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Thermo-viscoelastic Interaction In A Three-dimensional Problem Subjected To Fractional Heat Conduction

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Abstract

In capturing visco-elastic behavior, experimental tests play a fundamental role, since they allow in building up theoretical constitutive laws very useful for simulating their own behavior. In the present contribution, estimation is made to investigate the transient phenomena in a homogeneous isotropic three-dimensional medium whose surface is subjected to a time-dependent thermal loading and is free of traction, in the context of two-temperature three-phase-lag thermoelastic model with non-local fractional operators. The governing equations are formulated for Kelvin-Voigt two-temperature theory and are solved employing the normal mode analysis. Numerical estimates of the thermophysical quantities are depicted graphically for a copper-like material. This mathematical formulation is assessed by experimental tests and the effect of two-temperature theory and the non-local fractional operator is analyzed. Also, the effect of viscosity on the thermophysical quantities is reported in the literature.

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Keywords: Kelvin-Voigt model; generalized thermoelasticity; two-temperature theory; normal mode analysis; finite wave speed.

1. Introduction

Effect of internal friction on the propagation of plane waves in an elastic medium may be attributed to the fact that dissipation accompanies vibrations in solid media due to the conversion of elastic energy to heat energy [1]. Several mathematical models have been used by authors to accommodate the energy dissipation in vibrating solids where it is observed that internal friction produces attenuation and dispersion; hence, the effect of the viscoelastic nature of material medium in the process of wave propagation cannot be neglected. The study of viscoelastic behavior is of interest in several contexts. Materials used in engineering applications may exhibit viscoelastic behavior. Viscoelasticity is of interest in some branches of material science, metallurgy, and solid-state physics [2]. Further, the causal links between viscoelasticity and microstructure are exploited in the use of viscoelastic tests as inspection tools. The problem of moving heat source acting in a viscoelastic body is extremely important in engineering involving materials processing, case hardening, and boiling nucleation [3].

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Differential equations of fractional order have been the focus of many studies due to their frequent appearance in various applications in fluid mechanics, visco-elasticity, biology, physics and engineering. The most important advantage of using fractional differential equations in these and other applications is their non-local property. It is well known that the integer order differential operator is a local operator but the fractional order differential operator is non-local. This means that the next state of a system depends not only upon its current state but also upon all of its historical states. This is more realistic and it is one reason why fractional calculus has become more and more popular [4–6].

In the present analysis, we have considered an isotropic, three-dimensional thermoviscoelastic medium in which the bounding plane is subjected to prescribed time–dependent temperature and boundary is free of traction. The heat conduction equation has been formulated in the context of non–local fractional two–temperature three–phase–lag model of heat conduction. Introducing normal mode analysis, the governing equations have been expressed in Cartesian coordinates and are applied to a thermal shock problem which fills the half–space. The numerical estimates for the thermal stress, strain, displacement, conductive temperature and thermodynamic temperature are computed for a copper–like material and have been depicted graphically and most significant points arising from our analysis are highlighted. The effect of non–local fractional parameter and temperature discrepancy is carried out. Also, the effect of viscosity has been reported.

2. Formulation of the Problem

The stress-strain-temperature relations are

$$\sigma_{ij} = 2\mu^\star e_{ij} + [\lambda^\star \triangle - \beta_1^\star \theta]\delta_{ij}, \quad i, j = 1, 2, 3 \tag{1}$$

where σ_{ij} is the stress tensor, θ is the temperature field, the cubical dilatation, the parameters $\lambda^\star$, $\mu^\star$ and $\beta_1^\star$ are defined as

$$\lambda^\star = \lambda_e\left(1 + \alpha_0\frac{\partial}{\partial t}\right), \quad \mu^\star = \mu_e\left(1 + \alpha_1\frac{\partial}{\partial t}\right), \quad \beta_1^\star = \beta_{1e}\left(1 + \beta_1\frac{\partial}{\partial t}\right),$$

where $\beta_{1e} = (3\lambda_e + 2\mu_e)\alpha_t, \beta_1 = (3\lambda_e\alpha_1 + 2\mu_e\alpha_0)\alpha_t/\beta_{1e}$; λ_e, μ_e being Lame's constants, $\alpha_0, \alpha_1, \beta_{1e}$ being the mechanical relaxation times, α_t being the coefficient of linear thermal expansion, $\triangle = e_{ii}$ and e_{ij} is the strain tensor given by

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right). \tag{2}$$

Stress equation of motion in absence of body force is

$$\sigma_{ij,j} = \rho\ddot{u}_i, \quad i, j = 1, 2, 3 \tag{3}$$

The heat equation for the dynamic coupled generalized thermoviscoelasticity based on the two temperature fractional order three–phase–lag thermoelastic model is given by

$$\left[K^\star\left(1 + \frac{\tau_v^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right) + K\frac{\partial}{\partial t}\left(1 + \frac{\tau_T^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)\right]\nabla^2\phi = \left(1 + \frac{\tau_q^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha} + \frac{\tau_q^{2\alpha}}{2\alpha!}\frac{\partial^{2\alpha}}{\partial t^{2\alpha}}\right)\left(\rho c_v\ddot{\theta} + \beta_1^\star\theta_0\ddot{e}\right). \tag{4}$$

The relation between the conductive temperature (ϕ) and thermodynamic temperature (θ) is given by

$$\phi - \theta = a\nabla^2\phi, \tag{5}$$

where u_i $(i = 1, 2, 3)$ are the displacement component and ρ is the density, c_v is the specific heat at constant strain, θ_0 is the reference temperature and a is the temperature discrepancy.

We now consider an isotropic, homogeneous thermo-viscoelastic medium considered in the three dimensional space which fills the region Ω, which is defined by $\Omega = \{(x, y, z) : 0 \leqslant x < \infty, -\infty < y < \infty, -\infty < z < \infty\}$. We consider the rectangular Cartesian coordinate with the origin on the bounding plane $x = 0$ is traction free and is subjected to a time-dependent thermal loading. We now introduce the rectangular cartesian system (x, y, z) and having the components of

the displacement vector $\vec{u}$ as (u, v, w).
The constitutive relations are

$$\sigma_{xx} = 2\mu_e\left(1+\alpha_1\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial x} + \lambda_e\left(1+\alpha_0\frac{\partial}{\partial t}\right)e - \beta_{1e}\left(1+\beta_1\frac{\partial}{\partial t}\right)\theta, \tag{6}$$

$$\sigma_{yy} = 2\mu_e\left(1+\alpha_1\frac{\partial}{\partial t}\right)\frac{\partial v}{\partial y} + \lambda_e\left(1+\alpha_0\frac{\partial}{\partial t}\right)e - \beta_{1e}\left(1+\beta_1\frac{\partial}{\partial t}\right)\theta, \tag{7}$$

$$\sigma_{zz} = 2\mu_e\left(1+\alpha_1\frac{\partial}{\partial t}\right)\frac{\partial w}{\partial z} + \lambda_e\left(1+\alpha_0\frac{\partial}{\partial t}\right)e - \beta_{1e}\left(1+\beta_1\frac{\partial}{\partial t}\right)\theta, \tag{8}$$

$$\sigma_{xy} = \mu_e\left(1+\alpha_1\frac{\partial}{\partial t}\right)\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right), \tag{9}$$

$$\sigma_{yz} = \mu_e\left(1+\alpha_1\frac{\partial}{\partial t}\right)\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right), \tag{10}$$

$$\sigma_{zx} = \mu_e\left(1+\alpha_1\frac{\partial}{\partial t}\right)\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right), \tag{11}$$

The equations of motion in absence of body forces are

$$(\lambda^\star+2\mu^\star)\frac{\partial^2 u}{\partial x^2} + \mu^\star\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) + (\lambda^\star+\mu^\star)\left(\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial^2 w}{\partial x\partial z}\right) - \beta_1^\star\frac{\partial\theta}{\partial x} = \rho\ddot{u}, \tag{12}$$

$$(\lambda^\star+2\mu^\star)\frac{\partial^2 v}{\partial y^2} + \mu^\star\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial z^2}\right) + (\lambda^\star+\mu^\star)\left(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 w}{\partial y\partial z}\right) - \beta_1^\star\frac{\partial\theta}{\partial y} = \rho\ddot{v}, \tag{13}$$

$$(\lambda^\star+2\mu^\star)\frac{\partial^2 w}{\partial z^2} + \mu^\star\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right) + (\lambda^\star+\mu^\star)\left(\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial^2 v}{\partial y\partial z}\right) - \beta_1^\star\frac{\partial\theta}{\partial z} = \rho\ddot{w}, \tag{14}$$

The heat conduction equation for two-temperature three-phase-lag fractional thermoelastic model is given by

$$\left[K^\star\left(1+\frac{\tau_v^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right) + K\frac{\partial}{\partial t}\left(1+\frac{\tau_T^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)\right]\nabla^2\phi = \left(1+\frac{\tau_q^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}+\frac{\tau_q^{2\alpha}}{2\alpha!}\frac{\partial^{2\alpha}}{\partial t^{2\alpha}}\right)\left(\rho c_v\ddot{\theta}+\beta_1^\star T_0\ddot{e}\right), \tag{15}$$

Eqs. (12)-(14) can be rewritten in the following form

$$\rho\frac{\partial\ddot{u}}{\partial x} = \mu^\star\nabla^2\frac{\partial u}{\partial x} + (\lambda^\star+\mu^\star)\frac{\partial^2 e}{\partial x^2} - \beta_1^\star\frac{\partial^2\theta}{\partial x^2}, \tag{16}$$

$$\rho\frac{\partial\ddot{v}}{\partial y} = \mu^\star\nabla^2\frac{\partial v}{\partial y} + (\lambda^\star+\mu^\star)\frac{\partial^2 e}{\partial y^2} - \beta_1^\star\frac{\partial^2\theta}{\partial y^2}, \tag{17}$$

$$\rho\frac{\partial\ddot{w}}{\partial x} = \mu^\star\nabla^2\frac{\partial w}{\partial z} + (\lambda^\star+\mu^\star)\frac{\partial^2 e}{\partial z^2} - \beta_1^\star\frac{\partial^2\theta}{\partial z^2}, \tag{18}$$

Introducing the following non-dimensional variables

$$x' = \frac{\widetilde{\omega}}{c_1}x,\quad y' = \frac{\widetilde{\omega}}{c_1}y,\quad z' = \frac{\widetilde{\omega}}{c_1}z,\quad u' = \frac{\rho c_1\widetilde{\omega}}{\beta_{1e}T_0}u,\quad v' = \frac{\rho c_1\widetilde{\omega}}{\beta_{1e}T_0}v,\quad w' = \frac{\rho c_1\widetilde{\omega}}{\beta_{1e}T_0}w,\quad t' = \widetilde{\omega}t,$$

$$\tau_q' = \widetilde{\omega}\tau_q,\quad \tau_T' = \widetilde{\omega}\tau_T,\quad \tau_v' = \widetilde{\omega}\tau_v,\quad \theta' = \frac{\theta}{T_0},\quad \phi' = \frac{\phi}{T_0},\quad \sigma_{ij}' = \frac{\sigma_{ij}}{\beta_{1e}T_0},$$

where

$$c_1^2 = \frac{\lambda_e+2\mu_e}{\rho}\quad\text{and}\quad \widetilde{\omega} = \frac{\lambda_e+2\mu_e}{K}c_v,$$

and after removing primes, the above equations can be rewritten in non–dimensional form as follows

$$\frac{\partial \ddot{u}}{\partial x} = \delta^2\left(1+\alpha_1\frac{\partial}{\partial t}\right)\nabla^2\frac{\partial u}{\partial x} + \left[1-\delta^2+\delta_0\frac{\partial}{\partial t}\right]\frac{\partial^2 e}{\partial x^2} - \left(1+\beta_1\frac{\partial}{\partial t}\right)\frac{\partial^2\theta}{\partial x^2}, \tag{19}$$

$$\frac{\partial \ddot{v}}{\partial y} = \delta^2\left(1+\alpha_1\frac{\partial}{\partial t}\right)\nabla^2\frac{\partial v}{\partial y} + \left[1-\delta^2+\delta_0\frac{\partial}{\partial t}\right]\frac{\partial^2 e}{\partial y^2} - \left(1+\beta_1\frac{\partial}{\partial t}\right)\frac{\partial^2\theta}{\partial y^2}, \tag{20}$$

$$\frac{\partial \ddot{w}}{\partial z} = \delta^2\left(1+\alpha_1\frac{\partial}{\partial t}\right)\nabla^2\frac{\partial w}{\partial z} + \left[1-\delta^2+\delta_0\frac{\partial}{\partial t}\right]\frac{\partial^2 e}{\partial z^2} - \left(1+\beta_1\frac{\partial}{\partial t}\right)\frac{\partial^2\theta}{\partial z^2}, \tag{21}$$

$$\left[a_0\left(1+\frac{\tau_v^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)+\frac{\partial}{\partial t}\left(1+\frac{\tau_T^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)\right]\nabla^2\phi = \left(1+\frac{\tau_q^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}+\frac{\tau_q^{2\alpha}}{2\alpha!}\frac{\partial^{2\alpha}}{\partial t^{2\alpha}}\right)\frac{\partial^2}{\partial t^2}\left[\theta + a_1\left(1+\beta_1\frac{\partial}{\partial t}\right)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)\right], \tag{22}$$

$$\phi - \theta = \eta_1\nabla^2\phi, \tag{23}$$

where

$$a_0 = \frac{K^\star}{K\widetilde{\omega}},\quad a_1 = \frac{\beta_{1e}^2 T_0}{\rho c_v(\lambda_e+2\mu_e)}\quad \text{and}\quad \eta_1 = \frac{a\widetilde{\omega}^2}{c_1^2}.$$

In a similar manner, we can transform the constitutive relations in non–dimensional forms. The dimensionless expressions for the constitutive relations are obtained by adding (19)-(21) as follows

$$\ddot{e} = \left(1+a_2\frac{\partial}{\partial t}\right)\nabla^2 e - \left(1+\beta_1\frac{\partial}{\partial t}\right)\nabla^2\theta, \tag{24}$$

We shall consider the invariant stress σ to be the mean value of the principal stresses as follows

$$\sigma = \frac{\sigma_{xx}+\sigma_{yy}+\sigma_{zz}}{3}.$$

Substituting the expressions of σ_{xx}, σ_{yy} and σ_{zz} into the above expressions, we obtain

$$\sigma = \left(b_1+b_2\frac{\partial}{\partial t}\right)e - \left(1+\beta_1\frac{\partial}{\partial t}\right)\theta, \tag{25}$$

where

$$a_2 = \alpha_0 + 2\delta^2(\alpha_1-\alpha_0),\quad b_1 = \frac{3-4\delta^2}{3}\quad \text{and}\quad b_2 = \frac{3a_2-4\delta^2\alpha_1}{3}.$$

3. Normal mode analysis

In this method, the solutions of the physical variables can be decomposed in terms of normal modes in the following form

$$[u,v,w,e,\theta,\phi,\sigma_{ij}](x,y,z,t) = [u^\star,v^\star,w^\star,e^\star,\theta^\star,\phi^\star,\sigma_{ij}^\star](x)\exp[\omega t + i(my+nz)], \tag{26}$$

where $u^\star(x)$, $v^\star(x)$, $w^\star(x)$, $e^\star(x)$, $\theta^\star(x)$, $\phi^\star(x)$ and $\sigma_{ij}^\star(x)$ are the amplitudes of the functions $i=\sqrt{-1}$, ω is the angular frequency, m and n are the wave numbers in y and z directions respectively.
Using the normal modes to (22), (24) and (25) and then eliminating $e^\star(x)$ and $\theta^\star(x)$ from the resulting expressions, we obtain the system of ordinary differential equations

$$\left(D^2-m^2-n^2\right)\phi^\star(x) = \zeta_2[(\delta_0+\delta_1)\phi^\star(x)+\delta_1\delta_2\sigma^\star(x)], \tag{27}$$

$$\left(D^2-m^2-n^2\right)\sigma^\star(x) = \zeta_3\sigma^\star(x)+\zeta_4\phi^\star(x), \tag{28}$$

where

$$\delta_0 = a_1(1+b_1\omega)^2,\quad \delta_1 = b_1+b_2\omega,\quad \delta_2 = \frac{a_1(1+\beta_1\omega)}{b_1+b_2\omega},$$

$$\zeta_2 = \frac{\omega^2\left(1 + \frac{\tau_q^\alpha}{\alpha!}\omega^\alpha + \frac{\tau_q^{2\alpha}}{2\alpha!}\omega^{2\alpha}\right)}{\left[\delta_1\left\{a_0\left(1+\frac{\tau_\nu^\alpha}{\alpha!}\omega^\alpha\right) + \omega\left(1+\frac{\tau_T^\alpha}{\alpha!}\omega^\alpha\right)\right\} + \eta_1\omega^2(\delta_0+\delta_1)\left(1+\frac{\tau_q^\alpha}{\alpha!}\omega^\alpha + \frac{\tau_q^{2\alpha}}{2\alpha!}\omega^{2\alpha}\right)\right]}$$

$$\zeta_3 = \frac{\zeta_2^2\eta_1(1+\beta_1\omega)(\delta_1+a_2+1)(\delta_0+\delta_1)\delta_1\delta_2 - (1+\beta_1\omega)(\eta_1\omega^2+\delta_1+a_2+1)\zeta_2\delta_1\delta_2+\omega^2}{(1+a_2)-(1+\beta_1\omega)(\delta_1+a_2+1)\eta_1\zeta_2\delta_1\delta_2},$$

$$\zeta_4 = \frac{(1+\beta_1\omega)(\delta_0+\delta_1)\zeta_2[\eta_1\zeta_2(\delta_0+\delta_1)(\delta_1+a_2+1)-(\eta_1\omega^2+\delta_1+a_2+1)]}{(1+a_2)-(1+\beta_1\omega)(\delta_1+a_2+1)\eta_1\zeta_2(\delta_1\delta_2)},$$

Elimination of $\phi^\star(x)$ from eqs. (27) and (28) yield the following fourth-order differential equation

$$(D^4 - LD^2 + M)\sigma^\star(x) = 0, \tag{29}$$

and similarly it can be shown that

$$(D^4 - LD^2 + M)\phi^\star(x) = 0, \tag{30}$$

where

$$L = 2(m^2+n^2) + \zeta_3 + \zeta_2(\delta_0+\delta_1), \quad M = (m^2+n^2)^2 + (m^2+n^2)\{\zeta_3+\zeta_2(\delta_0+\delta_1)\} + \zeta_2\zeta_3(\delta_0+\delta_1) - \zeta_2\zeta_4\delta_1\delta_2.$$

The solution of eqs. (29) and (30) which satisfies the regularity conditions of the problem, are given by

$$\sigma^\star(x) = \sum_{i=1}^{2} R_i(m,n,\omega)e^{-\lambda_i x}, \tag{31}$$

$$\phi^\star(x) = \sum_{i=1}^{2} R_i'(m,n,\omega)e^{-\lambda_i x}, \tag{32}$$

where $\lambda_i^2, (i = 1,2)$ are roots of the equation

$$\lambda^2 - L\lambda^2 + M = 0. \tag{33}$$

Further, from eqs. (26), (23) and (32), the thermodynamic temperature can be represented as

$$\theta^\star(x) = \sum_{i=1}^{2} R_i''(m,n,\omega)e^{-\lambda_i x}. \tag{34}$$

Using eqs. (28) and (31), we have from (32) and (34)

$$R_i'(m,n,\omega) = p_iR_i(m,n,\omega), \quad R_i''(m,n,\omega) = q_ip_iR_i(m,n,\omega),$$

where

$$p_i = \frac{\lambda_i^2 - m^2 - n^2 - \zeta_3}{\zeta_4}, \quad q_i = 1 - \eta_1(\lambda_i^2 - m^2 - n^2).$$

4. Boundary conditions

The boundary conditions for the problem is stated as

(i) Thermal Boundary conditions

The bounding plane $x = 0$ is subjected to a time-dependent thermal loading as follows

$$\phi(0,y,z,t) = \phi_0 r(0,y,z,t), \tag{35}$$

(ii) Mechanical Boundary conditions
The bounding plane to the surface $x = 0$ is free of traction, so we have

$$\sigma(0, y, z, t) = \sigma_{xx}(0, y, z, t) = \sigma_{yy}(0, y, z, t) = \sigma_{zz}(0, y, z, t) = 0. \tag{36}$$

Employing the boundary conditions, we arrive at

$$R_1 = \frac{\phi_0 r^\star}{p_1 - p_2}, \quad R_2 = -\frac{\phi_0 r^\star}{p_1 - p_2}.$$

Therefore, from eqn. (25), using (26), the strain component for the problem is represented as

$$e^\star(x) = \sum_{i=1}^{2} s_i R_i(m, n, \omega) e^{-\lambda_i x}, \tag{37}$$

where

$$s_i = \frac{1 + (1 + \beta_1 \omega) q_i p_i}{b_1 + b_2 \omega}.$$

Thus, the final solutions for the considered physical variables in non–dimensional form can now be expressed as

$$\sigma(x, y, z, t) = e^{\omega t} \cos(my + nz)[R_1 e^{-\lambda_1 x} + R_2 e^{-\lambda_2 x}], \tag{38}$$

$$\phi(x, y, z, t) = e^{\omega t} \cos(my + nz)[p_1 R_1 e^{-\lambda_1 x} + p_2 R_2 e^{-\lambda_2 x}], \tag{39}$$

$$\theta(x, y, z, t) = e^{\omega t} \cos(my + nz)[p_1 q_1 R_1 e^{-\lambda_1 x} + p_2 q_2 R_2 e^{-\lambda_2 x}], \tag{40}$$

$$e(x, y, z, t) = e^{\omega t} \cos(my + nz)[s_1 R_1 e^{-\lambda_1 x} + s_2 R_2 e^{-\lambda_2 x}], \tag{41}$$

5. Numerical results and discussions

With an aim to illustrate the results obtained in the preceding section, we now present the analytical results numerically. In the numerical computation, we have taken a copper-like material with material constants as follows

$$\lambda_e = 7.76 \times 10^{10} \text{ Kg m}^{-1} \text{ s}^{-2}, \quad \mu_e = 3.86 \times 10^{10} \text{Kg m}^{-1} \text{ s}^{-2}, \quad \rho = 8954 \text{ kg m}^{-3}, \quad T_0 = 293\text{K},$$

$$K = 386\text{W m}^{-1} \text{ K}^{-1}, \quad K^\star = 200, \quad \alpha_t = 1.75 \times 10^{-5}\text{K}^{-1}, \quad c_v = 383.1\text{J Kg}^{-1} \text{ K}^{-1},$$

$$\alpha_0 = 0.05\text{s}, \quad \alpha_1 = 0.1\text{s}, \quad \beta_1 = 0.1\text{s}.$$

The relaxation time parameters are taken to be

$$\tau_v = 0.1\text{s}, \quad \tau_T = 0.15\text{s}, \quad \tau_q = 0.2\text{s},$$

which agree with the stability condition Since ω is the complex time constant, we have $\omega = \omega_0 + i\zeta$, then $e^{\omega t} = e^{\omega_0 t}(\cos \zeta t + i \sin \zeta t)$.

$$\phi_0 = 10, \quad r^\star = 1, \quad m = 1.2, \quad n = 1.3, \quad \eta_1 = 0.1, \quad \omega_0 = 1.0, \qquad \zeta = 0.2.$$

In order to study the effect of the nonlocal fractional parameter, Fig 1 depicts the variation of the conductive temperature ϕ against the distance x when $y = z = 1.0$ and $\eta_1 = 0.0, 0.1$ respectively for time $t = 0.2$ for a viscous medium. From the figure, it is seen that ϕ attains the maximum vale on $x = 0$ satisfying the thermal boundary condition of the problem as laid down in equation (35). Also, with the increase of the non–local fractional parameter α, the magnitude of ϕ decays sharply and finally diminishes to zero. Further, for $\eta_1 = 0.1$, the decay of ϕ is slower inside the body compared to that of $\eta_1 = 0.0$.

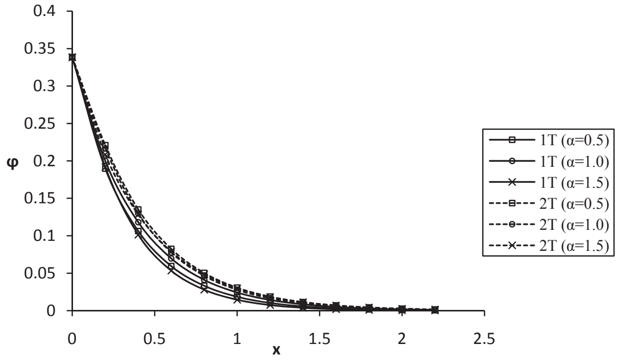

Fig. 1. ϕ versus x for $\eta_1 = 0.0,\ 0.1$ and $\alpha = 0.5,\ 1.0,\ 1.5$ for viscous medium.

Fig 2 is plotted to study the effect of viscosity on the elongation against the distance x and $y = z = 1.0$ for time $t = 0.2$ for fractional parameter $\alpha = 0.5,\ 1.0$ in presence of viscosity (VIS) and absence also (NVIS). From the figure, it is observed that the elongation is maximum in the body on the plane of application of the thermal loading. Further, with the increase of x, e decays sharply and diminishes to zero value. For different α, due to the effect of viscosity, the magnitude of e is lesser compared to the absence of viscous effect. Also, the decay of e is slower in the body due to the presence of viscosity.

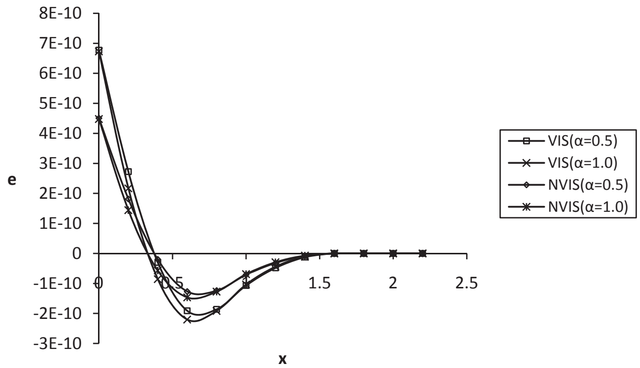

Fig. 2. Variation of e versus x for $\eta_1 = 0.1$ and $\alpha = 0.5,\ 1.0$.

Fig 3 depicts the variation of the profile of stress component σ against the distance x when $y = z = 1.0$, $\alpha = 0.5$ and $t = 0.2$ for a viscous medium. It is observed from the figure, that on the bounding plane $x = 0$, σ vanishes satisfying the mechanical boundary conditions of the problem, which also establishes the correctness of the problem. It is seen that near the plane of application of thermal loading, σ is compressive in nature and the maximum value attended near $x = 0.5$ and it approaches to $\sigma = 0$ as we move far from the boundary. Also, the figure reveals the fact that with the increase of the time t, the magnitude of the profile of σ increase at first and finally it reaches to a steady state which is quite plausible.

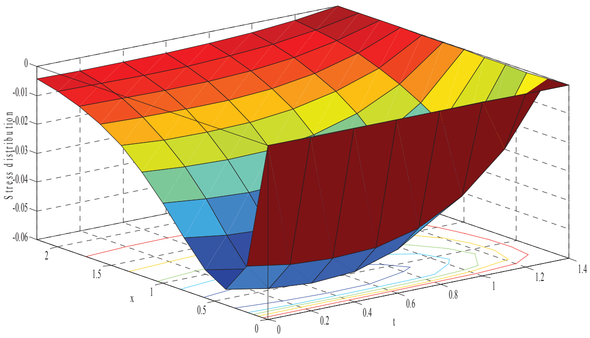

Fig. 3. Profile of σ versus x and t for $\eta_1 = 0.1$ and $\alpha = 0.5$ for viscous medium.

6. Conclusions

In this present analysis, a mathematical treatment has been presented to explore the wave propagation in a three–dimensional isotropic thermo–viscoelastic medium subjected to two temperature three–phase–lag heat conduction in the context of non–local fractional heat conduction. The problem has been solved theoretically and exemplified through specific models. All the figures plotted are self–explanatory in exhibiting the different peculiarities which occur in the propagation of waves, yet the following remarks may be added.

1. The magnitude of the thermophysical quantities increase with the increase of the non-local fractional parameter α.
2. The two temperature theory has a high significance on the thermophysical quantities. For one temperature theory, magnitude of the thermophysical quantities decays sharply. Therefore, 2TT has a high significance in maintaining the continuity of the profile of the thermophysical quantities. So, it is more advantageous to incorporate the 2TT in the problems of high heat flux arising for very short time intervals.
3. With the increase of the time t, magnitude of the thermophysical quantities increase at first and finally attain a steady state.
4. Here all the results for $\alpha = 1$ and $\eta_1 = 0$ agree with the existing literature [7].

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References

[1] R. Karmakar, A. Sur, M. Kanoria, Generalized thermoelastic problem of an infinite body with a spherical cavity under dual–phase–lags, J. Appl. Mech. Tech. Phys 57(4) (2016) 652–665.
[2] S.H. Mallik, M. Kanoria, Generalized thermoviscoelastic interaction due to periodically varying heat source with three–phase–lag effect, Euro. J. Mech. A/Solids 29 (2010) 695–703.
[3] A. Sur, M. Kanoria, Thermoelastic interaction in a viscoelastic functionally graded half–space under three–phase–lag model, Euro. J. Comput. Mech. 23(5–6) (2014) 179–198.
[4] A. Sur, M. Kanoria, Fractional order two–temperature thermoelasticity with finite wave speed, Acta Mech. 223 (2012) 2685–2701.
[5] I. Podlubny, Fractional differential equations, Academic Press, New York, 1999.
[6] M. Islam, M. Kanoria,One–dimensional problem of a fractional order two temperature generalized thermo–piezoelasticity, Math. Mech. Solids 19(6) (2013) 672693.
[7] K.K. Kalkal, S. Deswal, Effects of phase–lags on three–dimensional wave propagation with temperature dependent properties, Int. J. thermophys. 35 (2014) 952-969.