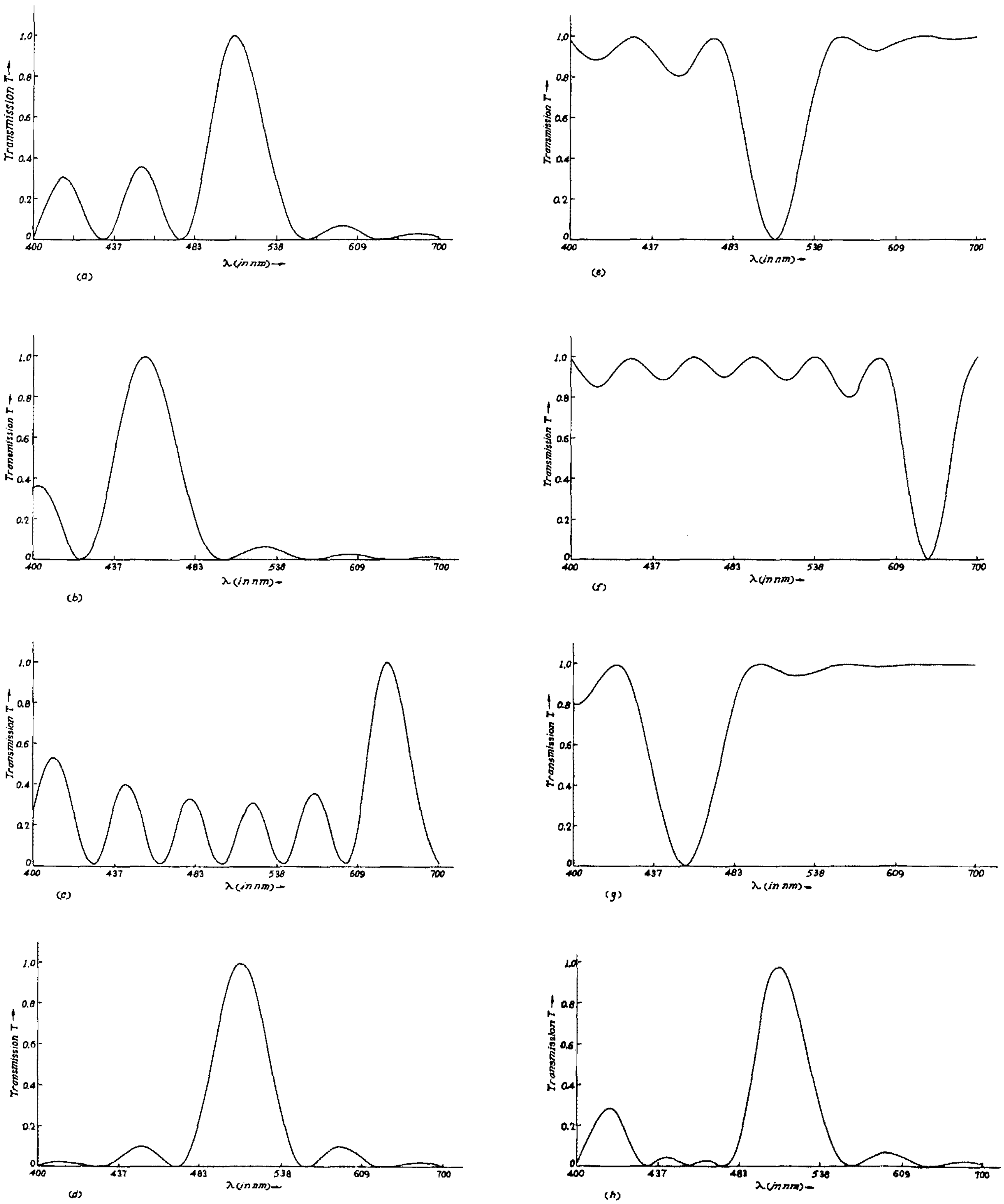

FIG. 2. Plots (a)–(f) show the variation of intensity transmittance T with wavelength λ of different filter configurations.

Plots (e), (f), and (g) of Fig. 2 show the variation of intensity transmittance T with wavelength λ of BBSF systems. Here, the rotator thicknesses are obtained with the help of the relation $\alpha_{0R} = m\pi/2n$ where α_{0R} is the amount of rotation in degrees introduced by each rotator for the principal rejection wavelength λ_{0R}, n is the number of the rotator, and $m = 1,3,5...$, etc. The transmission curves are plotted with $n = 10$ and $m = 1$. For plots (e), (f), and (g) the principal rejection wavelengths are 509, 640, and 453 nm, and the rotator thicknesses are 0.31, 0.50, and 0.24 mm, respectively. All three plots of BBSF show that the fluctuations on the left-hand side passband are much higher than the right-hand side passband of the principal rejection wavelength λ_{0R}. With increased values of m, i.e., with $m = 3$, 5, and 7, etc., the fluctuations on the passband of both sides increase.

The BBSF system may be used with other filter systems for modifying the spectral intensity transmittance characteristics. Plot (h) shows the transmittance characteristics of two filter systems in combination, whose transmission characteristics are represented by plots (a) and (g). Comparing plot (h) with plot (a), we note, that in plot (h) the secondary maxima on the left-hand side of the principal maxima is much reduced and the transmission at the principal pass wavelength occurs at 98% of its ideal value.

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[1]J. W. Evans, J. Opt. Soc. Am. **39**, 3 (1949).
[2]J. W. Evans, J. Opt. Soc. Am. **48**, 3 (1958).
[3]E. O. Amman, Prog. Opt. **9**, 123 (1971).
[4]A. K. Chakraborty and B. Mondal Adhikari, J. Opt. (India) **6**, 73 (1977).
[5]A. Ghosh and A. K. Chakraborty, Opt. Acta **29**, 1407 (1982).
[6]B. H. Billings, J. Opt. Soc. Am. **37**, 738 (1947).
[7]J. F. Lotspeich, R. R. Stephens, and D. M. Handerson, Appl. Opt. **22**, 7 (1983).
[8]M. Born and E. Wolf, *Principles of Optics*, 5th ed. (Pergamon, Oxford, 1975).
[9]B. Chakraborty and A. K. Chakraborty, J. Phys. D **18**, 145 (1985).
[10]A. K. Chakraborty, Opt. Commun. **10**, 374 (1974).