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The scattering of an obliquely incident surface wave by a submerged fixed vertical plate

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The problem of scattering of surface waves obliquely incident on a submerged fixed vertical plate is solved approximately for a small angle of incidence by reducing it to the solution of an integral equation. The correction to the reflection and transmission coefficients over their normal incidence values for a small angle of incidence are obtained. For different values of the incident angle these coefficients are evaluated numerically, taking particular values of the wave number and the depth of the plate, and represented graphically.

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1. INTRODUCTION

Dean[1] and Ursell[2] first considered the problems of scattering of surface waves normally incident on submerged and partially immersed fixed vertical plane barriers in deep water. These problems are subsequently studied by Williams[3], Goswami[4], and others by employing different mathematical techniques, e.g., reduction method, integral equation method, etc. The problem of scattering of surface waves normally incident on a submerged fixed vertical plate in water of finite depth was considered by Goswami,[5] the deep water case being earlier considered by Evans.[6]

The scattering of surface waves obliquely incident on a partially immersed or completely submerged vertical barrier in deep water was studied by Faulkner,[7,8] Jarvis and Taylor,[9] Evans and Morris.[10] In the present paper the problem of scattering of surface waves obliquely incident on a submerged fixed vertical plate in deep water is solved by reducing it to the solution of an integral equation involving the unknown difference of velocity potentials across the plate by a simple use of Green's integral theorem in the fluid medium. The kernel of this integral equation is expanded in a series involving different orders of the sine of the angle of incidence which is assumed to be small. This expansion of the kernel suggests the corresponding form of the expansion of the unknown function in the integral equation and this is then used to solve the integral equation approximately. A somewhat similar type of technique of solving the integral equation approximately was successfully used by Goswami[5,11,12] and Mandal and Goswami.[13]

2. STATEMENT OF THE PROBLEM

We consider the scattering of surface waves by a submerged fixed vertical plate in deep water and use a coordinate system in which the y axis is taken to be vertically downwards, the mean-free surface is the plane $y = 0$, and the position of the plate is given by $x = 0, a \leqslant y \leqslant b, -\infty < z < \infty$. Assuming the fluid to be inviscid and incompressible and the motion to be irrotational and simple harmonic in time with circular frequency σ and small amplitude, a velocity potential exists and it may be taken to be the real part of $\chi(x, y, z)\, e^{-i\sigma t}$ satisfying the equations

$$\nabla^2 \chi = 0, \text{ in the fluid region,}$$

$$\frac{\partial \chi}{\partial y} + K\chi = 0, \text{ on } y = 0,$$

$$\frac{\partial \chi}{\partial x} = 0, \text{ on } x = 0, \ a < y < b,$$

where $K = \sigma^2/g$.

A wave represented by $\chi_0 = \exp(-Ky + i\mu x + i\nu z)$, where $\mu = K\cos\alpha$, $\nu = K \sin\alpha$ is assumed to be incident at an angle α to the normal of the plate from negative infinity. Such a wave will be partially reflected and transmited by the plate, and in view of the geometry of the plate it is reasonable to assume $\chi(x, y, z) = \Phi(x, y) e^{i\nu z}$. Then Φ must satisfy

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} - \nu^2 \Phi = 0, \quad y \geqslant 0, \tag{2.1}$$

$$\frac{\partial \Phi}{\partial y} + K\Phi = 0, \quad y = 0, \tag{2.2}$$

$$\frac{\partial \Phi}{\partial x} = 0, \quad x = 0, \quad a < y < b, \tag{2.3}$$

and it and its derivatives are continuous everywhere except possibly across $x = 0, a < y < b$. We also require that Φ and its first derivatives are bounded everywhere away from the lines $x = 0, y = a$ and $x = 0, y = b$ and that near these lines $\{x^2 + (y - a)^2\}^{1/2}$ grad Φ and $\{x^2 + (y - b)^2\}^{1/2}$ grad Φ, respectively, are bounded. This is called the edge condition. Finally we assume that as $|x| \to \infty$, $\Phi(x, y)$ has the asymptotic forms

$$\Phi(x, y) \sim \exp(i\mu x - Ky) + R\exp(-i\mu x - Ky) \quad (x \to -\infty),$$
$$\Phi(x, y) \sim T\exp(i\mu x - Ky) \quad (x \to +\infty), \tag{2.4}$$

where R and T are the (complex) reflection and transmission coefficients, respectively. Let us write $\Phi(x,y) = \varphi_0(x,y) + \varphi(x,y)$, where $\varphi_0(x,y) = \exp(-Ky + i\mu x)$ and $\varphi(x,y)e^{i\nu z}$ is the scattered velocity potential. $\varphi(x,y)$ also satisfies (2.1), (2.2), and the edge conditions stated above, and by the conditions (2.4) $\varphi(x,y)$ represents an outgoing wave at infinity.

3. REDUCTION TO AN INTEGRAL EQUATION

Following Levine,[14] the generalized Green's function satisfying (2.1), (2.2), and G, grad G being bounded at a large distance, and G representing an outgoing wave at infinity, may be obtained as

$$G(x,y;\xi,\eta) = K_0(\nu\rho) - K_0(\nu\rho^*) + i\frac{2\pi K}{(K^2-\nu^2)^{1/2}}\exp\{-K(y+\eta) + i(K^2-\nu^2)^{1/2}|x-\xi|\} + 2\int_\nu^\infty \frac{(k^2-\nu^2)^{1/2}\cos[(k^2-\nu^2)^{1/2}(y+\eta)] - K\sin[(k^2-\nu^2)^{1/2}(y+\eta)]}{K^2+k^2-\nu^2}\exp\{-k|x-\xi|\}dk, \quad (3.1)$$

where $\rho^2, \rho^{*2} = (x-\xi)^2 + (y \mp \eta)^2$.

Now applying Green's theorem to $\varphi(x,y)$ and $G(x,y;\xi,\eta)$ in the fluid region we obtain

$$2\pi\varphi(\xi,\eta) = \int_a^b f(y)\frac{\partial G}{\partial x}(0,y;\xi,\eta)dy, \quad (3.2)$$

where

$$f(y) \equiv \varphi(+0,y) - \varphi(-0,y).$$

By (2.3) and (3.2), we have

$$-i2\pi K\cos\alpha e^{-K\eta} = \int_a^b f(y)\frac{\partial^2 G}{\partial\xi\partial x}(0,y;0,\eta)dy, \quad a<\eta<b. \quad (3.3)$$

Writing $\epsilon = \sin\alpha$ and following Mandal and Goswami[13] we can obtain

$$\frac{\partial^2 G}{\partial\xi\partial x}(0,y;0,\eta) = \frac{\partial^2 G_0}{\partial\eta^2}(0,y;0,\eta) - \frac{1}{2}K^2\epsilon^2 G_0(0,y;0,\eta) + O(\epsilon^4\ln\epsilon,\epsilon^4), \quad a<y,\ \eta<b, \quad (3.4)$$

where

$$G_0(0,y;0,\eta) = -\ln\left|\frac{y-\eta}{y+\eta}\right| - 2\int_0^\infty \frac{K\sin k(y+\eta) - k\cos k(y+\eta)}{K^2+k^2}dk + i2\pi\exp\{-K(y+\eta)\},$$

and $G_0(x,y;\xi,\eta)$ is the expression (3.1) when $\alpha = 0$. From the expansion (3.4) it is reasonable to expand $f(y)$ in the following form:

$$f(y) = f_0(y) + \epsilon^2 f_1(y) + O(\epsilon^4\ln\epsilon,\epsilon^4). \quad (3.5)$$

By (3.3), (3.4), and (3.5), and by noting the coefficients of terms involving a different order of ϵ, we obtain

$$-i2\pi Ke^{-K\eta} = \frac{d^2}{d\eta^2}\int_a^b f_0(y)G_0(0,y;0,\eta)dy, \quad a<\eta<b, \quad (3.6)$$

and

$$i\pi Ke^{-K\eta} = \frac{d^2}{d\eta^2}\int_a^b f_1(y)G_0(0,y;0,\eta)dy - \frac{K^2}{2}\int_a^b f_0(y)G_0(0,y;0,\eta)dy, \quad a<\eta<b. \quad (3.7)$$

Let us now define a function $\psi(y)$ by

$$\psi(y) = Kf(y) + f'(y)$$

so that

$$f(y) = e^{-Ky}\int_a^y e^{Ku}\psi(u)du. \quad (3.8)$$

Then $\psi(y)$ may be expanded as

$$\psi(y) = \psi_0(y) + \epsilon^2\psi_1(y) + O(\epsilon^4\ln\epsilon,\epsilon^4). \quad (3.9)$$

Now integrating both sides of (3.6) with respect to η, we have

$$A_0 + i2\pi e^{-K\eta} = \frac{d}{d\eta}\int_a^b f_0(y)G_0(0,y;0,\eta)dy, \quad a<\eta<b, \quad (3.6a)$$

and by (3.6) and (3.6a), we obtain

$$\int_a^b \psi_0(y)\frac{2y\,dy}{y^2-\eta^2} = KA_0. \quad (3.10)$$

The singular integral equation (3.10) is a Cauchy-type one and following Mikhlin,[15]

$$\psi_0(y) = \frac{D_0(d_0^2 - y^2)}{(y^2-a^2)^{1/2}(b^2-y^2)^{1/2}}, \quad a<y<b, \quad (3.11)$$

where $D_0 (= -KA_0/\pi), d_0^2$ are constants to be determined. Since $f(b) = 0$, by (3.8)

$$\int_a^b \psi(u)e^{Ku}\,du = 0,$$

and therefore,

$$\int_a^b \psi_0(u)e^{Ku}\,du = 0, \quad \int_a^b \psi_1(u)e^{Ku}\,du = 0, \text{ etc.}, \quad (3.12)$$

so that d_0^2 may be obtained from

$$\int_a^b \frac{(d_0^2 - u^2)e^{Ku}\,du}{(u^2-a^2)^{1/2}(b^2-u^2)^{1/2}} = 0. \quad (3.13)$$

By (3.6) and (3.8), we obtain

$$\int_a^b \psi_0(y)\left[2\int_0^\infty \frac{\sin ky(K\sin k\eta - k\cos k\eta)}{K^2+k^2}k\,dk - i\pi K\exp\{-K(y+\eta)\}\right]dy = i2\pi e^{-K\eta}, \quad a<\eta<b. \quad (3.14)$$

Now by (3.11) and (3.14), we obtain

$$D_0 = \frac{2i}{p_0 - q_0 - ir_0}, \quad (3.15)$$

where

$$p_0 = \int_{-a}^a \frac{(d_0^2 - u^2)e^{-Ku}\,du}{(a^2-u^2)^{1/2}(b^2-u^2)^{1/2}},$$

$$q_0 = \int_b^\infty \frac{(d_0^2 - u^2)e^{-Ku}\,du}{(u^2-a^2)^{1/2}(u^2-b^2)^{1/2}},$$

and

$$r_0 = \int_a^b \frac{(d_0^2 - u^2)e^{-Ku}\,du}{(u^2 - a^2)^{1/2}(b^2 - u^2)^{1/2}}.$$

Similarly from (3.7) and (3.11), we obtain

$$\psi_1(y) = \frac{K^2 D_0 (d_1^4 - y^4)}{4(y^2 - a^2)^{1/2}(b^2 - y^2)^{1/2}} + i\left(p_1 - q_1 - ir_1 + \frac{2e^{Ka}}{K^3}(1 - Ka)\right) \times \frac{K^2 D_0^2 (d_0^2 - y^2)}{8(y^2 - a^2)^{1/2}(b^2 - y^2)^{1/2}}, \quad a < y < b, \qquad (3.16)$$

where

$$p_1 = \int_{-a}^{a} \frac{(d_1^4 - u^4)e^{-Ku}\,du}{(a^2 - u^2)^{1/2}(b^2 - u^2)^{1/2}},$$

$$q_1 = \int_b^{\infty} \frac{(d_1^4 - u^4)e^{-Ku}\,du}{(u^2 - a^2)^{1/2}(u^2 - b^2)^{1/2}},$$

and

$$r_1 = \int_a^b \frac{(d_1^4 - u^4)e^{-Ku}\,du}{(u^2 - a^2)^{1/2}(b^2 - u^2)^{1/2}}$$

and d_1^4 is given by

$$\int_a^b \frac{(d_1^4 - u^4)e^{Ku}\,du}{(u^2 - a^2)^{1/2}(b^2 - u^2)^{1/2}} = 0.$$

4. REFLECTION AND TRANSMISSION COEFFICIENTS

As $\xi \to -\infty$ and $+\infty$, respectively, we have by (3.2) and (3.8) the complex reflection and transmission coefficients R and T as

$$R = -\frac{1}{2}\int_a^b \psi(y)e^{-Ky}\,dy,$$

$$T = 1 + \frac{1}{2}\int_a^b \psi(y)e^{-Ky}\,dy, \qquad (4.1)$$

so that $T = 1 - R$ and hence

$$T_0 = 1 - R_0, \quad T_1 = -R_1, \quad \text{etc.},$$

where

$$R = R_0 + \epsilon^2 R_1 + O(\epsilon^4 \ln \epsilon, \epsilon^4),$$

$$T = T_0 + \epsilon^2 T_1 + O(\epsilon^4 \ln \epsilon, \epsilon^4). \qquad (4.2)$$

Now by (4.1), (4.2), (3.11), (3.15), and (3.16), we have

$$R_0 = -\tfrac{1}{2} D_0 r_0,$$

$$R_1 = -i\tfrac{1}{16} K^2 D_0^2 \left(r_0(p_1 - q_1) - r_1(p_0 - q_0) + \frac{2}{K^3}(1 - Ka) r_0 e^{Ka} \right), \qquad (4.3)$$

etc.

5. DISCUSSION

The reflection and transmission coefficients obtained here are valid for all angles of incidence ($0° \leqslant \alpha < 90°$) of the surface wave and for all wavelengths other than short ones. We have calculated these coefficients numerically for different values of α in the range $0° \leqslant \alpha \leqslant 15°$ thus enabling us to retain only up to two terms in the different analytical approximations. However, by taking an appropriate number of terms in these approximations, these coefficients can be calculated for values of α beyond 15°.

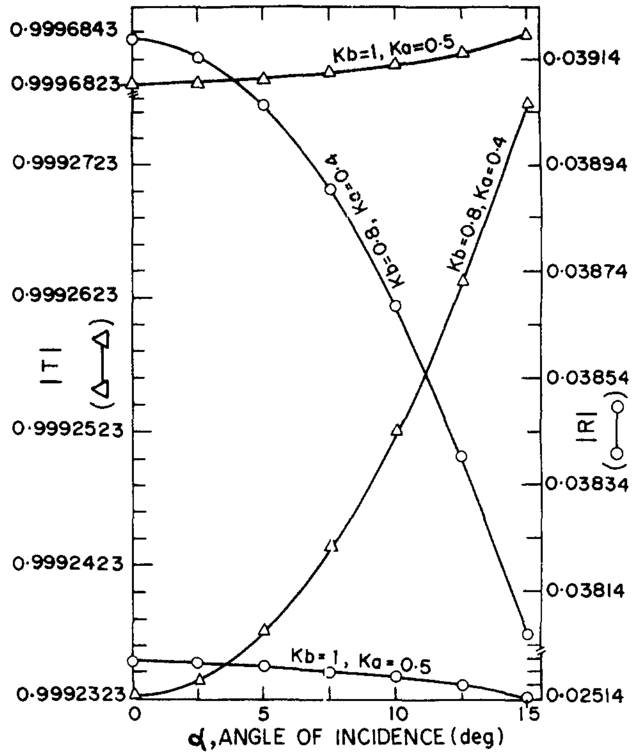

FIG. 1. $|T|$ and $|R|$ against α(deg).

Thus the integral equation method is found to be simple and straightforward and gives equally good results in all the cases of partially immersed as well as submerged fixed vertical barrier and plate in case of oblique incidence of a surface wave train. In contrast to this, Evans and Morris[10] found by using the method of variational principle that the results are not so good in the case of submerged fixed vertical barrier in comparison to those found in the case of a partially immersed fixed vertical barrier.

It should be noted that $|R_0|$, $|T_0|$ give the corresponding results for a normally incident wave train and are in complete agreement with those obtained earlier by Evans.[6] Taking $Kb = 2Ka = 0.8$ and $Kb = 2Ka = 1.0$, respectively, $|R|$ and $|T|$ are calculated for $\alpha = 0°$, 2.5°, 5°, 7.5°, 10°, 12.5°, and 15°, and plotted in Fig. 1.

It appears from the figure that for fixed Ka (and hence, Kb), transmission coefficient increases and reflection coefficient decreases with α, which is similar to the situation arising in the case of a partially immersed barrier (cf. Evans and Morris,[10] Mandal and Goswami[13]).

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