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We study theoretically the interband optical absorption coefficient of semiconductors described within the two-band Kane model in the presence of an external electric field for photon energy below and above the semiconductor band gap. The optical matrix element depends on the momentum vector, and this important aspect has been incorporated in the present analysis. For photon energies below the band gap, the absorption coefficient exhibits an exponential falloff with the electric field and the photon energy. For the opposite inequality, the same coefficient oscillates with the photon energy without consideration of the Wannier–Stark levels, which generally exists in a band due to the presence of an external electric field. Additional oscillations are expected in the above two cases because of the formation of the discrete Stark ladders. © *2001 American Institute of Physics.* [DOI: 10.1063/1.1330253]

I. INTRODUCTION

The effect of an electric field on the optical absorption of semiconductors, near the absorption edge, was investigated in early work by Franz[1] and Keldysh.[2] These authors have shown that in the presence of an electric field, the absorption occurs for photon energies lower than the semiconductor band gap (E_g). This phenomenon is known as the Franz–Keldysh effect and has been experimentally confirmed.[3,4] The theoretical analyses of Franz and Keldysh could not predict oscillations in the optical absorption in the presence of an external electric field. It may be noted here that the effect of an electric field on the band structure of a solid is needed for the interpretation of the experimental electro-optic observations. This has gained momentum after Wannier[5] predicted that a ladderlike splitting of the band levels of Bloch electrons occurs due to the presence of a uniform external electric field. This ladderlike structure of the band is known as ''Wannier–Stark ladders (WSL).'' The existence of WSL has been found in the experimental work of Koss and Lambert.[6] In addition to the lowering of the optical band gap of the semiconductor (i.e., the Franz–Keldysh effect) in the presence of an electric field, one expects oscillations in the absorption coefficient due to WSL. Callaway[7] and others[8–10] theoretically predicted the oscillations in the optical absorption coefficient in the presence of an external electric field, considering WSL. The oscillations occur both below and above E_g. Their calculations are based on the theory of tunneling as given by Kane[11] and Argyres,[12] considering WSL present in the semiconductor band structure when an external electric field is applied.

Considering WSL, Callaway formulated an expression for the interband transition rate between two bands, in the presence of a photon field. The transition probability rate, thus obtained,[7,8] contains a steady state term independent of WSL and oscillatory terms due to the same. The steady state term never shows oscillation for photon energy both below and above E_g. Besides, the oscillations that are obtained are due to consideration of WSL. Furthermore, Callaway considered the optical matrix element (OME) constant with respect to the momentum vector ($\bar{k}$) in evaluating the interband transition matrix element; an entity which cannot be taken as constant in the interband transition.[13] Incidentally, as shown later, if the OME is assumed to be constant with respect to the momentum vector, the optical absorption coefficient vanishes.

The main purpose of the present study is to re-analyze Callaway's formulation for the interband transition matrix element, considering the fact that the OME depends on the momentum vector, and to show that for photon energy greater than E_g oscillations are initiated in the optical absorption coefficient without any effect of WSL, whereas for photon energy below E_g, no oscillations occur. This observation could not be predicted by Callaway[7] and others.[8] The oscillations they observed are the additional ones due to WSL.

For the purpose of condensed completeness, we have given in Sec. II brief outlines of Callaway's derivations for the interband transition matrix element, which consists of the steady state term and oscillatory terms due to WSL. Thereafter, we start with the steady state term to evaluate the transition probability rates, for which no oscillations occur in accordance with Callaway[7] and others.[8] In Sec. II A we give a brief description of Callaway's derivation for the interband transition matrix element; whereas in Sec. II B, we evaluate the same for the steady state term. In Sec. II C, we calculate the interband transition probability rates for photon energy less than and greater than E_g, respectively. We also formulate the optical absorption coefficient for the above two

cases. In Sec. III, we discuss our results, taking GaAs as an example for the purpose of numerical computations. In the Appendix, we provide the derivation for the optical matrix element, $\bar{\varepsilon}\cdot\bar{p}_{11'}(\bar{k})$, as a function of electric field and band indices, respectively.

II. THEORETICAL ANALYSIS

A. Formulation of the optical absorption coefficient and general expression for the interband transition matrix element from Callaway's derivation

The interband optical absorption coefficient, α_0, can be written as[7]

$$\alpha_0=\frac{2\hbar P(\omega)}{\omega n\epsilon_0 CA_0^2}, \tag{1}$$

where $\hbar=h/2\pi$, h is the Planck constant, $p(\omega)$ is the rate of interband transition probability per unit time per unit volume, ω is the angular frequency of the incident radiation, n is the refractive index of the semiconducting material, ϵ_0 is the permitivity of the free space, C is the velocity of light in free space, and A_0 is the amplitude of incident light wave.

In the presence of a uniform electric field of force, F, applied along the x direction, the effective Schrödinger equation for one electron can be written as

$$\left[E_1(\bar{k})-iF\frac{\partial}{\partial k_x}\right]A_{1,\nu}=W_{1,\nu}A_{1,\nu}, \tag{2}$$

where $E_1(\bar{k})$ is the energy of a Bloch electron in band 1, $\bar{k}$ is the momentum vector, k_x is the x component of $\bar{k}$, $A_{1,\nu}$ is the amplitude of the wave function, and $W_{1,\nu}$ is the eigenenergy value of the Bloch electron. It is assumed that the direction of the electric field coincides with one of the reciprocal lattice vectors of the solid. The solution of Eq. (1) is given by

$$A_{1,\nu}=\frac{1}{G^{1/2}}\exp\left[\frac{i}{F}\int_0^{k_x}\{W_{1,\nu}-E_1(k_\perp,k_x')\}dk_x'\right], \tag{3}$$

where

$$W_{1,\nu}=\frac{2\pi\nu F}{G}+\frac{1}{G}\int_{-G/2}^{G/2}E_1(k_\perp,k_x)dk_x, \tag{4}$$

in which $k_\perp$ is the normal component of the wave vector $\bar{k}$, G is the width of the Brilloin's zone in the x direction, $i=(-1)^{1/2}$, and $\nu(=0,\pm1,\pm2,\ldots)$ is an integer showing the discrete ''Stark'' level. The effect of the electric field on the motion of the electron is neglected in this calculation, as there are off-diagonal matrix elements of the Hamiltonian between two bands. This off-diagonal matrix element is necessary for the description of the tunneling. In the present case of optical absorption, the perturbed term in the total Hamiltonian is given by[7]

$$H'=\frac{e}{m}(\bar{A}\cdot\bar{p}) \tag{5}$$

where e is the electronic charge, m is the free electron mass, $\bar{A}$ is the vector potential due to the photon, and $\bar{p}$ is the linear momentum operator. The matrix element of transition between the two bands (from $1'$ to 1 band) can be written as[7]

$$M_{11'}(k_\perp)=\frac{eA_0}{m}\int_{-G/2}^{G/2}A_{1,\nu}^*(\bar{k})\bar{\varepsilon}\cdot\bar{p}_{11'}(\bar{k})A_{1',\nu'}(\bar{k})dk_x, \tag{6}$$

where $\bar{\varepsilon}\cdot\bar{p}_{11'}(\bar{k})$ is the usual interband optical matrix element (OME) and is given by

$$\bar{\varepsilon}\cdot\bar{p}_{11'}(\bar{k})=\int\Psi_1^*(\bar{k},\bar{r})\bar{\varepsilon}\cdot\bar{p}\Psi_{1'}^*(\bar{k},\bar{r})d^3r, \tag{7}$$

in which $\bar{\varepsilon}$ indicates the direction of polarization of the light vector. Physically, the OME represents the probability amplitude of the transition of an electron from band $1'$ to band 1. When a time-dependent electromagnetic wave is incident on a solid, the electrons in the solid gain energy from the field. As a result, the electron in the valence band (VB) makes a transition to the conduction band (CB). This implies that there is an interaction between photons with VB and CB. This interaction must be dependent on $\bar{k}$. Therefore, the OME, which represents this interaction, cannot be assumed to be independent of $\bar{k}$. Incidentally, the said drastic assumption was made by Callaway.[7] In addition, as the bands split into discrete levels due to the presence of an electric field, OME depends on the band indices, the discrete levels, and the electric field, respectively.

From Eqs. (2) and (5), we get

$$M_{11'}=\frac{eA_0}{mG}\int_{-G/2}^{G/2}\bar{\varepsilon}\cdot\bar{p}_{11'}(\bar{k})\times\exp\left\{\frac{i}{F}\int_0^{k_x}\left[\frac{2\pi F}{G}(\nu'-\nu)+\Delta_{1'1}+E_{11'}\right]dk_x'\right\}dk_x, \tag{8}$$

where

$$\Delta_{1'1}=\frac{1}{G}\int_{-G/2}^{G/2}E_{1'1}(k_\perp,k_x)dk_x$$

and

$$E_{1'1}(\bar{k})=E_{1'}(\bar{k})-E_1(\bar{k}).$$

The rate of the interband transition probability ($\omega_{11'}$) per unit time per unit volume, which makes transitions between the band $1'$ and the band 1, is given by:[7]

$$\omega_{11'}=\frac{2G}{\hbar}\sum_{r=-\infty}^{\infty}\int\frac{dk_\perp}{(2\pi)^2}\int dx|M_{11'}|^2 .\delta\left(\frac{2\pi F}{G}\cdot x+\Delta_{11'}-\hbar\omega\right)\exp(2\pi irx), \tag{9}$$

where $x=\nu-\nu'$.

Substituting Eq. (A8) into Eq. (8) and carrying out the integration appearing in Eq. (9), we can write

$$\omega_{11'}=\frac{G^2}{\pi F\hbar}\left\{\int_{-G/2}^{G/2}\frac{dk_\perp}{(2\pi)^2}|M_{11'}(x_0)|^2 + 2\sum_{r=1}^{\infty}\int_{-G/2}^{G/2}\frac{dk_\perp}{(2\pi)^2}|M_{11'}(x_0)|^2\cos(2\pi r x_0)\right\}, \tag{10}$$

in which

$$x_0=\frac{G}{2\pi F}(\Delta_{11'}-\hbar\omega),$$

$\hbar\omega$ is the incident photon energy, and

$$M_{11'}(x_0)=\frac{eA_0E_g}{mG}|\bar{\varepsilon}\cdot\bar{p}_{11'}(0)| \times\int_{-G/2}^{G/2}dk_x\frac{\exp\left[\frac{i}{F}\int_0^{k_x}\{E_{11'}(k_x',k_\perp)-\hbar\omega\}dk_x'\right]}{E_{11'}(k_x,k_\perp)-\hbar\omega}. \tag{11}$$

Therefore, in the presence of a uniform electric field F, $P(\omega)$ is given [neglecting the oscillatory terms due to WSL appearing in Eq. (10)] by

$$P(\omega)=\frac{G^2}{\pi F\hbar}\int_{-G/2}^{G/2}\frac{dk_\perp}{(2\pi)^2}|M_{11'}(x_0)|^2, \tag{12}$$

where $M_{11'}(x_0)$ is expressed through Eq. (11).

Note that Eq. (11) is more general and quite different from Callaway[7] and others.[8] Also, Eq. (11) is independent of the Stark ladder index, ν. Although $\bar{\varepsilon}\cdot\bar{p}_{11'}(\bar{k})$ is a function of the momentum vector $\bar{k}$, the electric field, and band indices, including the ''Stark'' levels, ν, the overall effect of the photon vector on the interband transition probability rate, including the effect of OME, we find two factors, namely: a steady state term independent of the Stark ladder indices, and the oscillatory terms due to it. According to Callaway, this steady state term provides a nonoscillatory transition probability rate for both $\hbar\omega<E_g$ and $\hbar\omega\geqslant E_g$, respectively. The said predictions of Callaway is not accurate.

As the determination of $M_{11'}(x_0)$ is very much necessary in the present study, we have investigated the same in Sec. II B, for semiconductors whose energy band structures are defined by the two-band Kane model.

B. Evaluation of $M_{11'}(x_0)$

Considering the $\bar{k}\cdot\bar{p}$ interaction between the conduction band (CB=1) and the valence band (VB=1′), when WSL is absent, we can write:[11]

$$E_{cv}(\bar{k})=\left[E_g^2+\frac{E_g\hbar^2k^2}{m_r}\right]^{1/2}, \tag{13}$$

where $m_r(=(m_c^{-1}+m_v^{-1})^{-1})$ is the reduced effective mass, m_c and m_v are the effective masses at the edge of CB and VB, respectively.

It may be noted that in the evaluation of M_{cv} [Eq. (11)], various authors[7–10] have approximated $E_{cv}(\bar{k})$ by the effective mass approximations, when the two bands centered at $\bar{k}=0$ are considered. Therefore they used the expression: $E_{cv}(\bar{k})=E_g+(\hbar^2k^2/2m_r)$ in the subsequent analysis. We have not approximated $E_{cv}(\bar{k})$ in accordance with the above expression and have also taken into account the fact that the optical matrix element depends on $\bar{k}$. With these realistic considerations, the expression for the matrix element, $M_{cv}(x_0)$, obtained by us is quite different from others.[7–10] For this case, we have obtained:

$$M_{cv}(x_0)=\frac{eA_0}{mG}E_g|\bar{\varepsilon}\cdot\bar{p}_{cv}(0)|I, \tag{14}$$

where

$$I=\left[\frac{m_r}{E_g\hbar^2}\right]^{1/2}\exp\left[-\frac{\pi}{2}b|q^2|\right]\int_{-\infty}^{\infty}dk_x\frac{\exp\left[ib\left\{k_x(k_x^2-q^2)^{1/2}-q^2.\ln\left[\frac{k_x+(k_x^2-q^2)^{1/2}}{q}\right]-2Rk_x\right\}\right]}{(k_x^2-q^2)^{1/2}-R}, \tag{15}$$

in which

$$2b=\frac{1}{F}\left[\frac{E_g\hbar^2}{m_r}\right]^{1/2},$$

$$R=\hbar\omega\left[\frac{m_r}{E_g\hbar^2}\right],$$

$$-q^2=k_y^2+k_z^2+\frac{E_gm_r}{\hbar^2},$$

and k_y and k_z are the y and z components of $\bar{k}$, respectively. The limits of integration in Eq. (15) are taken from $-\infty$ to ∞. This is because the integrand is convergent over the limits. It may be noted that k_x in Eq. (15) is, in general, a complex quantity because of the transition the electron takes place through the forbidden band.

To carry out the integration in Eq. (15), we used the method of contour integration. For this, we substituted

$$k_x=q\cosh(t) \tag{16}$$

into Eq. (15). In general, t is also complex. With the limits of $k_x\rightarrow\pm\infty$, we have the limits of t as $(-\infty-i\pi/2)$ and $(\infty-i\pi/2)$, respectively. Therefore, in t-plane, the integral of Eq. (15) becomes

$$I=\left[\frac{m_r}{E_g\hbar^2}\right]^{1/2}\exp\left[-\frac{\pi}{2}b|q^2|\right]\int_{-\infty-i\pi/2}^{\infty-i\pi/2}dt\,\frac{\sinh(t)\exp\left[\frac{ibq^2}{2}\left\{(\sinh(2t)-2t)-\frac{4R}{q}\cosh(t)\right\}\right]}{\sinh(t)-R/q}. \tag{17}$$

The integral in Eq. (17) has poles in the t-plane, represented by t_p where

$$\sinh(t_p)=\frac{R}{q}. \tag{18}$$

Substituting the values of R and q into Eq. (18), we find, for small electric field, F and neglecting $E_\perp$ $[=[(\hbar^2k_\perp^2)/2m_r]$, $k_\perp^2=k_y^2+k_z^2]$ for $E_\perp \ll E_g$,

$$\sinh(t_p)=-ia, \tag{19}$$

where $a=\hbar\omega/E_g$.

As we have two possible cases of Eq. (19), so we have two types of poles, mentioned below:

Case (i): When $a=\dfrac{\hbar\omega}{E_g}<1$,

i.e., the photon energy ($\hbar\omega$) is less than the band gap E_g;

Case (ii): When $a=\dfrac{\hbar\omega}{E_g}\geqslant 1$,

i.e., the photon energy ($\hbar\omega$) is greater than or equal to the band gap E_g.

For these two possible cases, we have found t_p, in order to evaluate integral I in Eq. (17).

For case (i), we can write

$$I=2\pi\left[\frac{m_r}{E_g\hbar^2}\right]^{1/2}\tan(\theta)\exp\left[-\frac{b|q^2|}{2}\beta(\theta)\right], \tag{20}$$

where $\beta(\theta)=(\pi+2\theta+\sin(2\theta))$ and $\sin\theta=(\hbar\omega/E_g)$. It is clear from above that $\beta(\theta)$ is constant. Therefore, from Eqs. (14) and (20), the expression for the interband transition matrix element, $M_{cv}(x_0)$, is given by

$$M_{cv}(x_0)=\frac{2\pi eA_0}{mG}|\bar{\varepsilon}\cdot\bar{p}_{cv}(0)|\left[\frac{E_gm_r}{\hbar^2}\right]^{1/2}\tan(\theta)\times\exp\left[-\frac{b|q^2|}{2}\beta(\theta)\right], \tag{21}$$

when $\hbar\omega<E_g$.

For case (ii), integral I in Eq. (17) is given by:

$$I=4\pi\left[\frac{m_r}{E_g\hbar^2}\right]^{1/2}\cosh(\alpha)\sin\left[\frac{b|q^2|}{2}\gamma(\alpha)\right]\times\exp(-\pi b|q^2|), \tag{22}$$

where $\gamma(\alpha)=(\sinh(2\alpha)-2\alpha)$ and

$$\cosh\alpha=a=\hbar\omega/E_g. \tag{23}$$

It may be noted that the $\gamma(\alpha)$ is constant.

Therefore, from Eqs. (14) and (22), the expression for the interband transition matrix element, $M_{cv}(x_0)$, for $\hbar\omega\geqslant E_g$ can be written as

$$M_{cv}(x_0)=\frac{4\pi eA_0}{mG}|\bar{\varepsilon}\cdot\bar{p}_{cv}(0)|\left[\frac{E_gm_r}{\hbar^2}\right]^{1/2}\times\cosh\alpha\sin\left[\frac{b|q^2|\gamma(\alpha)}{2}\right]\exp(-\pi b|q^2|). \tag{24}$$

It may be noted that $M_{cv}(x_0)$, as derived in Eqs. (21) and (24) for the above two cases, is quite different from others.[7–10] This is due to the consideration of the functional dependence of OME on $\bar{k}$ in the presence of an electric field and taking into account of the realistic band structure. The effect of $\bar{k}$-independent OME on the interband transition matrix element and hence on the optical absorption coefficient is discussed in the Sec. II C.

C. Determination of the optical absorption coefficient

In this section, the transition probability rate, $P(\omega)$ and the optical absorption coefficient (α_0) are obtained for $\hbar\omega<E_g$ and $\hbar\omega\geqslant E_g$, respectively.

Case (i): $\hbar\omega<E_g$

By using Eqs. (12) and (21), we get the rate of transition probability, as

$$P(\omega)=\frac{e^2A_0^2}{m^2}\frac{|\bar{\varepsilon}\cdot\bar{p}_{cv}(0)|^2}{\pi\hbar F}\frac{E_gm_r}{\hbar^2}\tan^2\theta\int\int dk_y\,dk_z\times\exp(-b\beta(\theta)|q^2|). \tag{25}$$

Equation (25) can be written as

$$P(\omega)=\frac{2e^2A_o^2}{m^2}|\bar{\varepsilon}\cdot\bar{p}_{cv}(0)|^2\frac{m_r^{3/2}E_g^{1/2}}{\hbar^4}\frac{\tan^2\theta}{\beta(\theta)}\times\exp\left[-\beta(\theta)\frac{E_g^{3/2}m_r^{1/2}}{2\hbar F}\right]. \tag{26}$$

Therefore, using Eq. (26) in Eq. (1), the optical absorption coefficient (α_0) can be expressed as

$$\alpha_0=K\frac{(\hbar\omega)}{(E_g^2-(\hbar\omega)^2)}\frac{(2\pi m_r^{3/2}E_g^{1/2})}{(\pi+2\theta+\sin(2\theta))}\times\exp\left[-\beta(\theta)\frac{E_g^{3/2}m_r^{1/2}}{2\hbar F}\right], \tag{27}$$

where

$$K=\frac{2e^2|\bar{\varepsilon}\cdot\bar{p}_{cv}(0)|^2}{\pi nC\varepsilon_om^2\hbar^2}$$

is a constant and can be evaluated by using the physical parameters of semiconductors.

Note that the expression for the optical absorption coefficient (α_c), as given by Callaway,[7] on the basis of the mo-

mentum vector independent optical matrix element, for $\hbar\omega < E_g$ [Eq. (23) of Ref. 7] is given by

$$\alpha_c = K\frac{m_r\hbar F}{4\hbar\omega(E_g-\hbar\omega)}\exp\left[-\left(\frac{E_g-\hbar\omega}{E_0}\right)^{3/2}\right]f(\omega), \quad (28)$$

where

$$E_0^{3/2}=\frac{3\hbar F}{4(2m_r)^{1/2}},$$

and $f(\omega)=1$, for the case without consideration of WSL. The comparison between Eqs. (27) and (28) exhibits the contribution of the present paper as compared with that of Callaway.

Case (ii): $\hbar\omega \geqslant E_g$

The rate of transition probability can be written as

$$P(\omega)=\frac{e^2A_0^2}{m^2}|\bar{\varepsilon}\cdot\bar{p}_{cv}(0)|^2\frac{2E_g^{1/2}m_r^{3/2}}{\hbar^4}\cosh^2(\alpha) \times\exp(-2\chi)S_r, \quad (29)$$

where

$$\chi=\frac{\pi}{2}\frac{E_g^{3/2}m_r^{1/2}}{\hbar F}$$

and

$$S_r=\left[1-\frac{\cos\left[\frac{E_g^{3/2}m_r^{1/2}}{2\hbar F}\,\gamma(\alpha)+\tan^{-1}\left(\frac{\gamma(\alpha)}{2\pi}\right)\right]}{\left[1+\left(\frac{\gamma(\alpha)}{2\pi}\right)^2\right]^{1/2}}\right].$$

Therefore, the optical absorption coefficient (α_0) can be expressed as

$$\alpha_o=K\frac{\hbar\omega}{(\hbar\omega)^2-E_g^2}E_g^{1/2}m_r^{3/2}\exp\left(-\frac{\pi E_g^{3/2}m_r^{1/2}}{\hbar F}\right) \times\left\{1-\frac{\cos\left[\frac{E_g^{3/2}m_r^{1/2}}{2\hbar F}\,\gamma(\alpha)+\tan^{-1}\left(\frac{\gamma(\alpha)}{2\pi}\right)\right]}{\left[1+\left(\frac{\gamma(\alpha)}{2\pi}\right)^2\right]^{1/2}}\right\}. \quad (30)$$

It may be noted that the expression for the optical absorption coefficient (α_c), as given by Callaway,[7] on the basis of momentum vector independent of optical matrix element for $\hbar\omega \geqslant E_g$ [i.e., Eqs. (29) and (31) of Ref. 7] is given by

$$\alpha_c=K\frac{(2m_r)^{3/2}}{\hbar\omega}(\hbar\omega-E_g)^{1/2}g(\omega), \quad (31)$$

where

$$g(\omega)=\left[1+\Gamma\left(\frac{4}{3}\right)\left(\frac{E_0}{(\hbar\omega)-E_g}\right)^{1/2} -\frac{1}{3}\left(\frac{E_0}{(\hbar\omega)-E_g}\right)^{3/2}\cos\left\{\left(\frac{\hbar\omega-E_g}{E_0}\right)^{3/2}\right\}\right]. \quad (32)$$

The terms with WSL effect on $g(\omega)$ are omitted.

Equation (30) shows that, without consideration of WSL, it is possible to have oscillations in the optical absorption coefficient for $\hbar\omega \geqslant E_g$. This is due to the dependence of OME on $\bar{k}$ in the presence of an external electric field, a fact which was neglected by Callaway.[7] Besides, Roy and Mahapatra[8] incorporated drastic assumptions in the derivation of the same. Additional oscillations may be found in our formulation because of the consideration of the discrete ''Stark'' levels.

Note here that, according to Callaway, if we assume OME without its dependence on $\bar{k}$, the poles, as in Eq. (17), will not appear. As a result, the value of the contour integration in Eq. (17) will be zero. This implies that $M_{cv}(x_0)$ vanishes. This fact indicates that the transition probability rate and, therefore, the optical absorption coefficient will be zero for $\hbar\omega < E_g$ and $\hbar\omega \geqslant E_{g'}$, respectively. These observations are unlikely with that of Callaway; since he obtained[7] a finite value of optical absorption coefficient without including the dependence of OME on $\bar{k}$. This is because of the fact that Callaway further used the approximation: $E_{cv}(\bar{k})=E_g+[(\hbar^2k^2)/2m_r]$, rather than the correct expression of the dispersion relation as given in Eq. (13).

It may be noted that although the term

$$\frac{1}{3}\left[\frac{E_0}{(\hbar\omega)-E_g}\right]^{3/2}\cos\left[\left(\frac{(\hbar\omega)-E_g}{E_0}\right)^{3/2}\right]$$

has appeared in Eq. (32), as given by Callaway[7] for $\hbar\omega \geqslant E_g$, the amplitude of the cosine term is so small numerically compared with the other two terms, and the oscillation vanishes for the case when ''Stark levels'' not considered. This is discussed in Sec. III.

III. RESULTS AND CONCLUSIONS

Taking $E_g=1.424$ (eV), $m_c=0.067m_0$, $m_v=0.075m_0$, $n=3.3$, and $E(=F/e)=1.0\times10^9$ V/m, for GaAs, we have plotted in Fig. 1 α_0 [in cm^{-1}, solid curve, Eq. (27)] and α_c [in cm^{-1}, dotted curve, Eq. (28)] against $\hbar\omega$ (in eV) for $\hbar\omega < E_g$. It appears from Fig. 1 that α_0 increases exponentially with $\hbar\omega$, at a higher values of it (above 0.4 eV). But α_c initially decreases and then increases because of the $(1/\hbar\omega)$ term in the prefactor of the expression [Eq. (28)], followed by an exponential rise up. There is no oscillation in the optical absorption in these cases. In calculating α_c, the value of $|\bar{\varepsilon}\cdot\bar{p}_{nn'},(\bar{k})|^2$ of Callaway is taken as the value of $|\bar{\varepsilon}\cdot\bar{p}_{cv},(0)|^2$ of our case.

In Fig. 2, we plotted α_0 [in cm^{-1}, solid curve, Eq. (30)] and α_c [in cm^{-1}, dotted curve, Eqs. (31) and (32)] against $\hbar\omega$ (in eV) for $\hbar\omega \geqslant E_g$. It is evident from Fig. 2 that α_0 exhibits an oscillation as $\hbar\omega$ increases to comply with theoretical expectation. The amplitude of the oscillations decreases with increasing $\hbar\omega\cdot\alpha_c$ never shows oscillations, although it contains the oscillatory terms [Eq. (32)]. This is because the amplitude of the cosine term is very small numerically, compared with the other two terms. The oscillation vanishes for the case when the ''Stark level'' is not considered. The numerical values of α_c for both cases are larger than α_0. Experimental data for the purpose of verifi-

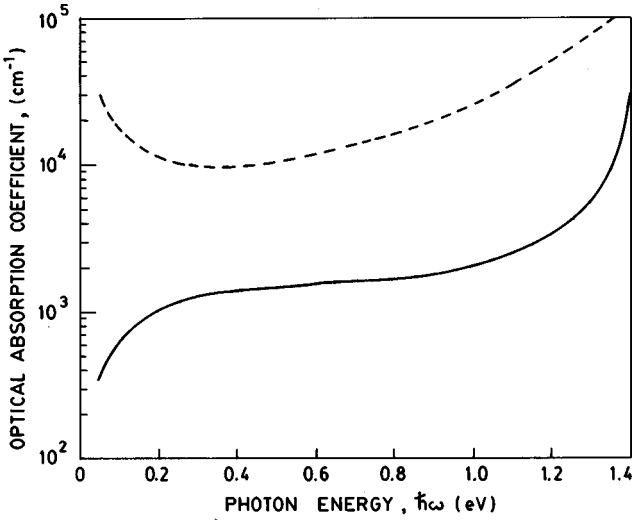

FIG. 1. The absorption coefficients, α_0 [in cm^{-1}, solid curve, Eq. (27)] and α_c [in cm^{-1}, dotted curve, Eq. (28)], plotted against the photon energy, $\hbar\omega$ (in eV) for GaAs at room temperature for $\hbar\omega < E_g$.

cation of the theoretical results (of α_0 vs. $\hbar\omega$) of this paper are still not available in the literature to the best of our knowledge. It may be noted in this context that our theoretical approach will be useful to analyze the experimental results when they appear.

From Eqs. (27) and (30), the following conclusions may be drawn for the optical absorption coefficient (α_0):

(i) As the electric field (F) approaches zero, the absorption coefficient (α_0) tends to zero. Physically, this implies that for $\hbar\omega < E_g$, and in the absence of an electric field, no optical absorption is possible. Therefore, the optical threshold value for absorption has been reduced by the presence of an external electric field, i.e., the phenomenon known as the Franz–Keldysh effect.[7]

(ii) In the absence of the incident photon field (i.e., $\hbar\omega \rightarrow 0$), the absorption coefficient approaches to zero value and is independent of the electric field. This is unlikely with the Callaway's observation for $\hbar\omega < E_g$.

(iii) As $\hbar\omega \rightarrow E_g$, α_0 approaches a very large value.

The following observations may be obtained from Eq. (30) for $\hbar\omega \geqslant E_g$:

(iv) The optical absorption coefficient (α_0) increases with the electric field. This is similar to Callaway's observations.

(v) The oscillatory nature of the optical absorption coefficient is not due to the presence of ''Stark'' levels. The absorption coefficient oscillates with $\hbar\omega$ (for $\hbar\omega \geqslant E_g$) because the optical matrix element depends on the momentum vector in the presence of an electric field. This observation is unlikely with that of Callaway.[7] The oscillations predicted by Callaway and others in the optical absorption coefficient are the additional ones due to the presence of WSL.

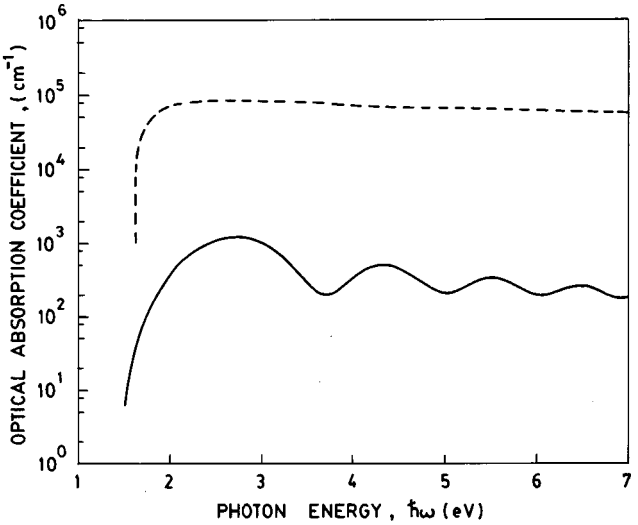

FIG. 2. The absorption coefficients, α_0 [in cm^{-1}, solid curve, Eq. (30)] and α_c [in cm^{-1}, dotted curve, Eqs. (31) and (32)], plotted against the photon energy, $\hbar\omega$ (in eV) for GaAs at room temperature for $\hbar\omega \geqslant E_g$.

APPENDIX: EVALUATION OF $\bar{\varepsilon}\cdot\bar{p}_{11}(\bar{k})$, FOR THE TWO-BAND MODEL

Consideration of the $\bar{k}\cdot\bar{p}$ interaction between the conduction band, $C(=1)$, and the valence band, $V(=1')$, leads to the term $p_{cv}(\bar{k})$ in Eq. (8). Physically, it represents the probability amplitude of transition of an electron from the valence band (VB) to the conduction band (CB), which in turn, can be represented,[14] following Eq. (7), as

$$p_{cv}(\bar{k}) = \int U_c^*.(\bar{k},\bar{r})(-i\hbar\bar{\nabla})U_v(\bar{k},\bar{r})d^3r. \tag{A1}$$

When $U_c(\bar{k},\bar{r})$ and $U_v(\bar{k},\bar{r})$ are the eigen wave functions of CB and VB, respectively. In the conventional treatment of optical absorption,[7] $p_{cv}(\bar{k})$ is approximated as a constant for direct transition. The effect of this approximation on the optical absorption has been discussed in Sec. II C.

To determine $p_{cv}(\bar{k})$, we follow the $\bar{k}\cdot\bar{p}$ perturbation method. According to this technique, the eigenfunctions, $U_c(\bar{k},\bar{r})$ and $U_v(\bar{k},\bar{r})$ can, respectively, be written as

$$U_c(\bar{k},\bar{r}) = \frac{1}{(2\eta)^{1/2}}\{(\eta+E_g)^{1/2}U_c(0,\bar{r}) + (\eta-E_g)^{1/2}U_v(0,\bar{r})\}, \tag{A2}$$

$$U_v(\bar{k},\bar{r}) = \frac{1}{(2\eta)^{1/2}}\{(\eta-E_g)^{1/2}U_c(0,\bar{r}) - (\eta+E_g)^{1/2}U_v(0,\bar{r})\}, \tag{A3}$$

where η is the difference between conduction band and valence band energy, without the effect of WSL, and is given by

$$\eta = E_c(\bar{k}) - E_v(\bar{k}).$$

From Eqs. (A1)–(A3), we find:

$$p_{cv}(\bar{k}) = -\frac{E_g}{\eta}\cdot p_{cv}(0), \tag{A4}$$

where

$$p_{cv}(0)=\frac{m}{2}\left(\frac{E_g}{m_r}\right)^{1/2}. \tag{A5}$$

Explicitly, η is given by Eq. (13) as:

$$\eta=\left(E_g^2+\frac{E_g\hbar^2k^2}{m_r}\right)^{1/2}. \tag{A6}$$

Considering the presence of WSL in a band, due to the presence of an external electric field, the energy difference (η) in Eq. (A4) can be written as:[7,12]

$$\eta=[\{(W_{c,\nu}-E_c(\bar{k}))-(W_{v,\nu'}-E_v(\bar{k}))]. \tag{A7}$$

Using Eq. (A7) in Eq. (A4), we obtain the interband optical matrix element (OME) as:

$$\bar{\varepsilon}\cdot\bar{p}_{11'}(\bar{k})=\frac{\bar{\varepsilon}\cdot\bar{p}_{11'}(0)E_g}{[(W_{1',\nu'}-E_{1'}(\bar{k}))-(W_{1,\nu}-E_1(\bar{k}))]}, \tag{A8}$$

where $W_{1,\nu}$ is given by Eq. (4).

Therefore, $\bar{\varepsilon}\cdot\bar{p}_{11'},(\bar{k})$ is a function of the electric field and band indices through the terms $W_{1,\nu}$ and $W_{1',\nu'}$.

It is worth remarking that the dependence of OME on the momentum vector, as in Eq. (A8), is quite different from earlier results.[8] Equation (A8) has been utilized in Eq. (8) in Sec. II A for the purpose of evaluation of the transition matrix element, $M_{11'}(k_\perp)$.

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