

SINP-TNP/95-23

November 1995

# Symmetry breaking for rho meson in neutron matter

Abhee K. Dutt-Mazumder <sup>1</sup>, Anirban Kundu,

Triptesh De and Binayak Dutta-Roy

Saha Institute of Nuclear Physics, 1/AF Bidhan Nagar

Calcutta - 700 064, India

## Abstract

Qualitative changes in the collective excitation spectra of the  $\rho$ -meson triplet in neutron matter is studied, with particular emphasis on the breaking of the discrete  $p \leftrightarrow n$  symmetry. The appearance of additional branches in the dispersion characteristics, the mass splitting among the charge states, the splitting between longitudinal and transverse modes of the  $\rho^\pm$  mesons and the appearance of 'island' modes (or loops) in the time-like region are some of the features that are exposed.

PACS numbers: 12.38.Mh, 11.30.Rd, 12.40Vv, 21.65.+f

*Keywords:*  $\rho$  meson, neutron matter, collective oscillation

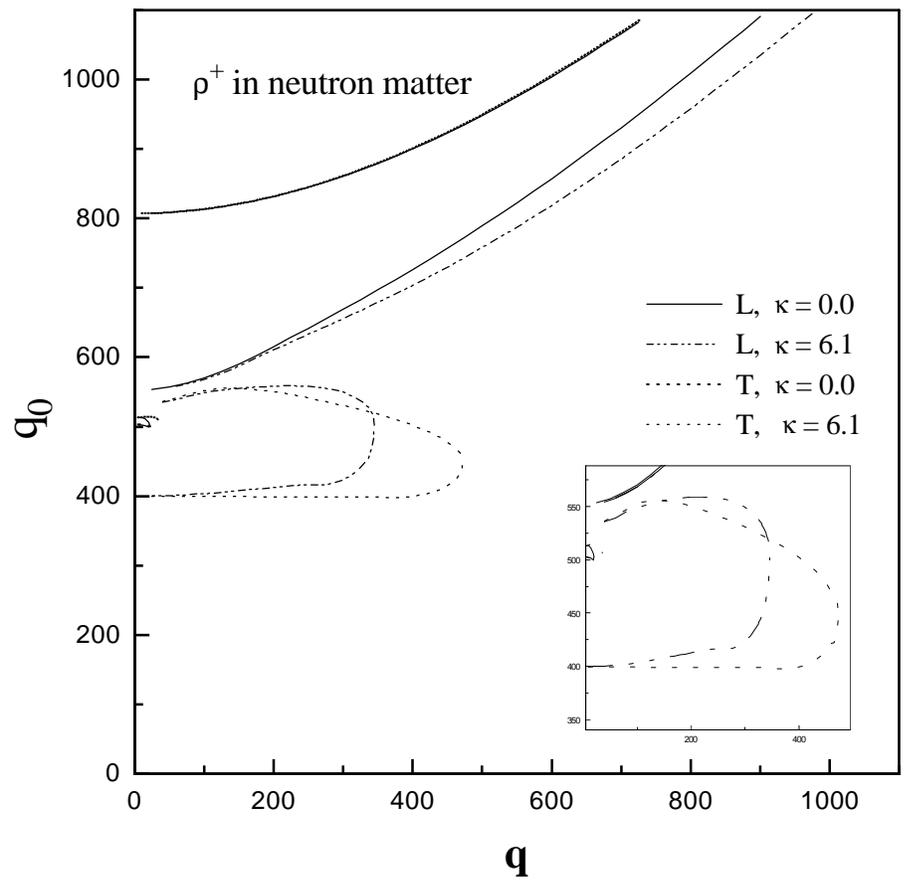
---

<sup>1</sup>E-mail: abhee@tnp.saha.ernet.in

In the recent past there have been several attempts both in the arena of theory and experiment to unravel the behaviour of different particles inside dense nuclear medium, particularly at densities 2-3 times higher than the normal nuclear matter density. Such studies are of cardinal importance both in the context of heavy ion collision and nuclear astrophysics. These apart, understanding of their properties inside hot and dense nuclear matter would also pave the way for extracting valuable informations from the various signals produced in the high energy heavy ion collisions needed to determine the equation of state of the system. Here we study the in-medium behaviour of the  $\rho$  meson in particular, as its significant role in heavy ion collisions in connection with the dilepton spectroscopy has been pointed out by several authors [1, 2]. Of specific interest here is to investigate the qualitative changes in the dispersion characteristics when the isospin symmetry is broken. Here the Lagrangian respects the symmetry but it is broken by the isospin asymmetric ground state or the ‘vacuum’.

Earlier, the in-medium mass shift of the  $\rho$  meson was investigated by several authors, both at zero and finite temperature within the framework of different theoretical models [2, 3, 4]. These studies were mostly confined to isospin symmetric nuclear matter. This letter, in contrast, as mentioned already, attempts to probe the properties of  $\rho$  meson propagator in nuclear matter where the so-called ‘vacuum’ or the ground state does not respect isospin symmetry, *i.e.* the densities of the neutron and proton are not equal. In order to bring out the effects in clear relief we consider the extreme limit of asymmetry viz., neutron matter. Our essential focus is on the collective oscillations of neutron matter set by the propagating  $\rho$  meson, popularly known as particle modes [5]. Among the features qualitatively different from what

are observed in symmetric nuclear matter, most remarkable is the splitting of longitudinal (L) and transverse (T) modes in the collective excitation spectra for  $\rho^\pm$  meson even in the static limit and degeneracy of the same for  $\rho^0$ . Also noteworthy is the appearance of the little ‘islands’ in the dispersion curves for  $\rho^\pm$  mesons. These branches represent two strongly interacting modes that survive only upto a finite value of momentum having a minimum at somewhat lower value of  $|\vec{q}|$  (Fig. 1). Also significant is the appearance of additional branches in the excitation spectra usually absent for symmetric nuclear mat-



The Lagrangian describing  $\rho$ -nucleon interaction may be written as

$$\mathcal{L}_{int} = g_v^j [\bar{N} \gamma_\mu \tau^j N - \frac{\kappa}{2M} \bar{N} \sigma_{\mu\nu} \tau^j N \partial^\nu] \rho_j^\mu \quad (1)$$

in presence of a tensor or magnetic interaction apart from the usual vector interaction of the original Walecka model (QHD-I). The index  $j$  runs from 1 to 3, and we use  $\rho^\pm = (\rho_1 \mp i\rho_2)/\sqrt{2}$  and  $\rho_0 = \rho_3$ ,  $\tau^j$  are the usual  $2 \times 2$  Pauli isospin matrices, and  $N$  denotes the two-component isospinor for the nucleon. The coupling constant  $g_v$  and  $\kappa$  may be estimated from the Vector Meson Dominance (VMD) of nucleon form factors or from the fitting of the nucleon-nucleon interaction data as done by the Bonn group [6]. From the estimates it is clear that in case of the vector-isovector  $\rho$  meson, the tensor interaction is substantial [7]. While VMD fits suggest the value  $\kappa = 3$ , we consider the larger estimates of the Bonn group again to emphasize the effects:

$$\frac{g_v^2}{4\pi} = 0.92 \quad \kappa = 6.1 \quad (2)$$

As the detailed treatment of the underlying formalism is available elsewhere, here, for brevity, we sketch only a brief outline of the same and present the final results of our calculations. The essential idea here is to determine the in-medium dressing of the  $\rho$  meson propagator due to the nucleon-hole and nucleon-antinucleon excitations (the former being the effect of the Fermi sea, the latter driven by the Dirac vacuum). Different  $\Pi$ -functions or vacuum polarizations are therefore calculated to study the in-medium effects on the  $\rho$  meson propagation inside the nuclear matter. We concentrate only on the effect of the Fermi sea which we refer as ‘dense part’, the contribution of the Dirac sea on the other hand is not included in the present analysis. The form of the nucleon propagator in nuclear matter may be expressed as

$$G(k) = G_F(k) + G_D(k) \quad (3)$$

where

$$G_F(k) = (\not{k} + M^*) \left[ \frac{1}{k^2 - M^{*2} + i\epsilon} \right] \quad (4)$$

and

$$G_D(k) = (\not{k} + M^*) \left[ \frac{i\pi}{E^*(k)} \delta(k_0 - E^*(k)) \theta(k_F - |\vec{k}|) \right] \quad (5)$$

with  $M^*$  denoting the effective mass of the nucleon in the medium and  $E^*(|\vec{k}|) = \sqrt{|\vec{k}|^2 + M^{*2}}$ . The first term in  $G(k)$ , namely  $G_F(k)$ , is the same as the free propagator of a spin  $\frac{1}{2}$  fermion, except for the fact that the effective mass of the nucleon is to be used, while the second part,  $G_D(k)$ , involving  $\theta(k_F - |\vec{k}|)$ , arises from Pauli blocking, describes the modifications of the same in the nuclear matter at zero temperature [8], as it deletes the on mass-shell propagation of the nucleon in nuclear matter with momenta below the Fermi momentum. Finite temperature effects may be incorporated by replacing the Heaviside  $\theta$ -function by the Fermi distribution, but the main features to be exposed in this paper are not modified significantly. In general, the self-energy or the second order polarization function may be written as

$$\Pi_{\mu\nu}^{lm} = \frac{-i}{(2\pi)^4} \int d^4k \text{Tr} [i\Gamma_\mu^l iG(k+q) i\bar{\Gamma}_\nu^m iG(k)] \quad (6)$$

where  $\Gamma$  represents appropriate vertex factors and  $l, m$  are the isospin indices. Just like the nucleon propagator, the polarization function can also be expressed as a sum of two parts, one coming from the dense part contribution and the other being the contribution of the Dirac sea. Here we consider only the first.

$$\Pi_{\mu\nu}(q) = \Pi_{\mu\nu}^F(q) + \Pi_{\mu\nu}^D(q) \quad (7)$$

The real part of the density dependent polarization tensor for  $\rho^+$  propagation is given by

$$\Pi_{\mu\nu}^D = \frac{g_v^2}{(2\pi)^3} \left[ \int_0^{k_F^{(p)}} \frac{d^4k}{E_p^*(k)} \delta(k_0 - E_p^*(k)) \frac{T_{\mu\nu}(k-q, k)}{(k-q)^2 - M_n^{*2}} \right]$$

$$+(k \rightarrow k + q, p \rightarrow n)] \quad (8)$$

where the superscript on  $k_F$  and the subscript on  $E^*$  and  $M^*$  denote the corresponding quantities for protons and neutrons.  $T_{\mu\nu}$  is the relevant trace taken over the fermion loop, the arguments being the fermionic momenta. Results for  $\rho^-$  are obtained by making the replacement  $q_0 \rightarrow -q_0$  and  $|\vec{q}| \rightarrow -|\vec{q}|$ . The angle integration of eq. (8) is performed analytically whereas the momentum integration is computed numerically.

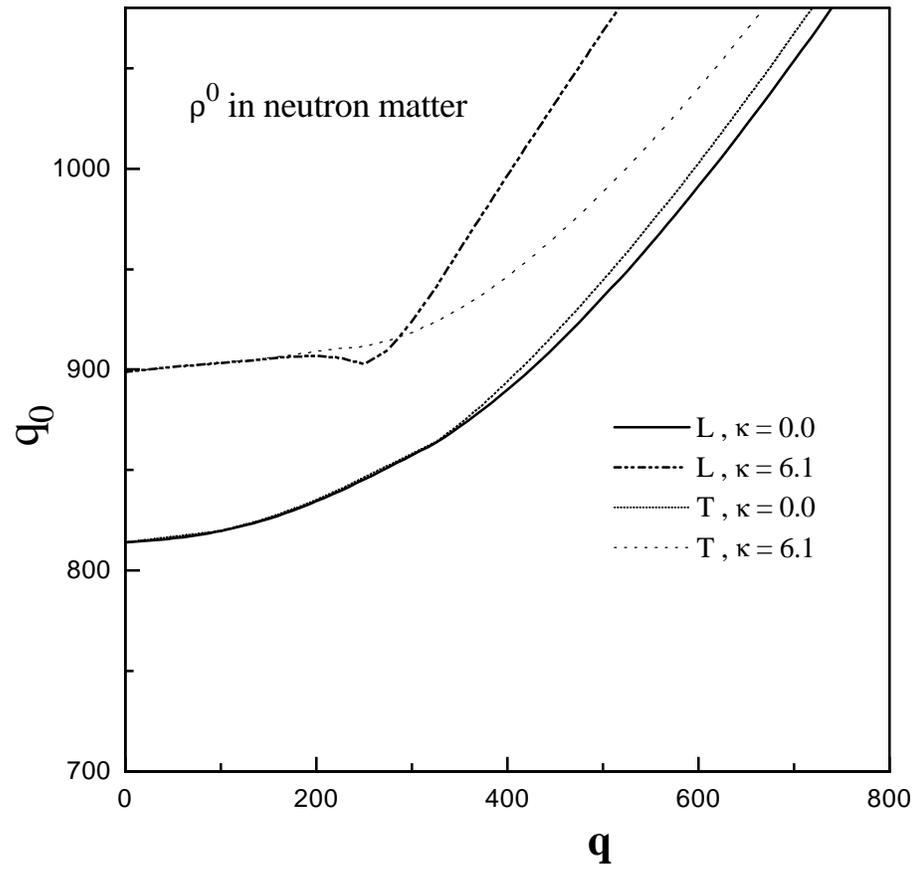
The collective modes arising out of the meson propagation in neutron matter is characterized by the zeros of the dielectric functions,  $\epsilon(q_0, |\vec{q}|)$ , which is calculated from the self energy [5]. Independent components of  $\Pi_{\mu\nu}$  are combined appropriately to give longitudinal and transverse modes of the dielectric functions. Therefore the eigenconditions for collective oscillations are given by

$$\epsilon_T^2(q)\epsilon_L(q) = 0 \quad (9)$$

Dispersion characteristics corresponding to the longitudinal and transverse modes are obtained by solving this equation for  $q_0$  and  $|\vec{q}|$  numerically.

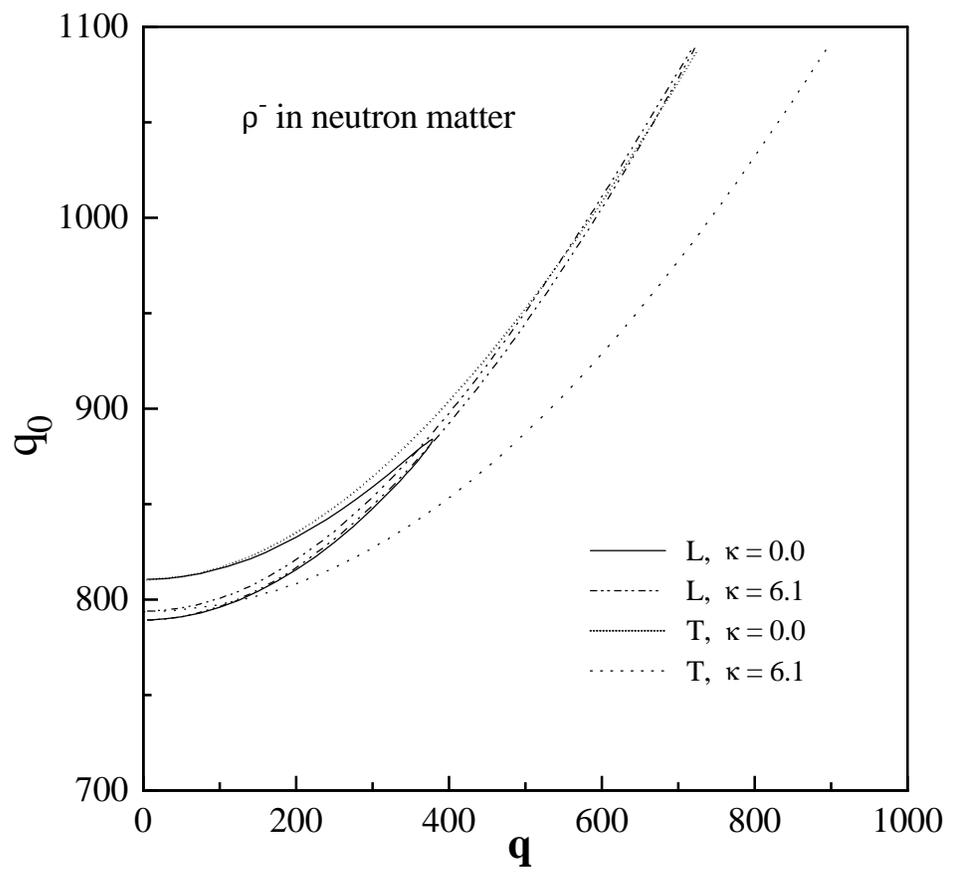
It is evident from eq. (8) that the density dependent part of the polarization involves two integrations having two different limit of integration corresponding to neutron and proton fermi momenta. In symmetric nuclear matter the radii of two fermi spheres (proton and neutron) are the same, in other words discrete isospin symmetry  $p \leftrightarrow n$  remains exact in this limit. Hence the effective masses of the neutron and proton are also equal. This ensures the conservation of current for all the isospin multiplets in symmetric nuclear matter, which in turn implies that to every longitudinal mode, there exists a corresponding transverse mode which coincide in the static limit ( $|\vec{q}| = 0$ ) [9].

This feature, however, is not totally present in nuclear matter having different number of neutron and proton, that is to say that the symmetry is broken resulting in the splitting of the longitudinal and transverse modes even in the static limit. This situation is noteworthy as in this case even though the Lagrangian is invariant under rotation in isospin space, the ‘vacuum’ is not, and hence particle spectrum also breaks the isospin symmetry. However, for  $\rho^0$  we have n-n and p-p loops, even for asymmetric nuclear matter having two fermi spheres of different sizes (one may be zero) and, therefore, in the loop integral effective masses involved are either  $M_p^*$  or  $M_n^*$  making it possible to combine the integrations so as to express it in a ‘gauge invariant’ form. It may be shown that in this case the requirement of current conservation is fulfilled *i.e.* ,  $q^\mu \Pi_{\mu\nu} = \Pi_{\mu\nu} q^\nu = 0$ , while in case of  $\rho^\pm$  this criteria is satisfied only if  $k_F^{(n)} = k_F^{(p)}$ . Hence, for  $\rho^0$  propagation, no matter whether the ground state of the nuclear system maintains this symmetry or not, in the static limit the branches corresponding to the longitudinal and transverse mode always coincide *i.e.*  $\rho_{trans}^0$  and  $\rho_{long}^0$  have same value for the mass even in neutron matter. We refer our readers to Fig. 2. However, this mass changes with the value of  $\kappa$  and density.



For the virtual proton propagator in neutron matter only the first part, viz.  $G_F$  survives whereas the second part,  $G_D$ , vanishes as  $k_F^{(p)} \rightarrow 0$ . Of course for the neutron propagator both the terms are nonvanishing. As in the present case we have only the neutron fermi sphere,  $\rho^+$  and  $\rho^-$  propagators will be modified differently. For the former the modification is due to proton-neutron hole, while the latter is being dressed by the neutron-antiproton excitation. In case of  $\rho^0$ , however, proton-antiproton and neutron-neutron hole will contribute as pointed out earlier. In the  $\rho^0$  spectra, it is observed that for  $\kappa = 6.1$ , though the longitudinal and transverse modes cross each other, mode conversion is not possible as the nature of polarization is different. It should also be noted that the degeneracy of these two modes in the static limit follows from our earlier remark.

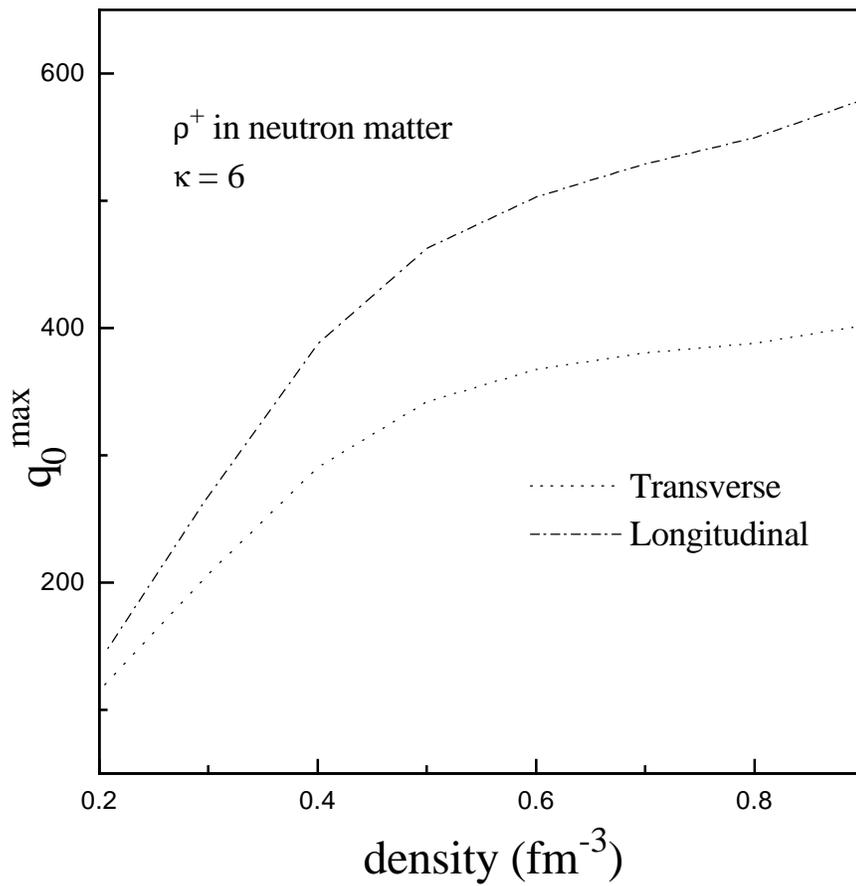
Next we focus on the  $\rho^-$  propagation. The dispersion characteristics in the region of stability, *i.e.*, for  $q_0 \leq (M_p + E_F^n)$  (beyond which the collective modes decay via particle-hole excitations), have been depicted in Fig. 3.



Unlike the case of  $\rho^0$ , here the longitudinal and transverse modes are not necessarily degenerate in the static limit. Even more interesting is the case of  $\rho^+$  propagation. Depending on the strength of magnetic interaction one can have at most four L, and three T modes in the region of stable collective oscillation (from the charge symmetry one can say that the situation is similar to that of  $\rho^-$  propagation in proton matter). This is evident from Fig. 2 where one of the longitudinal modes is unpaired. It is worth noting that the separation between the paired modes increases with the introduction of  $\kappa$ . The appearance of the so-called ‘island’ modes in the particle branch, which to the best of our knowledge has not been highlighted earlier (though such a situation has been encountered in the spacelike region [10]), is another feature that we want to emphasize. For  $\rho^+$ , the T and L branches corresponding to the ‘island’ modes are coincident in the static limit.

One of the interesting property of these modes is their presence even in the limit  $\kappa \rightarrow 0$  both for  $\rho^+$  and  $\rho^-$  propagation. However the islands are absent in  $\rho^0$  spectra. Such modes are not observed in case of symmetric nuclear matter, hence these are to be understood as the effect of asymmetry. In case of  $\rho^-$  in neutron matter again these ‘islands’ are not found for the transverse while for the longitudinal modes all of them form loops.  $\rho^+$  on the other hand shows loops both for longitudinal and transverse polarizations. The loops grow in size with  $\kappa$  and with the density of nuclear matter. It is appropriate to note the shape of the ‘islands’; for  $\rho^-$  these are narrow and sharp at the end while for  $\rho^+$  they are rounded. Coupling of the modes or the appearance of these loops indicate the onset of instability beyond a particular value of momenta ( $|\vec{q}|_{max}$ ). In fact beyond this limit the roots of  $q_0$  become complex. For completeness we have also shown

the variation of  $|\vec{q}|_{max}$  as a function of density. We find that  $|\vec{q}|_{max}$  approaches a saturation value with increasing density, which is shown in Fig. 4.



To conclude and summarize we note that the  $\rho$  meson propagation in neutron matter is qualitatively different from what is obtained in symmetric nuclear matter. However for  $\rho^0$ , results do not alter much as expected from the fact that  $\rho^0$  is blind to isospin asymmetry. While on the other hand  $\rho^+$  and  $\rho^-$  propagators are modified differently by two distinct mechanisms viz., proton-hole and neutron-antiproton pair creation respectively, and therefore corresponding dispersion curves also reflect this fact. It is observed that when the  $p \leftrightarrow n$  symmetry is broken, additional ‘island’ modes appear, together with the  $\rho^0$ ,  $\rho^\pm$  mass splitting and the removal of the degeneracy of L and T branches (even in the static limit). Of course for more reliable quantitative estimates, the contribution of the free part *i.e.*, the modifications due to the Dirac sea and the effect of mixing have to be incorporated. However, as we focus here only on the qualitative changes in the dispersion characteristics, the effect of Dirac sea is not expected to cause much difference as far as the nature of the collective oscillations are concerned. It can be easily shown that for the free part, the contribution to the self-energy is expressible as  $\Pi_{\mu\nu}^F = Q_{\mu\nu}\Pi$ , where  $Q_{\mu\nu} = -g_{\mu\nu} + q_\mu q_\nu/q^2$ . This evidently does not cause any splitting between the L and T branches in the static limit. Apart from that, and only for the diagram where both the  $\rho\bar{N}N$  couplings are of vectorial nature, there is an extra term which is proportional to  $g_{\mu\nu}\delta m$ , where  $\delta m$  is the mass difference between the two fermions in the loop, and is nonvanishing only for asymmetric matter. Studies are in progress to investigate dispersion characteristics in a varied range of  $p \leftrightarrow n$  asymmetry. Detailed results will be presented elsewhere.

## References

- [1] International school of heavy ion Physics. 3rd. Course: Probing the Nuclear Paradigm with Heavy Ion Reactions, Ed. R. A. Broglia, P. Kienle and P. F. Bartignon (World Scientific),1994, p.463.
- [2] M. Asakawa and C.M. Ko, Nucl. Phys. **A560** (1993) 399
- [3] H.C. Jean, J. Piekarewicz and A.G. Williams, Phys. Rev. **C49** (1994) 1981
- [4] H. Shiomi and T. Hatsuda, Phys. Lett. **B334** (1994) 281
- [5] S.A. Chin, Ann. Phys. **108** (1977) 301
- [6] J.J. Sakurai, Currents and Mesons (Univ. of Chicago Press, Chicago, 1969).
- [7] C. Song, P.W. Xia and C.M. Ko, Phys. Rev. **C52**, (1995) 408
- [8] B.D. Serot and J.D. Walecka, Adv. Nucl. Phys, (1986), vol. 16
- [9] A.K. Dutt- Mazumder, B. Dutta-Roy, A. Kundu and T. De, Saha Institute report no. SINP-TNP/95-08
- [10] J.C. Caillon and J. Labarsouque, Phys. Lett. **B352** (1995) 193

## Figure Captions

1. Dispersion characteristic for  $\rho^+$  meson (particle modes). The L and T modes for  $\kappa = 0.0$  are nearly coincident. Both  $q_0$  and  $\mathbf{q}$  are in MeV. The ‘island’ modes are shown in detail.
2. Dispersion characteristics for  $\rho^0$  meson (particle modes). For  $\kappa = 6.1$ , the L and T modes cross.
3. Dispersion characteristic for  $\rho^-$  meson (particle modes). The L mode for  $\kappa = 0$  form ‘island’, and for  $\kappa = 6.1$  form ‘island’ just outside the stable oscillation region.
4. Change of  $q_0^{max}$  (for ‘island’ modes and  $\kappa = 6.1$ ) with density.