SUPERUNIFIED MODEL WITH NATURALLY ULTRALIGHT DIRAC NEUTRINOS

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The recently proposed mechanism to naturally generate one ultralight Dirac neutrino per fermion family is implemented by construction of a globally supersymmetric SO(10) GUT superpotential featuring geometric hierarchy. A step forward is thus taken towards providing a firm theoretical basis for the relation $m_\nu = O(m_{3/2})M_{SUSY}/M_{GUT}$.

There is currently experimental as well as theoretical interest in ultralight Dirac neutrinos. The simplest conclusion from a comparison of tritium β-spectrum data implying [1,2] 20 eV $< m(\bar{\nu}_e) <$ 55 eV with negative searches [3,4] for nuclear double-β-decay requiring $(m_\nu)_{\mathrm{Majorana}} <$ 6–10 eV is that ν_e is a Dirac particle in the mass range 20–55 eV. A mechanism has recently been proposed [5,6] for naturally generating one such ultralight Dirac neutrino per fermion family in terms of a minimal [7] 4×4 neutral fermion Majorana mass-matrix. It is an important task to try to understand more deeply the dynamical origin of this mechanism.

The desired mass-matrix, mentioned above, is best obtained by enlarging [5,6] the usual set of nonsinglet gauge, matter and scalar field multiplets taken in ordinary GUTs to include three extra singlet fields (two fermionic and one scalar) as well as by requiring some global U(1) symmetries. There are models which can do without these extra fields by introducing nonsequential fermionic generations [8] or mirror fermions [9]. However, we shall concentrate here on the first approach because it can be linked with supersymmetry. This mechanism has been shown [5,7] to arise most naturally in an SO(10) SUSY–GUT [10] scenario with a geometric type [11–13] of hierarchy. With such a hierarchy, one has the SUSY-breaking scale M_{SUSY} and the GUT-breaking scale M_{GUT} around 10^{10} GeV and 10^{16} GeV [12] respectively. Identifying the VEV of the new scalar field with M_{SUSY}, the lightness of the Dirac neutrino relative to its charge 2/3 family member can be understood [5] via the simple but powerful formula $m_\nu = O(m_{2/3})M_{SUSY}/M_{GUT}$.

Notwithstanding the above nice feature, there was a serious gap in this scheme. It had not been successfully incorporated in any of the existing SO(10) SUSY–GUT models [10]; only the skeleton of an incomplete SUSY–GUT scenario leading to the scheme had been suggested [5,7]. There has thus been a need for credible and not totally unrealistic SUSY–GUT model for this purpose. Without it, the observation of an ultralight Dirac neutrino mass cannot be

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claimed to be (even indirect) experimental evidence for supersymmetry. In this note we fulfill this need by constructing precisely such a model or more specifically its superpotential W.

It is useful to enumerate the basic requirements involved in the construction of our W. First, one needs to implement the inverted or backwards hierarchy mechanism of Witten [14] as modified by Dimopoulos and Raby [11]. This means using an O'Raifearteigh type of piece in W to spontaneously generate M_{SUSY} at a scale $\mu \sim 10^{10}$ GeV with certain Higgs VEVs left undetermined at the tree level. Loop effects make the latter (as well as certain masses) slide to an exponentiated large value, thus leading to M_{GUT}. Second, radiative effects have to induce electroweak symmetry breakdown at the scale $V_W \equiv M_W/g$ (M_W being the W-mass and g the semiweak gauge coupling strength) with the order of magnitude geometric equality $M_{SUSY}^2 \sim V_W M_{GUT}$. The superpotential W must be chosen so that this is ensured, i.e. that [11, 13] two-loop supergraphs generate through $\ln(M_{SUSY}/M_{GUT})$ factors a negative sign for the appropriate Higgs mass squared leading to a minimum of the potential at V_W. Both these requirements necessitate the introduction of gauge-singlet superfields. Finally, these, together with the usual gauge-non-singlet superfields, must lead to the desired 4 × 4 Majorana mass-matrix in the neutral fermion sector.

Consider the chiral superfields necessary for constructing W. Let us stick to one fermionic generation. Call the 16-plet of matter superfields $\hat{\psi}_\alpha$ while the three singlet ones, mentioned earlier, can be designated $\hat{S}, \hat{S}'$ and $\hat{\sigma}$. (Throughout we use a caret to denote a superfield while caretlessness signifies the corresponding scalar component.) The following additional chiral superfields are introduced: a [‡1] 10-plet $\hat{H}_i$, a 16-plet $\hat{\chi}_\alpha$ as well as [‡2] a $\overline{16}$-plet $\hat{\bar{\chi}}'^\alpha$, four 54-plets $\hat{V}_{ij}, \hat{Y}_{ij}, \hat{Z}_{ij}, \hat{Z}'_{ij}$ and finally seven more singlets $\hat{X}_{1-6}$ as well as $\hat{A}$. Our superpotential can now be written as the sum of three parts $W = W_1 + W_2 + W_3$, where

$$W_1 = \alpha_1 \hat{X}_1(\mathrm{Tr}\,\hat{V}^2 - \hat{\sigma}^2) + \alpha_2 \hat{X}_2(\hat{\sigma}^2 - \mu^2) + \alpha_3 \,\mathrm{Tr}(\hat{Z} + \hat{Z}')(\hat{V}^2 + \mu'\hat{V}) + \alpha_4 \hat{X}_3 [\mathrm{Tr}\,\hat{Y}(\hat{Z} - \hat{Z}') + \hat{\chi}_\alpha \hat{\bar{\chi}}'^\alpha - \bar{\mu}^2] + (\alpha_5 \hat{X}_1 + \alpha_6 \hat{X}_4)^2 \hat{A} + \alpha_7 \hat{X}_5(\mathrm{Tr}\,\hat{Y}^2 - \hat{X}_4^2) + \alpha_8 \hat{X}_6 \,\mathrm{Tr}\,\hat{Z}'^2 , \tag{1a}$$

$$W_2 = \lambda_1 \hat{\sigma}\hat{A}^2 + \lambda_2 \hat{H}_i \hat{A} \hat{H}_i + \lambda_3 \hat{H}_i \hat{Y}_{ij} \hat{H}_j + \lambda_4 \hat{H}_i \hat{X}_4 \hat{H}_i \lambda_5 \hat{A}(\mathrm{Tr}\,\hat{V}^2 - \mu_1^2) , \tag{1b}$$

$$W_3 = h_1 \hat{\psi}_\alpha \hat{\bar{\chi}}'^\alpha \hat{S} + h_2 \hat{S}\hat{S}'\hat{\sigma} + h_3 \hat{\psi}_\alpha^{\mathrm{T}} (C^{-1}\Gamma_{i\alpha\beta}) \hat{\psi}_\beta \hat{H}_i . \tag{1c}$$

Here α's, λ's and h's are coupling strengths, Γ's are standard SO(10) matrices and C is the SO(10)-generalized charge-conjugation matrix. Further, μ, μ', $\bar{\mu}$, μ_1, are input mass parameters of the same order, namely M_{SUSY}, so that W contains only one scale.

The various terms of eq. (1) have been orchestrated to satisfy the requirements outlined above. W_1 causes both SUSY breakdown à la O'Raifearteigh and the GUT symmetry-breakdown $SO(10) \to SU(3)_C \times SU(2)_L \times U(1)_Y$. Further, it causes the Witten [14] inverted hierarchy to be generated leading to $M_{GUT} \gg M_{SUSY}$. The first three terms of W_1 constitute an SO(10)-generalization of Witten's [14] SU(5) superpotential with three important differences. (1) Symmetric 54-plets are chosen instead of the adjoint 45 which is antisymmetric here. (2) The SUSY-breaking scale μ is introduced through the second term along with $\hat{\sigma}$ which plays an important role in W_2 and W_3, as explained later. (3) There is a μ' part in the third term which is needed to solve a special problem in connection with the O'Raifearteigh mechanism which was absent in Witten's SU(5) example but arises in the SO(10) case; μ' prevents the occurrence of a SUSY-invariant solution in which [‡3] $SO(10) \to SO(5) \times SO(5)$ and further it plays a crucial part (see later) in ensuring the global minimality of the $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$-symmetric vacuum. At the minimum of the potential, $\langle X_1 \rangle$, $\langle X_2 \rangle$ and $\langle Z_{ij} \rangle$ become proportional to one another but remain

[‡1] This can easily be extended to include two 10-plets for more realistic charged fermion masses.

[‡2] The VEVs of χ_α and $\bar{\chi}'^\alpha$, chosen to be equal and parallel, ensure the absence of any D-term in the scalar potential.

[‡3] Correspondingly, $V_{ij} = \mu\,\mathrm{diag}(1,1,1,1,1,-1,-1,-1,-1,-1)/\sqrt{10}$ and $\sigma^2 = \mu^2$.

otherwise undetermined at the tree level; loop effects make them slide to M_{GUT}. The fourth term in W_1 ensures that the VEVs $\langle Y_{ij}\rangle$, $\langle \chi_\alpha\rangle$ and $\langle \bar{\chi}'^{\alpha}\rangle$ also acquire nonzero [‡4] values undetermined at the tree level; we will see that these also slide to M_{GUT} under loop effects. The last two of these three VEVs break $SU(4)_{PS} \times SU(2)_R$ down to $SU(3)_C \times U(1)_Y$ so that finally only the standard 3–2–1 symmetry is left invariant. The presence of Tr $\hat{Y}(\hat{Z} - \hat{Z}')$ along with $\hat{\chi}_\alpha \hat{\bar{\chi}}'^{\alpha} - \bar{\mu}^2$ makes all nonsinglets (except $\hat{H}_i$ and $\hat{\psi}$) heavy. Such nonsinglets, if light, may be allowable in the SU(5) case, but can simply not be accommodated in an SO(10) SUSY–GUT; apart from spoiling [15] asymptotic freedom at the GUT level, they also destroy [‡5] perturbative unification by forcing the evolving coupling strengths to shoot up beyond unity. The choice of identical (equal and opposite) coefficients for Z_{ij} and Z'_{ij} in the third (fourth) term has been carefully arranged to make the scalar potential a minimum with respect to X_3. (Of course, the SUSY nonrenormalization theorem ensures the preservation of this choice to all orders.) Then the fifth and sixth terms have been put in W_1 to allow $\langle X_4\rangle$ and $\langle Y_{ij}\rangle$ to slide proportionately to $\langle X_1\rangle$. Finally, the last term in W_1 ensures the vanishing of $\langle Z'_{ij}\rangle$.

We next focus on W_2. It is the link between the sector corresponding to W_1 and the low-energy sector described by W_3. The first term in W_2 has been included to attribute a mass $\sim\mu$ to A. The next term couples $\hat{H}_i$ and $\hat{A}$ which are from the low-energy and SUSY-breaking sectors respectively. This coupling, as demonstrated below, is essential (along with the λ_5 term) to the occurrence of the type of two loop supergraphs radiatively inducing the breakdown of electroweak symmetry. The third and fourth terms in W_2 are needed – with an appropriate tree-level fine tuning between λ_3 and λ_4 – to enforce the second hierarchy, namely to render superheavy all other components of H_i apart from the Weinberg–Salam weak doublet which is left to be massless at the tree level [‡6]. The fifth term is needed to couple $\hat{V}$ with $\hat{A}$ as needed in the supergraph mentioned above.

[‡4] This is guaranteed by a nonzero $\bar{\mu}$ as shown later in eq. (2c).

[‡5] This problem has not been noticed in the existing literature [10].

[‡6] It acquires a mass of order V_W under radiatively induced electroweak symmetry breakdown.

Lastly, W_3 describes the interactions among matter, Higgs and singlet superfields and is in particular responsible for generating fermion masses including ultralight Dirac neutrinos. The specific choice of W_3 – i.e. the absence of other possible cubic terms – is forced by the postulated global symmetries [7] $U_{a,b,c}(1)$ where $Q_a(\hat{\psi}) = Q_a(\hat{S}) = -Q_a(\hat{S}') = -\frac{1}{2}Q_a(\hat{H}) = 1$, $Q_a(\text{rest}) = 0$, $Q_b(\hat{S}) = -Q_b(\hat{S}') = Q_b(\hat{\bar{\chi}}') = 1$, $Q_b(\text{rest}) = 0$ and $Q_c(\hat{S}') = -Q_c(\hat{\sigma}) = 1$, $Q_c(\text{rest}) = 0$. As shown in ref. [7], W_3 is the minimal superpotential necessary to describe fermion masses including the desired 4 × 4 Majorana mass-matrix in the neutral fermion sector. In fact, with $h_1\langle\bar{\chi}'\rangle = A = O(m_{2/3})$, $h_3\langle H\rangle = B \sim M_{GUT}$, $h_2\langle\sigma\rangle = C \sim M_{SUSY}$ and using the notation of ref. [7], the desired mass-matrix [5–7]

	ν_L	s_L	n_L	s'_L
$\nu_L{}^C$	0	0	A	0
$s_L{}^C$	0	0	B	C
$n_L{}^C$	A	B	0	0
$s'_L{}^C$	0	C	0	0

emerges. This yields [5,7] an ultralight Dirac neutrino with a mass $AC/B = O(m_{2/3})M_{SUSY}/M_{GUT}$.

The superpotential (1) yields eighteen F-terms where $F_X = (\partial W/\partial \hat{X})_{\hat{X}=X}$. However, those involving derivatives of W with respect to superfields occurring only in W_3 (i.e. F_ψ, F_s, $F_{s'}$) automatically vanish at the minimum of the potential and need not be considered explicitly. The remaining fifteen split into a set of eleven F-terms which vanish at the global minimum of the potential $V_{pot} = \Sigma_i |F_i|^2$ and another set of four which cannot consistently be chosen to vanish at the said minimum. The two sets are displayed in table 1.

The solution for $(V_{pot})_{min}$ corresponds to two sets of conditions on the VEVs (N.B. each VEV is chosen to be real and positive). First, we list those corresponding to the part of V_{pot} that depends on those F's with vanishing VEVs.

$$0 = \langle H_i\rangle = \langle X_3\rangle = \langle X_5\rangle = \langle X_6\rangle = \langle A\rangle = \langle Z'_{ij}\rangle$$
$$= \mu_1\langle\sigma\rangle - \mu_2^2\,, \qquad (2a)$$

Table 1
Fifteen interesting F terms.

With vanishing VEVs		With nonzero VEVs	
Designation	Expression	Designation	Expression
$F_{V_{ij}}$	$2\alpha_1 X_1 V_{ij} + \alpha_3 [(Z+Z')V + V(Z+Z') - \frac{1}{5} I \,\mathrm{Tr}(Z+Z')V + \mu'(Z+Z')]_{ij}$	F_{X_1}	$\alpha_1(\mathrm{Tr}\, V^2 - \sigma^2) + 2\alpha_5(\alpha_5 X_1 + \alpha_6 X_4)A$
F_σ	$-2(\alpha_1 X_1 - \alpha_2 X_2)\sigma + \lambda_1(A^2 + \mu_1 X_4)$		
F_{X_3}	$\alpha_4[\mathrm{Tr}\, Y(Z-Z') - \bar{\chi}'^{\alpha} - \bar{\mu}^2]$	F_{X_2}	$\alpha_2(\sigma^2 - \mu^2)$
$F_{Y_{ij}}$	$\alpha_4 X_3 (Z-Z')_{ij} + 2\alpha_7 X_5 Y_{ij} + \lambda_3 H_i H_j$		
F_{X_4}	$2\alpha_6(\alpha_5 X_1 + \alpha_6 X_4)A + \lambda_4 H_i H_j - 2\alpha_7 X_5 X_4$	$F_{Z_{ij}}$	$\alpha_3(V^2 - \frac{1}{10} I \,\mathrm{Tr}\, V^2 + \mu' V)_{ij} + \alpha_4 X_3 Y_{ij}$
F_{X_5}	$\alpha_7(\mathrm{Tr}\, Y^2 - X_4^2)$		
F_A	$(\alpha_5 X_1 + \alpha_6 X_4)^2 + 2\lambda_1 \sigma A + \lambda_2 H_i H_i$		
F_{X_6}	$\alpha_8 \,\mathrm{Tr}\, Z'^2$		
F_{X_α}	$-\alpha_4 X_3 \bar{\chi}'^{\alpha}$	$F_{Z'_{ij}}$	$\alpha_3(V^2 - \frac{1}{10} \mathrm{Tr}\, V^2 + \mu' V)_{ij} - \alpha_4 X_3 Y_{ij} + 2\alpha_8 X_6 Z'_{ij}$
$F_{\bar{\chi}'^{\alpha}}$	$-\alpha_4 X_3 \chi_\alpha$		
F_{H_i}	$2(\lambda_2 A H_i + \lambda_3 Y_{ij} H_j + \lambda_4 X_4 H_i$		

$$\langle X_1 \rangle = (\alpha_2/\alpha_1)\langle X_2 \rangle = -(\alpha_6/\alpha_5)\langle X_4 \rangle$$

$$= -(\alpha_6/\alpha_5)\Big(\sum_i \langle Y_i \rangle^2\Big)^{1/2} , \tag{2b}$$

$$\sum_i \langle Y_i \rangle \langle Z_i \rangle = \langle \chi \rangle^2 - \bar{\mu}^2 , \tag{2c}$$

$$\alpha_1 \langle X_1 \rangle \langle V_k \rangle + \alpha_3 \Big(\langle Z_k \rangle \langle V_k \rangle - \frac{1}{10} \sum_i \langle Z_i \rangle \langle V_i \rangle + \frac{1}{2} \mu' \langle Z_k \rangle \Big) = 0 , \tag{2d}$$

$$\sum_i \langle V_i \rangle^2 + \mu_1^2 = 0 . \tag{2e}$$

In eqs. (2) we have chosen diagonal bases for $\langle Z_{ij} \rangle$, $\langle Y_{ij} \rangle$ and $\langle V_{ij} \rangle$, i.e. $\langle Z_{ij} \rangle = \langle Z_i \rangle \delta_{ij}$ etc; further $\langle \chi_\alpha \rangle$ and $\langle \bar{\chi}'^{\alpha} \rangle$ have been rotated [‡2] to the configuration $\langle \chi_\alpha \rangle = \langle \chi \rangle \delta_{\alpha,16} = \langle \bar{\chi}'^{\alpha} \rangle$. Next, we derive the conditions connected with the explicit minimization of $|F_{X_1}|^2 + |F_{X_2}|^2 + |F_{Z_{ij}}|^2 + |F_{Z'_{ij}}|^2$. The nontrivial minimizations are w.r.t. σ and V_{ij}. These respectively correspond to

$$\langle \sigma \rangle^2 = (\alpha_1^2 + \alpha_2^2)^{-1} \Big(\alpha_1^2 \sum_i \langle V_i \rangle^2 + \alpha_2^2 \mu^2 \Big) \tag{3a}$$

and

$$2\alpha_3^2 \langle V_k \rangle^3 + 3\mu' \alpha_3^2 \langle V_k \rangle^2 + \Big(\alpha_3^2 \mu'^2 - \alpha_1^2 \alpha_2^2 (\alpha_1^2 + \alpha_2^2)^{-1} \mu^2 + [\alpha_1^2 \alpha_2^2 (\alpha_1^2 + \alpha_2^2)^{-1} - \tfrac{1}{5}\alpha_3^2] \sum_i \langle V_i \rangle^2 \Big) \langle V_k \rangle - \tfrac{1}{5}\alpha_3^2 \sum_i \langle V_i \rangle^2 (\tfrac{3}{2}\mu' + \langle V_i \rangle) = 0 . \tag{3b}$$

The cubic equation (3b) for $\langle V_k \rangle$ together with the condition (3a) on $\Sigma_i \langle V_i \rangle^2$ and the coupled constraints vis-a-vis $\langle Z \rangle_k$, as given in (2d), puts strong restrictions on these tree level VEVs thereby limiting the possible different subgroups of SO(10) that can be left invariant. For a suitable range of the parameters $\alpha_{1,2,3}$, μ, μ', $\bar{\mu}$, μ_1, μ_2 and particularly the choices

$$\alpha_3^2 \mu'^2 = [\alpha_1^2 \alpha_2^2 / (\alpha_1^2 + \alpha_2^2)] \mu^2 ,$$

$$\mu_1^2 = (\alpha_2^2/\alpha_1^2)\mu^2 ,$$

the global minimum of V_{pot} can be shown to correspond, with $\Delta_{ij} \equiv \mathrm{diag}(2, 2, 2, 2, 2, 2, -3, -3, -3, -3)$, to the following VEV equalities:

$$\langle V_{ij} \rangle = \tfrac{3}{2}\mu'(1 + 30\mu'^2/\mu^2)^{-1} \Delta_{ij} , \tag{4a}$$

$$\langle Z_{ij} \rangle = \tfrac{3}{2}\alpha_1 \alpha_3^{-1} \langle X_1 \rangle (1 - 15\mu'^2/\mu^2)^{-1} \Delta_{ij} , \tag{4b}$$

$$\langle Y_{ij}\rangle = \langle X_4\rangle\Delta_{ij}/\sqrt{60} = -\alpha_5\alpha_6^{-1}\langle X_1\rangle\Delta_{ij}/\sqrt{60}\,, \qquad (4c)$$

$$\langle\chi\rangle^2 = \bar{\mu}^2 - 30\alpha_1\alpha_5\lambda_3(\alpha_3\alpha_6\lambda_2)^{-1}\langle X_1\rangle^2 \times (1 - 15\mu'^2/\mu^2)^{-1}\,. \qquad (4d)$$

A clarification is necessary for (4c). Although $\langle F_{H_i}\rangle$ vanishes through $\langle H_i\rangle$ being zero, the relation $3\lambda_2 = \sqrt{60}\,\lambda_3$, leading to the vanishing of $\lambda_2\langle Y_{7\,-10}\rangle + \lambda_3\langle X_4\rangle$, is the aforesaid fine-tuned choice to ensure the second hierarchy. Finally, it is the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ which is left invariant by the above VEVs. Further, $\langle F_{X_1}\rangle, \langle F_{X_2}\rangle, \langle F_{Z_{ij}}\rangle$ and $\langle F_{Z'_{ij}}\rangle$ are of order of M^2_{SUSY}.

The above minimization of V_{pot} has left $\langle X_{1,2,4}\rangle$ and relatedly $\langle Z_{ij}\rangle$, $\langle Y_{ij}\rangle$ as well as $\langle\chi_\alpha\rangle$, $\langle\bar{\chi}'^{\alpha}\rangle$, undetermined at the tree level. Their dominant terms, all mutually proportional, become subject – under loop effects – to Witten's [14] backwards hierarchy mechanism and generate a large VEV $\langle X\rangle \sim M_{GUT}$. That is true provided there is a total negative contribution from the corresponding superfields to the Coleman–Weinberg (mass)^4 coefficient [16] $G \equiv \mathrm{Tr}[3m_V^4 - 2(m_F^\dagger m_F)^2 + m_S^4]$, the subscripts V, F and S referring respectively to the masses of the vector, spin-half fermion and scalar components. Recall that the dominant term in the one-loop part of the scalar potential is

$$\Delta V = G(64\pi^2)^{-1}\ln(\langle X\rangle^2/\mu_0^2)\,, \qquad (5)$$

where $\mu_0^2 \sim \mu^2$. It is easy to check the sign of G by use of the formula ‡7 [17]

$$G = 2\langle\bar{F}^{abc}\rangle\langle F_c\rangle\langle F_{abd}\rangle\langle\bar{F}^d\rangle - 8e^2\langle F_c\rangle(T^2)^c_d\langle\bar{F}^d\rangle\,. \qquad (6)$$

In (6) $F_{abc} = (\partial^3 W/\partial\hat{\phi}^a\partial\hat{\phi}^b\partial\hat{\phi}^c)_{\hat{\phi}=\phi}$, $F_a = (\partial W/\partial\hat{\phi}^a)_{\hat{\phi}=\phi}$, $\bar{F}^{abc} = (F_{abc})^*$, $\bar{F}^a = (F_a)^*$, and e is the gauge coupling. Let us define the following dimensionless combination of massparameters $a \equiv [135\mu'^2\mu^{-2}(1 + 30\mu'^2\mu^{-2})^{-2} - 1]^2$ and $b = (1 - 60\mu'^2\mu^{-2})^2(1 + 30\mu'^2\mu^{-2})^{-4}$. Now direct evaluation yields

$$G = 2\mu^2\alpha_1^2\alpha_2^2(\alpha_1^2 + \alpha_2^2)^{-1}\{2\mu^2\alpha_2^2(\alpha_1^2 + \alpha_2^2)^{-1} \times [a\alpha_5^2 + 4\alpha_2^2(22a + 189b)] + 15\mu'^2 b(9\alpha_8^2 - 216e^2)\}\,. \qquad (7)$$

Thus, for sufficiently large values of e and μ'/μ, specifically $e^2 > \frac{9}{216}\alpha_8^2$ and $\mu'^2/\mu^2 > \frac{2}{15}\alpha_2^2[(\alpha_5^2 + 8\alpha_2^2)ab^{-1} + 756\alpha_2^2](\alpha_1^2 + \alpha_2^2)^{-1}(216e^2 - 9\alpha_8^2)^{-1}$, $G < 0$ and the occurrence of Witten's backwards hierarchy mechanism is guaranteed.

Once M_{GUT} is generated by a sliding $\langle X\rangle$, the VEVs $\langle X_{1,2,4}\rangle$ are of that order and so are $\langle Z_{ij}\rangle$, $\langle Y_{ij}\rangle$, $\langle\chi_\alpha\rangle$ and $\langle\bar{\chi}'^{\alpha}\rangle$. Thus the symmetry breakdown $SO(10) \to SU(3) \times SU(2)_L \times U(1)_Y$ for this model takes place at M_{GUT}. In contrast, $\langle V_{ij}\rangle$ and $\langle\sigma\rangle$ get fixed around $\mu \sim \mu' \sim M_{SUSY}$. Particles, corresponding to all superfields apart from $\hat{A}$, $\hat{\psi}$, $\hat{S}'$ and H, acquire masses $\sim M_{GUT}$. Those contained in $\hat{A}$, $\hat{\sigma}$ and $\hat{S}'$ have masses $\sim\mu$. The fifteen fermions of $\hat{\psi}_\alpha$($\hat{\psi}_{16}$ is superheavy) and the colourless scalars of $\hat{H}_i$ (the coloured ones are superheavy via the second hierarchy) remain massless at the tree level.

Loop diagrams induce a negative mass-squared term for H_i in the scalar potential with a coefficient $\sim\mu^4/M^2_{GUT}$. Consequently, the colourless doublets of H_i are led to acquire VEVs $\sim\mu^2/M_{GUT}$, thus radiatively driving the electroweak symmetry breaking scale, as desired. This can be seen by explicit computation of the contributions to the squared mass of H from the possible supergraphs of fig. 1. The result is:

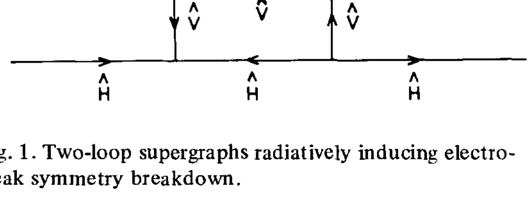

Fig. 1. Two-loop supergraphs radiatively inducing electroweak symmetry breakdown.

‡7 Such is the case only in theories obeying the decoupling [11] condition. The latter stipulates that among the participant supermultiplets only those with masses of the order of the VEV ($\langle X\rangle$) sliding to a very large value feel SUSY-breaking, i.e. develop mass-splits $\sim M_{SUSY}$. One can explicitly check from our mass-matrices that this is indeed true in our model.

$$\delta m^2 = -\tfrac{1}{3}\lambda_1^2\lambda_2^2(16\pi^2)^{-2}$$

$$\times \ln(M_\sigma^2/M_A^2)\,|\alpha_1\langle F_{X_1}\rangle - \alpha_2\langle F_{X_2}\rangle|^2 M_\sigma^{-2}\,, \qquad (8)$$

where $M_\sigma \sim M_{GUT}$, $M_A \sim M_{SUSY}$ and $F_{X_1} \sim F_{X_2} \sim (M_{SUSY})^2$. The negative value of the RHS is the important feature. Despite positive contributions to the coefficient of H^2 from other supergraphs, this negative term can be made dominant for a suitable range of parameters – thus generating the M_W scale.

We have checked that perturbative unification is not destroyed in our model by the contributions to the renormalization group equations from the extra particles brought in by supersymmetry. For three quark/lepton families and $\alpha_{QCD}(M_W) \sim 0.18$, we find that $M_{GUT} \sim 10^{16}$ GeV and $\alpha_{GUT} \sim 0.05$; further, the sine squared of the Weinberg angle θ_W can be maintained within the experimental limits. Of course, our purpose here has not been to present a complete supergrand unified theory. Untouched are several issues tackling of which will require considerable enlargement of the scope of the present work. We may mention among them the strong *CP* problem as well as the attainment of realistic proton decay rates. A third very important question is the extension of this model to include local SUSY since a globally supersymmetric geometric hierarchy *sans* supergravity is not completely realistic [17]; work is in progress towards constructing an SO(10) local SUSY–GUT with naturally ultralight Dirac neutrinos.

Another point in connection with this type of theory is the status of the neutrinos of the heavier families. It has been argued [5] that, if the effect of the generation hierarchy on the ratio C/B is not too strong, the three neutrinos should be spaced in mass as $m(\nu_e) : m(\nu_\mu) : m(\nu_\tau) = m_u : m_c : m_t$ leading to $m(\nu_\mu) = O$ (10 KeV) and $m(\nu_\tau) = O$ (100 KeV). The $\nu_{\mu,\tau}$ then have to be unstable, given the existing cosmological limits on stable neutrino masses. The most likely decay modes are $\nu_\mu \to \nu_e f$ and $\nu_\tau \to \nu_e f$ where f is a (generic) Goldstone boson arising from the spontaneous breakdown of a global symmetry, e.g. family [18] or $U_c(1)$. The relevant coupling is $\sim F^{-1}\bar{\nu}_e\gamma^\lambda\frac{1}{2}(1-\gamma_5)\,\nu_{\mu,\tau}\partial_\lambda f$ where the denominator F is of the order of the global symmetry breaking scale. If F is less than $10^{10}h^2$ GeV, h being the Hubble constant in units of 100 kms^{-1} Mpc^{-1}, the cosmological problem is evaded [19,20]. Further, any conflict with the experimental upper bound on $\Gamma(\mu \to ef)$ – which gets related to F by an $SU(2)_L$ rotation on the flavor changing current – can be avoided [19] by making the choice $F > 10^9$ GeV. A possible conflict can arise [20] for a O (10 KeV) $m(\nu_\mu)$ from the more stringent experimental limit on $\Gamma(K^+ \to \pi^+ f)$. However, to relate the quark and lepton processes $s \to df$ and $\nu_\mu \to \nu_e f$, one needs a nontrivial rotation under the Pati–Salam SU(4) subgroup of SO(10). The renormalization effects in evolving from M_{GUT} where $SU(4)_{PS}$ is broken in our model down to low energies could easily lead to a factor of three between the two F's effective in the two processes; that is what is needed to avoid [20] any such conflict for a O (10 KeV) $m(\nu_\mu)$. The point is that there is a permitted narrow window for F around 10^{10} GeV. One could also evade the constraints from charged particle decays by considering a generation symmetry involving only the singlet $\hat{S}$ superfields. Though we have not given any detailed account of such horizontal symmetries and their breaking, nothing prevents such scenarios [18] from occurring in our model.

To sum up, we have shown how – in a globally supersymmetric SO(10) GUT model featuring geometric hierarchy – naturally ultralight Dirac neutrinos emerge from supersymmetry in a believable manner. We believe that the above model serves an important purpose as a first step. By strengthening the link between an observable neutrino Dirac mass and supersymmetry, it brings the latter close to experiment.

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