ELSEVIER

23 November 1995

PHYSICS LETTERS B

Physics Letters B 363 (1995) 162–165

Temperature dependence of π, K and η meson masses

Abhijit Bhattacharyya [1], Sibaji Raha [2]

Department of Physics, Bose Institute, 93/1 A.P.C. Road, Calcutta 700 009, India

Received 30 December 1994; revised manuscript received 13 September 1995
Editor: C. Mahaux

Abstract

The temperature dependence of π, K and η meson masses have been studied from an effective chiral Lagrangian, where the chiral symmetry is realised nonlinearly. Effective masses have been obtained from the pole positions of the full propagator. The π, K and η masses are found to decrease with temperature.

PACS: 12.38.Mh; 12.40.Vv; 14.40.Aq
Keywords: Hadrons; Finite-temperature; Chiral; Effective masses; SU(3)

Hot and/or dense hadronic matter has been an intense field of research in recent times. The expected formation of a new phase of strongly interacting matter called Quark Gluon Plasma (QGP) [1–5], the restoration of the spontaneously broken chiral symmetry [6,7] and other such tantalizing possibilities make this a vibrant area of activity. The study of effective meson masses at finite temperature is of current interest since this might provide some very important information about the state of the matter and possible signatures of the putative phase transition [8].

Since hadrons cannot yet be described by QCD, due to our limited knowledge of its non-perturbative features, we have to depend on models. Hence all the calculations, so far, are model dependent and the results vary widely from model to model [8]. The temperature dependence of Goldstone boson (π, K and η) masses may be interesting to study as they arise due to the spontaneous breakdown of the $SU(3)_L \times SU(3)_R$ symmetry of the chiral representation of QCD. The temperature dependence of the pion mass has been studied in great detail by different authors [8–11]. The temperature dependence of the K and η masses have been studied from the NJL model [12]. The elementary particles in the NJL model are quarks and the confinement-deconfinement mechanism should play a crucial role there. The study of these hadronic masses from a purely hadronic model, where there is no question of deconfinement, should be interesting. With this motivation we calculate the temperature dependence of π, K and η meson masses from the chiral nonlinear σ-model. Though K and η are heavy compared to the pion, their role becomes important as the temperature increases. The nonlinear σ-model reproduces the low energy structure of QCD successfully. A very good discussion of the low energy structure of QCD may be obtained in Refs. [13–21].

There are two popular approaches to the chiral effective model; the Hidden Gauge Symmetry Approach (HGSA) [21–23] and the Massive Yang Mills Approach (MYMA) [19]. In this paper we will follow

[1] Electronic mail: phys@boseinst.ernet.in.
[2] Electronic mail: sibaji@boseinst.ernet.in.

SSDI 0370-2693(95)01202-8

the HGSA. In this approach the vector mesons are incorporated as the dynamical gauge bosons of the hidden local symmetry contrary to the MYMA where the masses of the vector mesons are put in by hand. In the HGSA the Lagrangian, at the lowest order, can be written as a linear combination $\mathcal{L}_A + a\mathcal{L}_V$, a being an arbitrary parameter which is fixed from the condition of reproducing the Vector Meson Dominance (VMD). This condition leads to $a = 2$. The Lagrangian is given by [21–23]

$$\mathcal{L}_A = -\tfrac{1}{8} f_\pi^2 \mathrm{Tr}(D_\mu \xi_L \cdot {\xi_L}^\dagger - D_\mu \xi_R \cdot {\xi_R}^\dagger)^2 ,$$
$$\mathcal{L}_V = -\tfrac{1}{8} f_\pi^2 \mathrm{Tr}(D_\mu \xi_L \cdot {\xi_L}^\dagger + D_\mu \xi_R \cdot {\xi_R}^\dagger)^2 , \qquad (1)$$

where

$$D_\mu \xi_{L(R)} = (\partial_\mu - igV_\mu)\xi_{L(R)} . \qquad (2)$$

g is the gauge coupling constant and V_μ is the vector meson nonet consisting of ρ, ω, K^* and ϕ.

In Eq. (1) the terms $\xi_{L(R)}$ are given by

$$\xi_{L(R)} = \exp(i\sigma P) \cdot \exp(\pm iP/f) , \qquad (3)$$

where P is the pseudoscalar matrix which arises due to the spontaneous breakdown of the $SU(3)_L \times SU(3)_R$ symmetry and σ is an unphysical scalar field.

The $SU(3)$ breaking in the Lagrangians $\mathcal{L}_A$ and $\mathcal{L}_V$ has been extensively studied in two very recent papers by Bramon et al. [22,23]. This breaking is mediated by a term proportional to $(\xi_L \epsilon {\xi_R}^\dagger + \xi_R \epsilon {\xi_L}^\dagger)$ and the broken Lagrangians are given by

$$\begin{aligned}
&\mathcal{L}_A + \Delta\mathcal{L}_A \\
&\quad = -\tfrac{1}{8} f_\pi^2 \mathrm{Tr}\{(D_\mu \xi_L \cdot {\xi_L}^\dagger - D_\mu \xi_R \cdot {\xi_R}^\dagger)^2 \\
&\quad \times [1 + (\xi_L \epsilon_A {\xi_R}^\dagger + \xi_R \epsilon_A {\xi_L}^\dagger)]\} , \\
&\mathcal{L}_V + \Delta\mathcal{L}_V \\
&\quad = -\tfrac{1}{8} f_\pi^2 \mathrm{Tr}\{(D_\mu \xi_L \cdot {\xi_L}^\dagger + D_\mu \xi_R \cdot {\xi_R}^\dagger)^2 \\
&\quad \times [1 + (\xi_L \epsilon_V {\xi_R}^\dagger + \xi_R \epsilon_V {\xi_L}^\dagger)]\} . \qquad (4)
\end{aligned}$$

The matrix $\epsilon_{A(V)}$ is given by $\epsilon_{A(V)} = \mathrm{diag}(0, 0, c_{A(V)})$, where $c_{A(V)}$ is some constant determined from the experimental results.

Let us now fix the gauge $\xi_L^\dagger = \xi_R = \xi = \exp(iP/f)$. This eliminates the unphysical scalar fields which in turn gives mass to the vector mesons. Expanding in terms of the pseudoscalar fields one can see that the $SU(3)$ breaking effect leads to the renormalization of the kinetic term in $\mathcal{L}_A$ which can be achieved by simply rescaling the pseudoscalar fields as [22,23]

$$\sqrt{1 + c_A} K \to K , \quad \sqrt{1 + 2c_A/3}\eta \to \eta . \qquad (5)$$

The term $a(\mathcal{L}_V + \Delta\mathcal{L}_V)$ leads to the mass splitting in the vector meson sector:

$$M_\rho^2 = M_\omega^2 = 2g^2 f^2 ,$$
$$M_{K^*}^2 = M_\rho^2 (1 + c_V) , \quad M_\phi^2 = M_\rho^2 (1 + 2c_V) . \qquad (6)$$

The rescaling of the pseudoscalar fields leads to a redefinition of the relations between different pseudoscalar decay constants:

$$f_K = \sqrt{1 + c_A} f_\pi , \quad f_\eta = \sqrt{1 + 2c_A/3} f_\pi . \qquad (7)$$

We neglect the QCD anomaly (Wess-Zumino term) which only affects the $V - V - P$ interactions. These contributions are neglected here, as we discuss later on in this letter.

So, we have three parameters in the model g, c_A and c_V. We determine them from the experimental results giving $g = 4.15$, $c_A = 0.47$ and c_V has two values 0.33 and 0.36 [21–23]. We now proceed to calculate the self energy of π, K and η.

In the effective Lagrangian approach at zero temperature, it is assumed that the properties of the system are describable at the tree level, where the masses and the coupling constants are to be regarded as the physical ones. Loop diagrams, which are neglected here, produce only renormalization effects on them.

There are three sets of interactions, hence diagrams, which contribute to the self energy of the pseudoscalar mesons at the one loop level; the tadpole diagram (Fig. 1a), the $V - P - P$ interaction which results in a $P - V$ loop (Fig. 1b) and the $V - V - P$ interaction which gives a $V - V$ loop (Fig. 1c). We have neglected the last set of diagrams as it contains heavy mesons on both the internal lines, hence Boltzmann suppressed.

The calculation of the effective mass is done using the following principle:
The Dyson equation relates the free and the full propagator as

$$D(p) = D_0(p) + D_0(p)\Pi D(p) , \qquad (8)$$

where D_0 is the free propagator and D is the full propagator. The effect of interaction is embedded in the

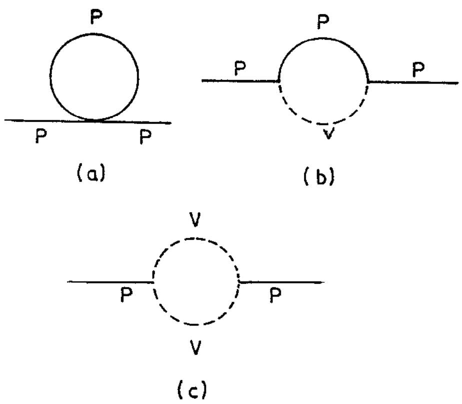

Fig. 1. Feynman diagrams contributing to the self energy of a pseudoscalar meson. The P represents a pseudoscalar meson and the V represents a vector meson.

polarisation function π. The temperature dependent polarisation has two parts, a real part and an imaginary part. The real part contributes to the mass of the corresponding meson whereas the imaginary part contributes to the decay width. For our purpose, i.e., for the calculation of the effective masses we will consider only the real part. Hence the self consistent solution of the equation

$$\omega^2 - m_{\rm ps}^2 - {\rm Re}\left(\prod_{\rm ps}(\omega, |\boldsymbol{k}| \to 0)\right)\bigg|_{k_0 = m_{\rm ps}^*} = 0 \qquad (9)$$

gives the temperature dependence of a pseudoscalar meson mass. In Eq. (9) $\omega = \sqrt{|\boldsymbol{k}|^2 + m_{\rm ps}^{*2}}$ is the effective energy and $m_{\rm ps}^*$ is the effective mass of the pseudoscalar meson. We have not shown the calculation of the self energy. The standard method for doing that can be obtained in Refs. [9,24,25]. The π, K and η pole masses as a function of temperature have been plotted in Figs. 2–4.

As a passing remark we would like to mention that this is just one of the definitions of the effective masses which is called the pole mass. This is the most commonly used definition. There are other definitions of the effective masses also; for example the screening mass [26] which is very popular in the context of the lattice gauge theory calculations.

To summarize, we have calculated the temperature dependence of π, K and η meson masses from the nonlinear σ-model. This is a purely hadronic model and the confinement does not play any role here compared to the Nambu-Jona-Lasinio model or other ef-

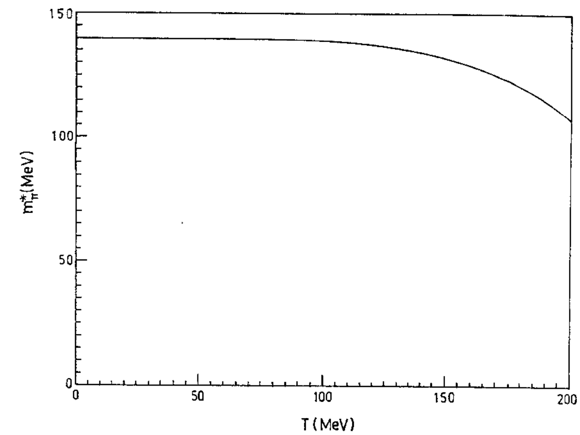

Fig. 2. Temperature dependence of the π mass.

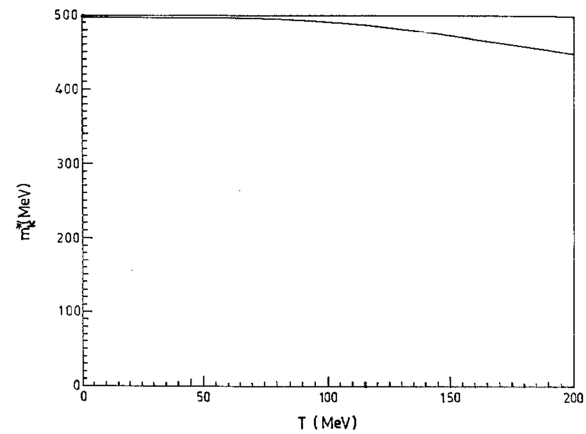

Fig. 3. Temperature dependence of the K mass.

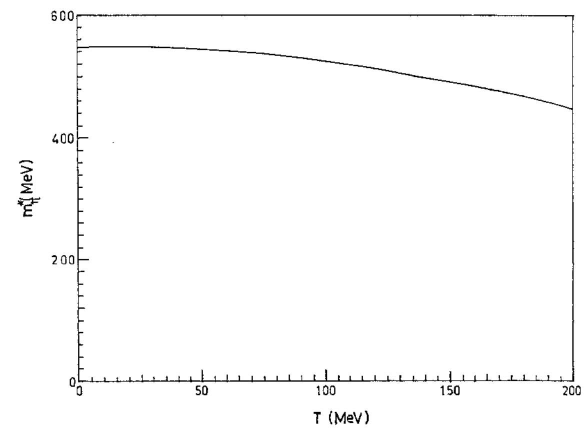

Fig. 4. Temperature dependence of the η mass.

fective quark models. Furthermore, this model reproduces the low energy structure of QCD successfully. We have adopted the Hidden Gauge Symmetry Approach in which the vector mesons are treated as the dynamical gauge bosons of the hidden local symmetry. The model has been properly modified to include the $SU(3)_V$ breaking effect. The three parameters of the model have been fixed from the experimental values of the masses and the decay widths.

The self energy of the pseudoscalar mesons has been calculated at the one loop level self consistently. Only the real part of the self energy contributes to the mass. The pole mass has been calculated from the pole of the full propagator in the limit $|\boldsymbol{k}| \to 0$. It can be seen from the figures that all the three masses decrease with temperature. Recently, the temperature dependence of the π mass has been calculated from the same model in the MYMA both from the $SU(2)$ and $SU(3)$ versions [9,24]. There also the pion mass has been found to decrease with temperature and the numerical values are also very close (the difference is less than 3%.)

In Ref. [12] the temperature dependence of these masses have been calculated from the NJL model for two cases. In case 1 the K mass decreases with temperature whereas, in case 2, it increases. In both the cases the η mass first decreases with temperature and then starts increasing and the pion mass increases with temperature. These results are quite different from ours, both quantitative and qualitatively. The pion mass has also been calculated at finite temperature from the linear σ-model where it has been found to increase with temperature. The Walecka model [27] reproduces an abnormal increase of the pion mass at high temperature. Recently, in a newly proposed model, the Zimanyi-Moskowski model [28], this problem of abnormal increase of pion mass is found to be overcome [29] although it still increases with temperature.

The above discussion suggests that studies of hadronic properties from different models are necessary and the problem is yet to be settled as different models predict widely different results. Also, one has to keep in mind that different models refer to different physical situations. The chiral effective model or the non-linear σ model is used as a low energy representation of QCD whereas the Walecka model or the Zimanyi-Moskowski model is used to reproduce the nuclear matter. The qualitative difference between the NJL and the non linear σ model may be due to the confinement effects, although much more work is needed to shed light on this issue. Calculation of π, K and η masses from some other models may give totally different results. Investigations in these directions are in progress.

The work of Abhijit Bhattacharyya has been supported partially by the Department of Atomic Energy (Government of India) through a Dr. K.S. Krishnan fellowship.

References

[1] E.V. Shuryak, Phys. Rep. 61 (1980) 71.
[2] L. McLerran, Rev. Mod. Phys. 58 (1986) 1021.
[3] B. Sinha and S. Raha, eds., Physics and Astrophysics of Quark Gluon Plasma (World Scientific, Singapore, 1988).
[4] B. Sinha Y.P. Viyogi and S. Raha, eds., Physics and Astrophysics of Quark Gluon Plasma (World Scientific, Singapore, 1994).
[5] T.C. Awes et al., eds., Proc. Quark Matter '91 (Gatlinburg, TN, November 1991), Nucl. Phys. A 544 (1992).
[6] E.V. Shuryak, Phys. Lett. B 107 (1981) 103.
[7] R.D. Pisarski, Phys. Lett. B 110 (1982) 155.
[8] A. Bhattacharyya and S. Raha, J. Phys. G 21 (1995) 741.
[9] C. Song, Phys. Lett. B 329 (1994) 312.
[10] J. Gasser and H. Leutwyler, Phys. Lett. B 188 (1987) 477.
[11] T. Matsui and B.D. Serot, Ann. Phys. (N.Y.) 144 (1982) 107.
[12] T. Hatsuda and T. Kunihiro, Phys. Rep. 247 (1994) 221.
[13] S. Weinberg, Physica A 96 (1979) 327.
[14] H. Leutwyler, Proc. XXVI Intern. Conf. on High Energy Physics (Dallas, TX, 1992), University of Bern preprint no: BUTP-92/42.
[15] J. Sakurai, Currents and Mesons (University of Chicago Press, Chicago, IL, 1969).
[16] B.W. Lee, Chiral Dynamics (Gordon and Breach, New York, 1972).
[17] N. Knoll, T.D. Lee and W. Zumino, Phys. Rev. 157 (1967) 1376.
[18] S. Gasiorowicz and D. Geffen, Rev. Mod. Phys. 41 (1969) 531.
[19] U.G. Meissner, Phys. Rep. 161 (1988) 213.
[20] A. Bhattacharyya, J. Alam, S. Raha and B. Sinha, submitted for publication.
[21] M. Bando, T. Kugo and K. Yamawaki, Phys. Rep. 164 (1988) 217.
[22] A. Bramon, A. Grau and G. Pancheri, Phys. Lett. B 344 (1995) 240.
[23] A. Bramon, A. Grau and G. Pancheri, Phys. Lett. B 345 (1995) 263.
[24] C. Song, Phys. Rev. D 49 (1994) 1556.
[25] K. Kajantie and J. Kapusta, Ann. Phys. 160 (1985) 477.
[26] C. DeTar, Phys. Rev. D 32 (1985) 276.
[27] J.D. Walecka, Ann. Phys. (N.Y.) 83 (1974) 491.
[28] J. Zimanyi and S.A. Moskowski, Phys. Rev. C 42 (1990) 1416.
[29] A. Bhattacharyya and S. Raha, submitted for publication.