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The role of negative ions on the Jeans instability in a complex plasma in the presence of nonthermal positive ions

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The role of negative ions on Jeans instability in the presence of nonthermal positive ions has been investigated in this paper. Electrons and negative ions are considered Boltzmann-distributed, while the positive ions follow nonthermal velocity distribution. A negatively charged dust component is modeled by continuity and momentum equations. The linear dispersion relation shows that the presence of negative ions reduces the critical value of the nonthermal parameter a_{cr} , which consequently increases the unstable region and hence pronounces the Jeans instability. The growth rate of this Jeans instability decreases with the increasing negative ion temperature.

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I. INTRODUCTION

Dusty plasmas, or complex plasmas as they have become more recently known, consist of massive charged grains in addition to electrons and ions appearing in the usual two-component plasma. Such a complex plasma is usually found in the interstellar clouds, circumstellar clouds, interplanetary medium, cometary tails, planetary rings, and the Earth's magnetosphere.¹⁻³ In the laboratory, dusty plasma occurs as a result of high Z impurities from the Tokamak walls during plasma etching and impurity generation in magnetohydrodynamic (MHD) power generators.

The Jeans instability of a self-gravitating dusty plasma system is a mode of particular interest to space and astrophysical plasma situations. This Jeans instability itself is a well known phenomenon,^{4,5} and in a dusty plasma it is of particular interest. It has been studied by several authors in recent decades.⁶⁻¹² A modification of the Jeans instability has been reported by Delzano *et al.*¹³ considering a Lenard-Jones-like shielding potential. Recently, Jacobs and Shukla¹⁴ reported that the magnetic field and ion-neutral collisions are stabilizing factors for the Jeans instability in a partially ionized astrophysical plasma. In all of the above studies, ion behavior has been modeled through a Maxwellian description.

However, nonthermal electron-ion distributions are turning out to be a characteristic feature of space plasmas where the Jeans instability plays an important role. The observations of nonthermal ions in the Earth's bow-shock region by the Vela satellite have been noted.¹⁵ On the Phobos 2 satellite, ASPERA has observed the loss of energetic ions from the upper ionosphere of Mars. Due to the absence of a strong

magnetic field, the impact of the solar wind with the planetary atmosphere results in nonthermal ion fluxes.¹⁶ There remains a wealth of other sources that point toward the existence of nonthermal electron-ion fluxes. Pillay and Verheest¹⁰ used the nonthermal ion distribution proposed by Cairns *et al.*¹⁷ to investigate the influence of nonthermal ions on the linear behavior of the Jeans instability in a dusty plasma. They have shown that for increasing values of the nonthermal parameter, there is a decrease in the critical Jeans wave number, corresponding to an increase in the space scale over which the Jeans instability exists, for both low and high ion-electron temperature ratios and for both weak and strong Jeans interactions.

Negative ions are found to be an extra component (which may occur naturally or may be injected from external sources) in most space and laboratory plasmas.¹⁸⁻²² The role of negative ions on the charging of dust grains in a plasma has also been investigated.²³ The presence of negative ions along with the negatively charged dust grains can play a very important role in a complex plasma. Vladimirov *et al.*²⁴ pointed out that the equilibrium state of plasma as well as the ion acoustic wave propagation are significantly modified in the complex plasma in the presence of negative ions due to relevant processes such as ionization, electron attachment, diffusion, positive-negative ion recombination, plasma particle collision, as well as elastic Coulomb and inelastic dust charging collisions. Negative ions are also very important for reactive laboratory and technological plasmas.^{25,26} The role of negative ions on dust acoustic wave (DAW) propagation has been recently reported based on the orbit motion limited (OML) theory of dust grain charging.^{27,28} Recently, the impact of negative ions on the development of the Jeans instability in a self-gravitating complex plasma has been investigated.²⁹ In this paper, we have investigated the role of negative ions on the Jeans instability in a complex plasma in the presence of nonthermal positive ions. The results of this investigation are of paramount importance in space plasmas as there are both negative ions and nonthermal positive ions.

Our results show that the presence of negative ions reduces the critical values of the nonthermal parameter of the

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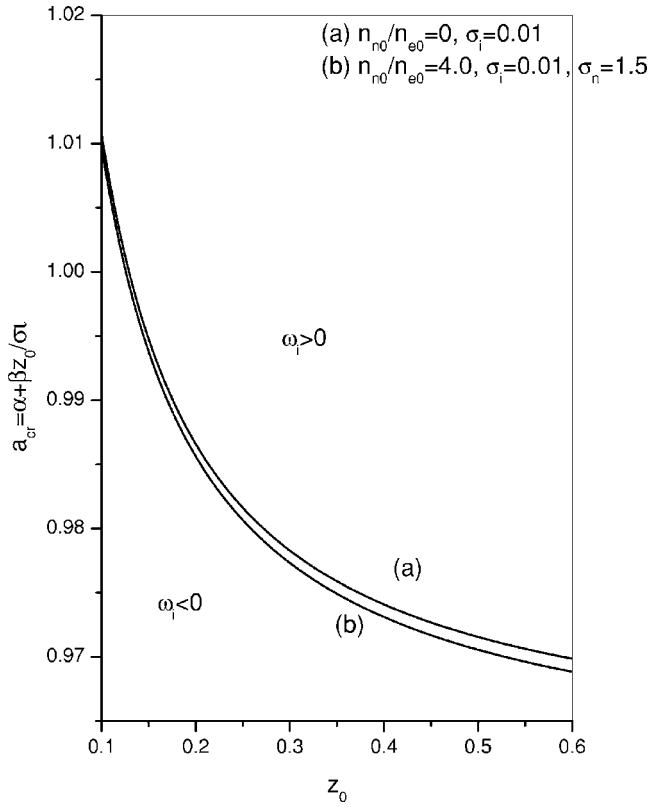


FIG. 1. For curve (a) (absence of negative ions), $a > / < a_{cr}$ corresponds to $\omega_i > / < 0$, i.e., instability/stability, and the same for curve (b) in the presence of negative ions.

positive ions, which increases the unstable region of the Jeans instability. Moreover, it has been shown that the growth rate of this Jeans instability decreases with increasing negative ion temperature.

II. FORMULATION OF THE PROBLEM

We consider a four-component dusty plasma, comprised of negatively charged warm adiabatic dust grains, Boltzmann-distributed electrons, negative ions, and nonthermal positive ions. The dust charge variations are also considered. Thus at equilibrium we have

$$n_{i0} = n_{e0} + n_{n0} + Z_{d0}n_{d0}, \quad (1)$$

where n_{j0} ($j=e, i, n, d$) are the equilibrium number densities of the j th (e =electron, i =positive ion, n =negative ion, d =dust) species and $-Z_{d0}e$ is the equilibrium charge on the dust surface.

As the positive ions are assumed to be nonthermally distributed, a three-dimensional equilibrium state positive ion velocity distribution function satisfying the collisionless Vlasov equation with a population of fast (energetic) particles can be written as

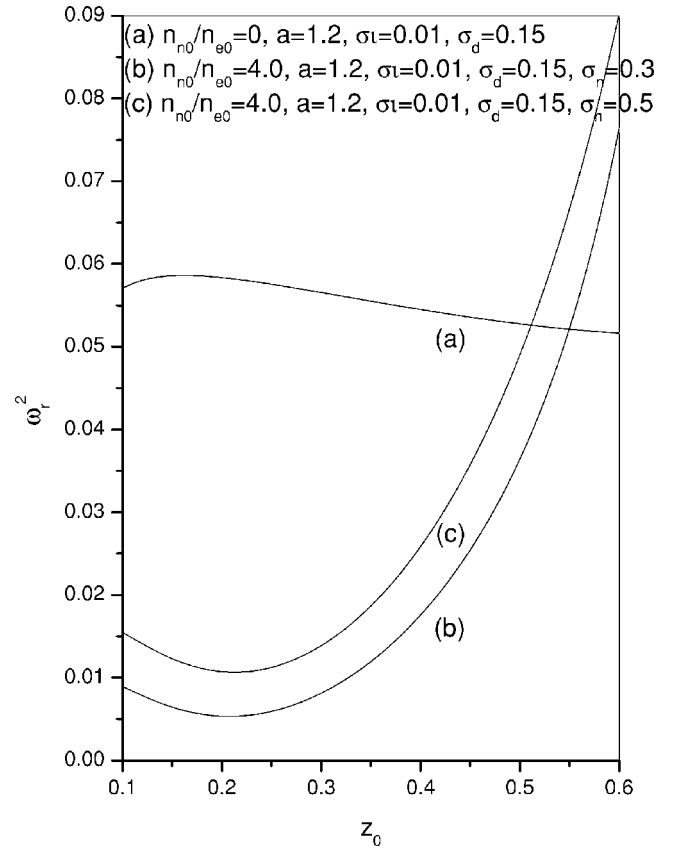


FIG. 2. Plot of the square of the normalized real frequency $\omega_r^2/(k^2 Z_{d0} k_B T_e / m_d)$ against z_0 of DA wave $\omega_{jd}=0$ for different plasma parameters.

$$\begin{aligned} F_i(v_i) &= F_i(v_x, v_y, v_z) \\ &= \frac{n_{i0}}{(1+3a)} \left(\frac{1}{2\pi V_{ii}^2} \right)^{3/2} \left[1 + 4a \left(\frac{1}{2} \frac{v_x^2}{V_{ii}^2} + \frac{\Phi}{\sigma_i} \right)^2 \right] \\ &\quad \times \exp \left(- \frac{v_x^2 + v_y^2 + v_z^2}{2V_{ii}^2} - \frac{\Phi}{\sigma_i} \right), \end{aligned} \quad (2)$$

where a is the positive ion nonthermal parameter that determines the number of fast (energetic) positive ions, v_x, v_y, v_z are the three components of the positive ion velocity v_i , $V_{ii} = \sqrt{k_B T_i / m_i}$ is the positive ion thermal velocity, T_i (T_e) is the positive ion (electron) temperature, m_i is the positive ion mass, and $\Phi = e\phi / k_B T_e$, $\sigma_i = T_i / T_e$, where ϕ is the electrostatic potential and k_B is the Boltzmann constant.

Integrating the positive ion distribution function (2) over the velocities v_y and v_z , the steady-state one-dimensional positive ion velocity distribution with a population of fast positive ion was obtained by Cairns *et al.*^{17,30} with $\phi=0$.

In the presence of a nonzero potential Φ , integration of the distribution function (2) gives the following positive ion number density:³¹

$$n_i = n_{i0} \left[1 + \frac{4a}{1+3a} \left(\frac{\Phi}{\sigma_i} + \frac{\Phi^2}{a_i^2} \right) \right] \exp \left(\frac{-\Phi}{\sigma_i} \right). \quad (3)$$

The velocity distribution function of electrons and negative ions is assumed to be Maxwellian, so that the electron and negative ion number densities are

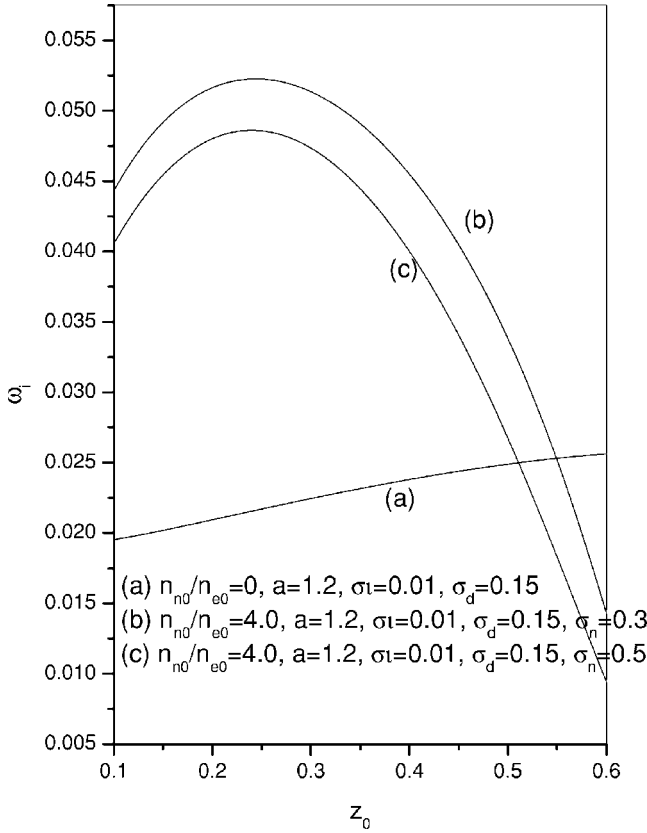


FIG. 3. Plot of normalized imaginary frequency $\omega_i/(k^2 C_{da}^2)$ as given by Eq. (29) against z_0 of DA wave for different plasma parameters.

$$n_e = n_{e0} \exp(\Phi), \quad (4)$$

$$n_n = n_{n0} \exp\left(\frac{\Phi}{\sigma_n}\right), \quad (5)$$

where $\sigma_n = T_n/T_e$, T_n is the negative ion temperature.

For analytic studies considering the effect of negative ions on dust acoustic waves in a self-gravitating dusty plasma with nonthermal positive ions, it is imperative that we use an analytic expression for negative ion current as well as for electron and positive ion current to the dust grain surface. For a quantitative description of the dust charging in gas discharge plasma, one of the most frequently used approaches is the orbit motion limited (OML) theory.³² This approach allows us to determine the cross sections for electron and ion collection by the dust particle from the laws of conservation of energy and angular momentum. According to OML theory, the dust charging currents due to the flow of nonthermal positive ions, negative ions, and electrons of the ambient plasma to the grain surface are, respectively,

$$I_i = \pi r_0^2 e \sqrt{\frac{8k_B T_i}{\pi m_i}} \frac{n_{i0}}{1 + 3a} \times \left[\left((1 + 24a/5) + 16a/3 \frac{\Phi}{\sigma_i} + 4a \frac{\Phi^2}{\sigma_i^2} \right) - \frac{eq_d}{r_0 k_B T_e \sigma_i} \right] \exp\left(\frac{-\Phi}{\sigma_i}\right); \quad q_d < 0, \quad (6)$$

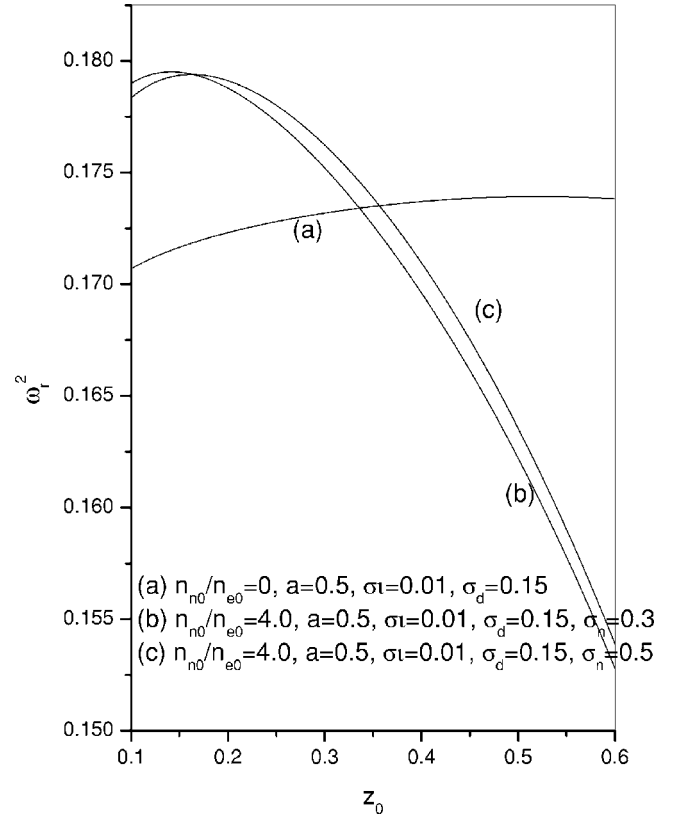


FIG. 4. Plot of the square of the normalized real frequency $\omega_r^2/(k^2 z_0 k_B T_e / m_d)$ against z_0 of DA wave $\omega_{jd}=0$ for different plasma parameters.

$$I_n = -\pi r_0^2 e \sqrt{\frac{8k_B T_n}{\pi m_n}} n_{n0} \exp\left(\frac{\Phi}{\sigma_n}\right) \exp\left(\frac{eq_d}{r_0 k_B T_n}\right); \quad q_d < 0, \quad (7)$$

$$I_e = -\pi r_0^2 e \sqrt{\frac{8k_B T_e}{\pi m_e}} n_{e0} \exp(\Phi) \exp\left(\frac{eq_d}{r_0 k_B T_e}\right); \quad q_d < 0, \quad (8)$$

where r_0 is the radius of the dust grains and m_n is the mass of the negative ion. In the above expression, the parameter a arises due to the effects of nonthermal positive ions (if we set $a=0$, we get the usual expression of positive ion current for Maxwellian positive ions).

Considering electron, positive ion, and negative ion currents due to collisions with plasma particles, the dust grain charging equation becomes

$$\frac{\partial q_d}{\partial t} + \nu_D \frac{\partial q_d}{\partial x} = I_e + I_i + I_n = I_{\text{tot}}. \quad (9)$$

The dust charging frequency ν_D is

$$\nu_D = -\left(\frac{\partial I_{\text{tot}}}{\partial q_d}\right)_{\text{eq}} = \frac{r_0}{\sqrt{2\pi}} \frac{\omega_{pi}^2 (5 + 8a)}{V_{ti} (5 + 15a)} \left[1 + \frac{z_0}{\alpha_n} + \left(\frac{5 + 24a}{5 + 8a}\right) \frac{\sigma_i}{\alpha_n} + \frac{1}{\sigma_n} \left(1 - \frac{1}{\alpha_n}\right) \left(\frac{z_0}{\sigma_i} + \frac{5 + 24a}{5 + 8a}\right) \right] \quad (10)$$

and the ion-electron number density ratio is given by

$$\frac{n_{i0}}{n_{e0}} = \sqrt{\frac{m_i}{m_e \sigma_i}} \frac{\left(\frac{5+15a}{5+8a}\right)}{\left(\frac{z_0}{\sigma_i} + \frac{5+24a}{5+8a}\right)} \alpha_n e^{-z_0} \quad (11)$$

with

$$\alpha_n = 1 + \frac{n_{n0}}{n_{e0}} \sqrt{\frac{\sigma_n m_e}{m_n}} e^{(1-1/\sigma_n)z_0}. \quad (12)$$

To derive the relation (11), we use the current balance equation $I_{e0} + I_{i0} + I_{n0} = 0$.

The dust component is modeled as a warm fluid, the dynamics of which is governed by the dust continuity equation

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d v_d) = 0 \quad (13)$$

and the dust momentum equation

$$\frac{\partial v_d}{\partial t} + v_d \frac{\partial v_d}{\partial x} = -\frac{q_d}{m_d} \frac{\partial \phi}{\partial x} - \frac{k_B T_d}{m_d n_d} \frac{\partial n_d}{\partial x} - \frac{\partial \psi}{\partial x}. \quad (14)$$

Here, n_d , m_d , v_d , q_d , and T_d represent the dust number density, mass, velocity, charge, and temperature, respec-

tively, where we specifically consider negatively charged particles for the purpose of this study. The gravitational potential is represented by ψ .

The above system is combined with the Poisson equations for the overall charge balance,

$$\epsilon_0 \frac{\partial^2 \phi}{\partial x^2} = -(en_i - en_e - en_n + q_d n_d) \quad (15)$$

and mass densities

$$\frac{\partial^2 \psi}{\partial x^2} = 4\pi G m_d (n_d - n_{d0}), \quad (16)$$

where we have neglected the gravitational effects of nonthermal positive ions, negative ions, and electrons.

It should be noted that for a gravitating plasma, the assumption of an equilibrium value n_{d0} of the dust number density n_d is a consequence of what is known as Jeans Swindle.

III. LINEARIZED FIELD SOLUTIONS OF THE PROBLEM

Considering perturbation $(\delta n_e, \delta n_i, \delta n_n, \delta n_d, Z_{d0} e q_1) e^{i(kx - \omega t)}$ about the equilibrium state $(n_{e0}, n_{i0}, n_{n0}, n_{d0}, -Z_{d0} e)$, Eq. (9) yields

$$q_1 = \frac{\beta_d}{\left(1 - i \frac{\omega}{\nu_D}\right)} \frac{(\alpha + \beta z_0 / \sigma_i)}{\left[\frac{z_0}{\alpha_n} + \frac{5+24a}{5+8a} \frac{\sigma_i}{\alpha_n} + \frac{1}{\sigma_n} \left(\frac{\alpha_n - 1}{\alpha_n}\right) \left(\frac{z_0}{\sigma_i} + \frac{5+24a}{5+8a}\right)\right]}, \quad (17)$$

where

$$\beta_d = \frac{\frac{z_0}{\alpha_n} + \frac{5+24a}{5+8a} \frac{\sigma_i}{\alpha_n} + \frac{1}{\sigma_n} \left(\frac{\alpha_n - 1}{\alpha_n}\right) \left(\frac{z_0}{\sigma_i} + \frac{5+24a}{5+8a}\right)}{z_0 \left[1 + \frac{z_0}{\alpha_n} + \frac{5+24a}{5+8a} \frac{\sigma_i}{\alpha_n} + ac1 \sigma_n \left(\frac{\alpha_n - 1}{\alpha_n}\right) \left(\frac{z_0}{\sigma_i} + \frac{5+24a}{5+8a}\right)\right]}, \quad (18)$$

$$\alpha = \frac{(8a - 15) - (15 + 72a)\sigma_i \tau_n}{(15 + 24a)}, \quad (19)$$

$$\beta = \frac{(16a - 15) - (15 + 24a)\sigma_i \tau_n}{(15 + 24a)}, \quad (20)$$

with

$$\tau_n = \frac{1}{\alpha_n} \left(1 + \frac{\alpha_n - 1}{\sigma_n}\right). \quad (21)$$

For the dust fluid, the dust density perturbation is related to Φ by

$$\frac{\delta n_d}{n_{d0}} = -\frac{k^2 C_{da}^2}{\omega^2 + \omega_{jd}^2 - k^2 V_{id}^2} \Phi, \quad (22)$$

where $C_{da} = \sqrt{Z_{d0} k_B T_e / m_d}$ is the dust acoustic speed, $\omega_{jd}^2 = 4\pi G m_d n_{d0}$ is the squared Jeans frequency, and $V_{id} = \sqrt{k_B T_d / m_d}$ is the dust thermal speed. To derive the above relation, we use Eqs. (13), (14), and (16). The corresponding electron, positive ion, and negative ion density fluctuations are given by

$$\frac{\delta n_e}{n_{e0}} = \Phi, \quad \frac{\delta n_i}{n_{i0}} = \frac{a-1}{1+3a} \frac{\Phi}{\sigma_i}, \quad \frac{\delta n_n}{n_{n0}} = \frac{\Phi}{\sigma_n}. \quad (23)$$

In the long-wavelength approximation, Poisson's equation reads

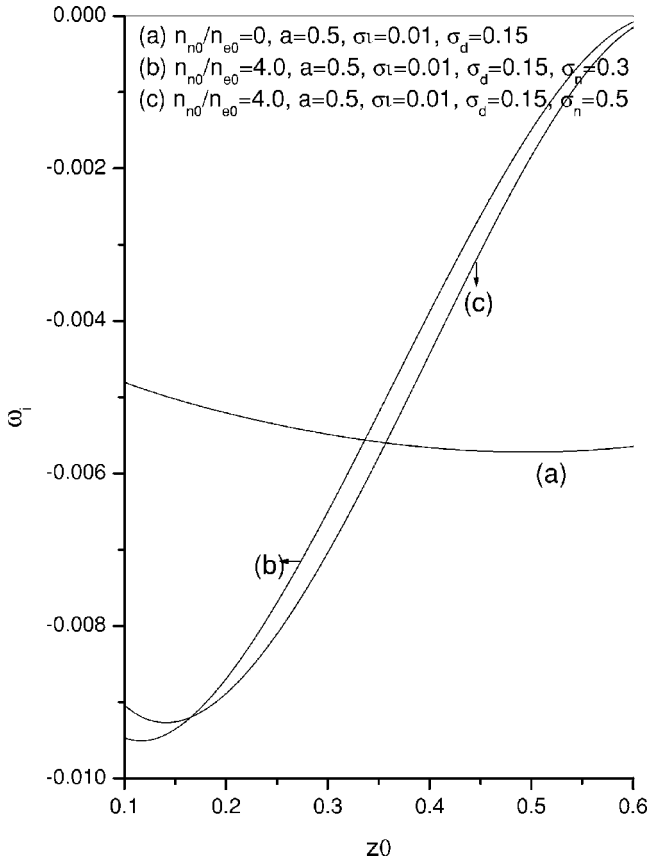


FIG. 5. Plot of normalized imaginary frequency $\omega_i/(k^2 C_{da}^2)$ as given by Eq. (29) against z_0 of DA wave for different plasma parameters.

$$\frac{n_{i0}}{n_{e0}} \frac{\delta n_i}{n_{i0}} - \frac{\delta n_e}{n_{e0}} - \frac{n_{n0}}{n_{e0}} \frac{\delta n_n}{n_{n0}} - \frac{Z_{d0} n_{d0}}{n_{e0}} \frac{\delta n_d}{n_{d0}} + \frac{Z_{d0} n_{d0}}{n_{e0}} q_1 = 0.$$

(24) with

$$\bar{\beta}_d = \frac{Z_{d0} n_{d0}}{n_{e0}} \beta_d \frac{(\alpha + \beta z_0 / \sigma_i)}{\left[\frac{z_0}{\alpha_n} + \frac{5 + 24a}{5 + 8a} \frac{\sigma_i}{\alpha_n} + \frac{1}{\sigma_n} \left(\frac{\alpha_n - 1}{\alpha_n} \right) \left(\frac{z_0}{\sigma_i} + \frac{5 + 24a}{5 + 8a} \right) \right]}, \quad (27)$$

where $\sigma_d = T_d / Z_{d0} T_e = V_{td}^2 / C_{da}^2$ arises due to the presence of warm dust grains.

The decay/growth rate works out to be

$$\omega_i = - \frac{\Im \epsilon(\omega_r, k)}{\frac{\partial}{\partial \omega} \Re \epsilon(\omega_r, k)}, \quad (28)$$

which gives

$$\frac{\omega_i}{k^2 C_{da}^2} = \frac{\beta_d}{2\nu_D} \frac{\left(\frac{Z_{d0} n_{d0}}{n_{e0}} \right)^2 \left(\alpha + \beta \frac{z_0}{\sigma_i} \right)}{\left(1 - \frac{1}{\sigma_i} \frac{n_{i0}}{n_{e0}} \frac{a-1}{1+3a} + \frac{1}{\sigma_n} \frac{n_{n0}}{n_{e0}} - \bar{\beta}_d \right)^2 \left[\frac{z_0}{\alpha_n} + \frac{5 + 24a}{5 + 8a} \frac{\sigma_i}{\alpha_n} + \frac{1}{\sigma_n} \frac{(\alpha_n - 1)}{\alpha_n} \left(\frac{z_0}{\sigma_i} + \frac{5 + 24a}{5 + 8a} \right) \right]}. \quad (29)$$

IV. DISPERSION RELATION

Now substituting (22), (23), and (17) in (24), we find the dispersion relation for dust acoustic waves due to nonadiabatic dust charge variation, where $\omega_{pd} / \nu_D \neq 0$ is a finite but small quantity in a self-gravitating complex plasma in the presence of nonthermal positive ions, Boltzmann negative ions, and Boltzmann electrons. The dispersion relation is

$$\begin{aligned} \epsilon(\omega, k) = & \left(\frac{\omega^2}{k^2} - V_{td}^2 + \frac{\omega_{Jd}^2}{k^2} \right) \\ & \times \left[\left(1 - \frac{1}{\sigma_i} \frac{n_{i0}}{n_{e0}} \frac{a-1}{1+3a} + \frac{1}{\sigma_n} \frac{n_{n0}}{n_{e0}} \right) - \frac{Z_{d0} n_{d0}}{n_{e0}} \frac{\beta_d}{\left(1 - i \frac{\omega}{\nu_D} \right)} \right] \\ & \times \frac{(\alpha + \beta z_0 / \sigma_i)}{\left[\frac{z_0}{\alpha_n} + \frac{5 + 24a}{5 + 8a} \frac{\sigma_i}{\alpha_n} + \frac{1}{\sigma_n} \frac{(\alpha_n - 1)}{\alpha_n} \left(\frac{z_0}{\sigma_i} + \frac{5 + 24a}{5 + 8a} \right) \right]} \\ & - \frac{Z_{d0} n_{d0}}{n_{e0}} C_{da}^2 = 0. \end{aligned} \quad (25)$$

To study the Jeans instability, we turn to the determination of roots of this dispersion relation as follows:

The real part of the frequency ω_r , satisfying $\Re \epsilon(\omega, k) = 0$ is given by

$$\begin{aligned} \omega_r^2 = & \frac{k^2 Z_{d0} k_B T_e}{m_d} \left(\sigma_d + \frac{\frac{Z_{d0} n_{d0}}{n_{e0}}}{1 - \frac{1}{\sigma_i} \frac{n_{i0}}{n_{e0}} \frac{a-1}{1+3a} + \frac{1}{\sigma_n} \frac{n_{n0}}{n_{e0}} - \bar{\beta}_d} \right) \\ & - \omega_{Jd}^2 \end{aligned} \quad (26)$$

Both the expressions (26) and (29) show that the presence of negative ions and self-gravitating force modify the propagation characteristics of linear DAW, which are shown graphically in Figs. 2–5 or different plasma parameters. From Eq. (29) it is clear that DAW may grow or may be damped depending on the algebraic sign of ω_i provided $\omega_r^2 > 0$. The condition for growth $\omega_i > 0$ is

$$p(z_0) = \alpha + \frac{z_0}{\sigma_i} \beta > 0. \quad (30)$$

From (19) and (20) it implies that

$$a > \frac{15}{8} \frac{(1 + \sigma_i \tau_n)(1 + z_0/\sigma_i)}{(1 + 2z_0/\sigma_i) - 3(3 + z_0/\sigma_i)\sigma_i \tau_n} = a_{cr} \quad (31)$$

provided $\tau_n < 1/3\sigma_i\sigma_i + 2z_0/3\sigma_i + z_0$.

V. NUMERICAL RESULTS

Equation (1) gives us the condition for $n_{i0}/n_{e0} > n_{n0}/n_{e0}$. We have chosen ratio n_{n0}/n_{e0} accordingly and obtain the value of n_{i0}/n_{e0} given by Eq. (11), which in turn depends on different values of $z_0 = Z_{d0}e^2/r_0k_B T_e$, $\sigma_i = T_i/T_e$, $\sigma_n = T_n/T_e$, and a . Further, we have considered the case in which the negative ion temperature is greater than the nonthermal positive ion temperature, i.e., $\sigma_n > \sigma_i$, which explains the choice of the values of $\sigma_i = 0.01$ and $\sigma_n = 0.3, 0.5$. As $V_{td} < C_{da}$, i.e., dust thermal speed < dust acoustic speed, the ratio $V_{td}^2/C_{da}^2 = \sigma_d < 1$.

Figure 1 shows the plot of $a_{cr} = \alpha + \beta z_0/\sigma_i$ versus z_0 at $n_{n0}/n_{e0} = 0, 4.0$, $\sigma_i = 0.01$, and $\sigma_n = 0, 1.5$. From this figure, it is clear that the presence of negative ions ($n_{n0}/n_{e0} = 4.0$, $\sigma_n = 0.01$) reduces the critical value a_{cr} of the nonthermal parameter of the positive ions at the same $\sigma_i = 0.01$. This effect consequently increases the unstable region of the Jeans instability.

Figures 2 and 3 are plotted for ω_r^2 against z_0 for $n_{n0}/n_{e0} = 0, 4.0$, $\sigma_i = 0.01$, $\sigma_d = 0.15$, and $\sigma_n = 0.3, 0.5$. In Fig. 2, nonthermal parameter $a = 1.2$, whereas in Fig. 3 it is $a = 0.5$. Figure 2 shows that for $a = 1.2$, ω_r^2 increases with z_0 beyond $z_0 \approx 0.22$. On the other hand, Fig. 3 shows that for $a = 0.5$, ω_r^2 decreases with z_0 beyond $z_0 \approx 0.16$. In both cases, ω_r^2 increases with increasing negative ion temperature.

Figures 4 and 5 are plotted for ω_i against z_0 for $n_{n0}/n_{e0} = 0, 4.0$, $\sigma_i = 0.01$, $\sigma_d = 0.15$, and $\sigma_n = 0.3, 0.5$. Figure 4 is drawn for $a = 1.2$ and Fig. 5 is drawn for $a = 0.5$. Figure 4 shows that for $a = 1.2$, growth rate ω_i decreases beyond $z_0 \approx 0.22$. It is interesting to note that for $a = 0.5$, Fig. 5 shows no Jeans instability as ω_i is then negative. Also for $a = 1.2$, the increase in negative ion temperature reduces ω_i . Hence, the increase in nonthermal parameter induces the increase in the Jeans instability, which is evident from Figs. 4 and 5.

VI. CONCLUSION

From the results obtained in this paper and from the curves, it is seen that the presence of nonthermal positive ions in a dusty plasma enhances the Jeans instability, which is further enhanced due to the presence of Boltzmann-distributed negative ions. However, the growth rate of this instability is reduced due to the increasing negative ion temperature. Our results can be useful in understanding the behavior of DA waves in space and astrophysical plasmas.^{3,17,30}

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