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Stability of solitary kinetic Alfvén waves and ion-acoustic waves in a nonthermal plasma

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The KdV-ZK (Korteweg–de Vries–Zakharov–Kuznetsov) equations for kinetic Alfvén and ion-acoustic waves in a nonthermal magnetized plasma have been derived. The coefficient of the nonlinear term of this equation for an ion-acoustic wave can vanish along a curve in a two-dimensional parameter space. In this case, two coupled equations constituting a modified KdV-ZK equation describing nonlinear behavior of an ion-acoustic wave are derived. The solitary wave solutions of all these equations are obtained and their stabilities are investigated by the Rowlands–Infeld method. It is found that the kinetic Alfvén solitons are stable. The instability conditions and the maximum growth rate of the instability for ion-acoustic solitons are determined.
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I. INTRODUCTION

The existence and stability of solitary waves in a plasma with external magnetic field have been investigated by Ladke and Spatschek,[1] Infeld,[2] and Das and Verheest,[3] in which electrons have been taken as isothermal. Recently, Cairns *et al.*[4] have shown that it is possible to obtain both compressive and rarefactive solitions when considering electrons as nonthermal with an excess of energetic particles. This investigation was motivated by the recent observation of solitary structures with density depletion made by Freja Satellite (Dovner *et al.*).[5] The effects of the external magnetic field have been included in these investigations by Cairns *et al.*[6,7] and Mamun and Cairns.[8] In the latter paper, the authors have studied the stability of ion-acoustic waves in a magnetized nonthermal plasma. Recently, Bandyopadhyay and Das[9] have extended this paper of Mamun and Cairns[8] in the following two directions: (i) instead of considering ions as cold, the ion temperature has been included and (ii) the case has been considered when the coefficient of the nonlinear term in the KdV-ZK (Korteweg–de Vries–Zakharov–Kuznehsor) equation derived in the first case vanishes. In all the abovementioned investigations for magnetized nonthermal plasma only electrostatic perturbations have been considered. If magnetic field perturbations are also taken into account, then in addition to the ion-acoustic mode the Alfvén mode will also appear, which becomes dispersive, if kinetic effects like ion-drift velocity are included in the ion continuity equation and in the ion fluid equation of motion. Due to the inclusion of kinetic effects, the Alfvén mode is termed the kinetic Alfvén mode. Solitary kinetic Alfvén waves have been investigated by many authors.[10–16] In the present investigation we extend the analysis of Mamun and Cairns[8] by including magnetic field perturbations and by including kinetic effects mentioned above.

The governing equations from which we start consist of the ion continuity equation, the parallel component of ion fluid equation of motion, the distribution for nonthermal electrons, the electron continuity equation, the quasineutrality condition, Ampere's law in the parallel direction, and the adiabatic pressure law for ions in which we keep both **E×B** and polarization drift terms in the expression for ion drift velocity and use two potentials to represent the electric field intensity. The linear dispersion relation obtained from these governing equations is

$$\left[1-\frac{\omega^2(1-\beta')}{k_z^2c_s^2}\left\{1+\frac{5}{3}\sigma(1-\beta')\right\}^{-1}\right]\left[1-\frac{\omega^2}{k_z^2V_{\mathrm{A}}^2}\right] =\frac{\omega^2k_\perp^2}{k_z^2\omega_{ci}^2}\left\{1+\frac{5}{3}\sigma(1-\beta')\right\}^{-1}, \tag{1}$$

where c_s is the ion-acoustic speed, V_{A} is the Alfvén velocity, ω_{ci} is the ion-cyclotron frequency, k_z and $k_\perp$ are the parallel and perpendicular components of the wave vector, ω is the frequency of the wave, σ is the ratio of ion and electron temperatures, and β' is a parameter of the nonthermal electron distribution. The dispersion relation (1) shows a coupling between ion-acoustic and kinetic Alfvén waves, and this dispersion relation reduces to the dispersion relation of Hasegawa,[10] if electrons are considered isothermal (i.e., $\beta'=0$) and the ion temperature is set equal to zero (i.e., $\sigma=0$).

From the above-mentioned governing equations, three-dimensional KdV equations are derived for both the ion-acoustic wave and kinetic Alfvén wave. The equations are found to admit kinetic Alfvén soliton and ion-acoustic soliton solutions propagating obliquely to the external magnetic field. The stability of these solitons is investigated by the small-k perturbation expansion method of Rowlands and

Infeld.[17–19] It is found that the coefficient of the nonlinear term of the three-dimensional KdV equation for the ion-acoustic wave vanishes on a curve in the $\beta'\sigma$-parameter plane. In this case, two coupled equations constituting a modified KdV-ZK equation describing ion-acoustic waves are derived, which are found to admit soliton solutions propagating obliquely to the external magnetic field. The stability of these solitons is also investigated by the same Rowlands–Infeld method. The paper is organized as follows. In Sec. II the basic equations are given. In Sec. III the KdV-ZK equations for both kinetic Alfvén and ion-acoustic waves are derived. In Sec. IV two coupled equations constituting a modified KdV-ZK equation are derived for the case when the coefficient of the nonlinear term of the KdV-ZK equation derived in Sec. III vanishes. The solitary wave solutions are obtained in Sec. V. The stability of these solitons is investigated in Sec. VI. Finally, brief conclusions are given in Sec. VII.

II. GOVERNING EQUATIONS

We consider a plasma consisting of warm adiabatic ions and nonthermal electrons immersed in an uniform external magnetic field B_0 directed along the z axis. We assume that the ratio of the particle pressure to the magnetic pressure is small and the characteristic frequency is much smaller than the ion-cyclotron frequency. The nonlinear behavior of kinetic Alfvén waves and ion-acoustic waves may be described by the following set of equations, which consist of the ion continuity equation, the parallel component of ion fluid equation of motion, the electron continuity equation, the quasineutrality condition, Ampere's law in the parallel direction, and the adiabatic pressure law for ions. These equations are to be supplemented by the distribution of nonthermal electrons. In the expression for ion-drift velocity both $\mathbf{E}\times\mathbf{B}$ and polarization drift terms are retained, and the electric field intensity is represented by two potentials according to Kadomtsev,[21]

$$\frac{\partial n_i}{\partial t}+\nabla_\perp\cdot(n_i\mathbf{v}_{id})+\frac{\partial}{\partial z}(n_i v_{iz})=0, \tag{2}$$

$$\frac{\partial v_{iz}}{\partial t}+(\mathbf{v}_{id}\cdot\nabla_\perp)v_{iz}+v_{iz}\frac{\partial v_{iz}}{\partial z}=E_z-\frac{\sigma}{n_i}\frac{\partial P}{\partial z}, \tag{3}$$

$$\frac{\partial n_e}{\partial t}+\frac{\partial}{\partial z}(n_e v_{ez})=0, \tag{4}$$

$$n_i=n_e, \tag{5}$$

$$\frac{\partial}{\partial z}\nabla_\perp^2(\varphi-\psi)=\frac{1}{2}\beta\frac{\partial j_z}{\partial t}, \tag{6}$$

$$P=n_i^{\gamma'}, \tag{7}$$

where the ion drift velocity and parallel current density j_z are given by

$$\mathbf{v}_{id}=\mathbf{E}_\perp\times\hat{z}+\frac{d\mathbf{E}_\perp}{dt}, \tag{8}$$

$$j_z=n_i v_{iz}-n_e v_{ez}. \tag{9}$$

The electric field intensity components are determined from the two potentials φ and ψ according to

$$\mathbf{E}_\perp=-\nabla_\perp\varphi,\quad E_z=-\frac{\partial\psi}{\partial z} \tag{10}$$

(Kadomtsev[21]), where $\perp$ and z indicate components perpendicular and parallel to the ambient magnetic field. In the above, n_e and n_i are the electron and ion number densities, v_{iz} and v_{ez} are the parallel ion and electron fluid velocities, E_z and $\mathbf{E}_\perp$ are the parallel and perpendicular components of electric field intensity vector, β is the ratio of particle and magnetic pressure, $\gamma'(=5/3)$ is the ratio of two specific heats. The above equations have been written in dimensionless form by normalizing the space coordinates (x,y,z), time (t), velocities $(v_{iz},v_{ez},\mathbf{v}_{id})$, pressure (P), electric potentials $(\varphi,\ \psi)$, parallel electric current density (j_z), electron and ion number densities (n_i,n_e) by ρ_s, ω_{ci}^{-1}, c_s, $n_0K_BT_i$, K_BT_e/e, en_0c_s, and n_0, respectively, where $\rho_s=c_s/\omega_{ci}$ is the equivalent ion gyroradius, K_B is the Boltzmann's constant, T_e and T_i are the electron and ion temperature, n_0 is the unperturbed number density of electrons and ions, and e is the electronic charge.

Since the electrons are assumed to be nonthermally distributed, their velocity distribution function is taken as[4–9]

$$f_e(v)=\frac{n_0}{(1+3\alpha')(2\pi v_e^2)^{1/2}}\left[1+\alpha'\left(\frac{v^2}{v_e^2}-2\psi\right)^2\right]\times\exp\left[-\frac{1}{2}\left(\frac{v^2}{v_e^2}-2\psi\right)\right], \tag{11a}$$

where $v_e=(K_BT_e/m_e)^{1/2}$. Consequently, the electron number density appearing in the above equations normalized to n_0 is given by

$$n_e=(1-\beta'\psi+\beta'\psi^2)e^{\psi}, \tag{11b}$$

where $\beta'=4\alpha'/(1+3\alpha')$. The parameter β' determines the proportion of fast electrons.

III. DERIVATION OF THE KdV-ZK EQUATION

To derive the KdV-ZK equation, we make the following stretchings of space coordinates and time:

$$\zeta=\epsilon^{1/2}(z-Vt),\quad \xi=\epsilon^{1/2}x,\quad \eta=\epsilon^{1/2}y,\quad \tau=\epsilon^{3/2}t, \tag{12}$$

where ϵ is a small parameter measuring the weakness of dispersion and V is a constant. We also make the following perturbation expansion of the field quantities;

$$\begin{aligned}f&=f^{(0)}+\epsilon f^{(1)}+\epsilon^2 f^{(2)}+\epsilon^3 f^{(3)}+\cdots,\\ \varphi&=\varphi^{(1)}+\epsilon\varphi^{(2)}+\epsilon^2\varphi^{(3)}+\cdots,\end{aligned} \tag{13}$$

where f stands for n, ψ, v_{iz}, v_{ez} and $n^{(0)}=1$, $\psi^{(0)}=v_{iz}^{(0)}=v_{ez}^{(0)}=0$, and $n=n_i=n_e$, which is according to the quasineutrality condition.

Substituting the stretchings (12) and the perturbation expansions (13) in the governing equations (2)–(4), (6), and (11) and then equating coefficients of different powers of ϵ on both sides of each equation, we get a sequence of equations for the perturbed quantities.

Applying the same procedure as followed by Das and Verheest[3] in their paper, we get from the above-mentioned sequence of equations the following two coupled equations, constituting the KdV-ZK equation for kinetic Alfvén wave and ion-acoustic wave:

$$\nabla_{\perp\xi}^2\varphi^{(1)}=D_I(V)n^{(1)}, \tag{14}$$

$$-2V[D_I(V)+D_A(V)]\frac{\partial n^{(1)}}{\partial\tau}+V^2\Bigg[D_I(V)+D_A(V)\times\left\{\frac{5\sigma}{9V^2}+\frac{1}{(1-\beta')^2}\left(1-\frac{5\sigma}{3V^2}\right)-3\right\}\Bigg]n^{(1)}\frac{\partial n^{(1)}}{\partial\zeta}+V^2[1-D_A(V)]\left[\frac{\partial n^{(1)}}{\partial\xi}\frac{\partial^2\varphi^{(1)}}{\partial\xi\partial\zeta}+\frac{\partial n^{(1)}}{\partial\eta}\frac{\partial^2\varphi^{(1)}}{\partial\eta\partial\zeta}\right]+\frac{V^2}{1-\beta'}\frac{\partial}{\partial\zeta}(\nabla_{\perp\xi}^2n^{(1)})=0, \tag{15}$$

where

$$\nabla_{\perp\xi}^2=\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\eta^2},$$

$$D_I(V)=1-\frac{1}{V^2(1-\beta')}\left\{1+\frac{5}{3}\sigma(1-\beta')\right\}, \tag{16}$$

$$D_A(V)=\frac{1}{2}\beta V^2-1.$$

The constant V is determined from the linear dispersion relation

$$D_I(V)D_A(V)=0. \tag{17}$$

Equation (17) gives two values of V,

$$V=[(5/3)\sigma+(1-\beta')^{-1}]^{1/2},\quad V=(2/\beta)^{1/2}, \tag{18}$$

and these are obtained, respectively, from $D_I(V)=0$ and $D_A(V)=0$. These values of V correspond, respectively, to the ion acoustic wave and the kinetic Alfvén wave.

The two coupled equations (14) and (15) constitute the KdV-ZK equation for the kinetic Alfvén wave or ion-acoustic wave according to whether the V appearing in the equations is determined from $D_A(V)=0$ or $D_I(V)=0$. Therefore, to describe the nonlinear behavior of kinetic Alfvén waves, we have the following two coupled equations:

$$\frac{\partial n^{(1)}}{\partial\tau}-ABn^{(1)}\frac{\partial n^{(1)}}{\partial\zeta}-AC\frac{\partial}{\partial\zeta}(\nabla_{\perp\xi}^2n^{(1)})-AD\left(\frac{\partial n^{(1)}}{\partial\xi}\frac{\partial^2\varphi^{(1)}}{\partial\xi\partial\zeta}+\frac{\partial n^{(1)}}{\partial\eta}\frac{\partial^2\varphi^{(1)}}{\partial\eta\partial\zeta}\right)=0, \tag{19}$$

$$n^{(1)}=AE\nabla_{\perp\xi}^2\varphi^{(1)}, \tag{20}$$

where

$$A=1/[2VD_I(V)],\quad B=V^2D_I(V),\quad C=V^2/(1-\beta'),\quad D=V^2,\quad E=2V, \tag{21}$$

and V is given by the second equation of (18).

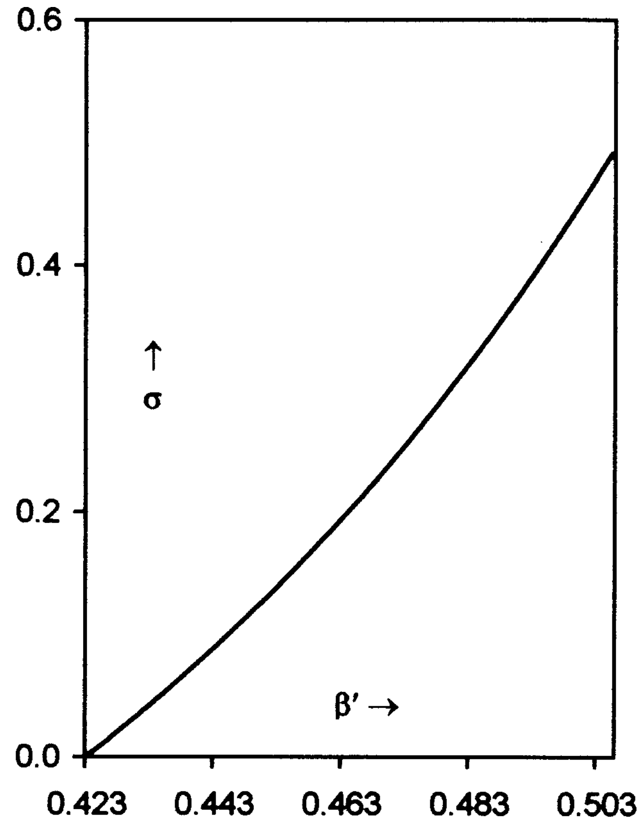

FIG. 1. σ plotted against β' when $B'=0$.

Now, since for the ion-acoustic wave $D_I(V)=0$, we have $\varphi^{(1)}\equiv0$ according to Eq. (14), and therefore (15) assumes the following form:

$$\frac{\partial n^{(1)}}{\partial\tau}-A'B'n^{(1)}\frac{\partial n^{(1)}}{\partial\zeta}-A'C'\frac{\partial}{\partial\zeta}(\nabla_{\perp\xi}^2n^{(1)})=0, \tag{22}$$

where

$$A'=\frac{1}{2VD_A(V)},\quad B'=D_A(V)\left\{\frac{1-3(1-\beta')^2}{(1-\beta')^3}-\frac{40}{9}\sigma\right\},\quad C'=\frac{V^2}{1-\beta'}, \tag{23}$$

and V is given by the first of the two equations in (18).

From the above-mentioned expression of B', we find that the coefficient of the nonlinear term of Eq. (22) vanishes along a particular curve in the $\beta'\sigma$ plane (Fig. 1). Therefore, for the values of β' and σ lying on this curve, it is not possible to study the nonlinear behavior of the ion-acoustic waves. In Sec. IV we shall derive the modified KdV-ZK equation to discuss the nonlinear behavior of the ion-acoustic waves when $B'=0$.

IV. DERIVATION OF THE MODIFIED KdV-ZK EQUATION

To derive the modified KdV-ZK equation we make the same stretchings of space coordinates and time as given in Eq. (12), but we shall use the following perturbation expansion of dependent variables:

$$f=f^{(0)}+\epsilon^{1/2}f^{(1)}+\epsilon f^{(2)}+\epsilon^{3/2}f^{(3)}+\cdots,$$
$$\varphi=\varphi^{(1)}+\epsilon^{1/2}\varphi^{(2)}+\epsilon\varphi^{(3)}+\cdots, \tag{24}$$

where f stands for the same variables as stated after Eq. (13) and $f^{(0)}$'s also have the same values stated there. Substituting the stretchings (12) and the perturbation expansions (24) into the governing equations (2)–(4), (6), and (11) and then equating coefficients of different powers of ϵ on both sides of each equation, we get a sequence of equations for the perturbed quantities. From the first few members of this sequence of equations we get

$$\begin{aligned}
&v_{iz}^{(1)}=Vn^{(1)}, \quad \psi^{(1)}=\left(V^2-\frac{5\sigma}{3}\right)n^{(1)},\\
&v_{ez}^{(1)}=Vn^{(1)}, \quad \varphi^{(1)}\equiv 0,\\
&v_{iz}^{(1)}=v_{ez}^{(1)}=Vn^{(2)}-Vn^{(1)^2},\\
&\psi^{(2)}=(1-\beta')^{-1}n^{(2)}-\tfrac{1}{2}(1-\beta')^{-3}n^{(1)^2},
\end{aligned} \tag{25}$$

together with the linear dispersion relation $D_I(V)=0$, which determines the constant V.

Equations (3), (11), and (4) at the orders $\epsilon^2,\epsilon^2,\epsilon^{3/2}$, respectively, are now solved for $\partial v_{iz}^{(3)}/\partial\zeta$, $\partial v_{ez}^{(3)}/\partial\zeta$, and $\partial\psi^{(3)}/\partial\zeta$ to express them in terms of $\partial n^{(3)}/\partial\zeta$ and first- and second-order perturbed quantities. Substituting these solutions in Eqs. (2) and (6), each at the order ϵ^2, we get the following two equations, in which the terms $\partial n^{(3)}/\partial\zeta$ and $(\partial/\partial\zeta)(n^{(1)}n^{(2)})$ do not appear in either of the two equations, since their coefficients in each equation vanish due to the linear dispersion relation $D_I(V)=0$ and the critical condition $B'=0$:

$$\frac{\partial n^{(1)}}{\partial\tau}-A'B''(n^{(1)})^2\frac{\partial n^{(1)}}{\partial\zeta}-A'C'\frac{\partial}{\partial\zeta}(\nabla_{\perp\xi}^2 n^{(1)})+\frac{1-D_A(V)}{2D_A(V)}\left(\frac{\partial n^{(1)}}{\partial\xi}\frac{\partial\phi^{(2)}}{\partial\eta}-\frac{\partial n^{(1)}}{\partial\eta}\frac{\partial\phi^{(2)}}{\partial\xi}\right)=0, \tag{26}$$

$$\frac{\partial}{\partial\zeta}\left[\nabla_{\perp\xi}^2\left(A'C'n^{(1)}+\frac{1}{2}V\phi^{(2)}\right)\right]-\frac{1+D_A(V)}{2D_A(V)}\left(\frac{\partial n^{(1)}}{\partial\xi}\frac{\partial\phi^{(2)}}{\partial\eta}-\frac{\partial n^{(1)}}{\partial\eta}\frac{\partial\phi^{(2)}}{\partial\xi}\right)=0, \tag{27}$$

where

$$B''=-\frac{1}{(1-\beta)^4}D_A(V)\left[(1-\beta')^4\left\{\frac{10\sigma}{27}-6V^2+\frac{3}{2}(1-\beta')\left(3V^2-\frac{5\sigma}{9}\right)^2\right\}-\frac{1}{2}(1+3\beta')\right]. \tag{28}$$

The coupled equations (26) and (27) constitute the modified KdV-ZK equation for ion-acoustic waves when $B'=0$.

V. SOLITARY WAVE SOLUTIONS

For solitary waves propagating in a direction making an angle α with the ζ axis, i.e., with the direction of the external magnetic field, we make the following change of variables:

$$\begin{aligned}
&\xi'=\xi\cos\alpha-\zeta\sin\alpha, \quad \eta'=\eta,\\
&\zeta'=\xi\sin\alpha+\zeta\cos\alpha, \quad \tau'=\tau, \quad z=\zeta'-U\tau'.
\end{aligned} \tag{29}$$

Under this change of variables, Eqs. (19), (20), (22), (26), and (27) assume the following form, in which we drop the primes on the variables (but not on the coefficients) to simplify the notations:

$$\begin{aligned}
&-U\frac{\partial n^{(1)}}{\partial z}+\frac{\partial n^{(1)}}{\partial\tau}+a_1n^{(1)}\frac{\partial n^{(1)}}{\partial z}+a_2n^{(1)}\frac{\partial n^{(1)}}{\partial\xi}+a_3\frac{\partial^3 n^{(1)}}{\partial z^3}+a_4\frac{\partial^3 n^{(1)}}{\partial\xi\partial z^2}+a_5\frac{\partial^3 n^{(1)}}{\partial\xi^2\partial z}+a_6\frac{\partial^3 n^{(1)}}{\partial\eta^2\partial z}+a_7\frac{\partial^3 n^{(1)}}{\partial\xi^3}+a_8\frac{\partial^3 n^{(1)}}{\partial\xi\partial\eta^2}\\
&+b_1\frac{\partial n^{(1)}}{\partial z}\frac{\partial^2\varphi^{(1)}}{\partial z^2}+b_2\frac{\partial n^{(1)}}{\partial\xi}\frac{\partial^2\varphi^{(1)}}{\partial z^2}+b_3\frac{\partial n^{(1)}}{\partial z}\frac{\partial^2\varphi^{(1)}}{\partial\xi\partial z}+b_4\frac{\partial n^{(1)}}{\partial z}\frac{\partial^2\varphi^{(1)}}{\partial\xi^2}+b_5\frac{\partial n^{(1)}}{\partial\xi}\frac{\partial^2\varphi^{(1)}}{\partial\xi\partial z}\\
&+b_6\frac{\partial n^{(1)}}{\partial\eta}\frac{\partial^2\varphi^{(1)}}{\partial\eta\partial z}+b_7\frac{\partial n^{(1)}}{\partial\xi}\frac{\partial^2\varphi^{(1)}}{\partial\xi^2}+b_8\frac{\partial n^{(1)}}{\partial\eta}\frac{\partial^2\varphi^{(1)}}{\partial\xi\partial\eta}=0,
\end{aligned} \tag{30}$$

$$n^{(1)}=c_1\frac{\partial^2\varphi^{(1)}}{\partial z^2}+c_2\frac{\partial^2\varphi^{(1)}}{\partial\xi\partial z}+c_3\frac{\partial^2\varphi^{(1)}}{\partial\xi^2}+c_4\frac{\partial^2\varphi^{(1)}}{\partial\eta^2}, \tag{31}$$

$$\begin{aligned}
&-U\frac{\partial n^{(1)}}{\partial z}+\frac{\partial n^{(1)}}{\partial\tau}+d_1n^{(1)}\frac{\partial n^{(1)}}{\partial z}+d_2n^{(1)}\frac{\partial n^{(1)}}{\partial\xi}\\
&+d_3\frac{\partial^3 n^{(1)}}{\partial z^3}+d_4\frac{\partial^3 n^{(1)}}{\partial\xi\partial z^2}+d_5\frac{\partial^3 n^{(1)}}{\partial\xi^2\partial z}+d_6\frac{\partial^3 n^{(1)}}{\partial\eta^2\partial z}\\
&+d_7\frac{\partial^3 n^{(1)}}{\partial\xi^3}+d_8\frac{\partial^3 n^{(1)}}{\partial\xi\partial\eta^2}=0,
\end{aligned} \tag{32}$$

$$\begin{aligned}
&-U\frac{\partial n^{(1)}}{\partial z}+\frac{\partial n^{(1)}}{\partial\tau}+\bar{d}_1n^{(1)^2}\frac{\partial n^{(1)}}{\partial z}+\bar{d}_2n^{(1)^2}\frac{\partial n^{(1)}}{\partial\xi}\\
&+d_3\frac{\partial^3 n^{(1)}}{\partial z^3}+d_4\frac{\partial^3 n^{(1)}}{\partial\xi\partial z^2}+d_5\frac{\partial^3 n^{(1)}}{\partial\xi^2\partial z}+d_6\frac{\partial^3 n^{(1)}}{\partial\eta^2\partial z}\\
&+d_7\frac{\partial^3 n^{(1)}}{\partial\xi^3}+d_8\frac{\partial^3 n^{(1)}}{\partial\xi\partial\eta^2}+\frac{1-D_A(V)}{2D_A(V)}\left[\left(\frac{\partial n^{(1)}}{\partial\xi}\frac{\partial\varphi^{(2)}}{\partial\eta}-\frac{\partial n^{(1)}}{\partial\eta}\frac{\partial\varphi^{(2)}}{\partial\xi}\right)\cos\alpha+\left(\frac{\partial n^{(1)}}{\partial z}\frac{\partial\varphi^{(2)}}{\partial\eta}-\frac{\partial n^{(1)}}{\partial\eta}\frac{\partial\varphi^{(2)}}{\partial z}\right)\sin\alpha\right]=0,
\end{aligned} \tag{33}$$

$$\frac{\partial^3}{\partial z^3}(d_3 n^{(1)}+\bar{d}_3\varphi^{(2)})+\frac{\partial^3}{\partial\xi\partial z^2}(d_4 n^{(1)}+\bar{d}_4\varphi^{(2)})$$
$$+\frac{\partial^3}{\partial\xi^2\partial z}(d_5 n^{(1)}+\bar{d}_5\varphi^{(2)})+\frac{\partial^3}{\partial\eta^2\partial z}(d_6 n^{(1)}+\bar{d}_6\varphi^{(2)})$$
$$+\frac{\partial^3}{\partial\xi^3}(d_7 n^{(1)}+\bar{d}_7\varphi^{(2)})+\frac{\partial^3}{\partial\xi\partial\eta^2}(d_8 n^{(1)}+\bar{d}_8\varphi^{(2)})$$
$$+\frac{1+D_{\mathrm{A}}(V)}{2D_{\mathrm{A}}(V)}\left[\left(\frac{\partial n^{(1)}}{\partial\xi}\frac{\partial\varphi^{(2)}}{\partial\eta}-\frac{\partial n^{(1)}}{\partial\eta}\frac{\partial\varphi^{(2)}}{\partial\xi}\right)\cos\alpha\right.$$
$$\left.+\left(\frac{\partial n^{(1)}}{\partial z}\frac{\partial\varphi^{(2)}}{\partial\eta}-\frac{\partial n^{(1)}}{\partial\eta}\frac{\partial\varphi^{(2)}}{\partial z}\right)\sin\alpha\right]=0, \tag{34}$$

where the coefficients a_j, b_j, c_j, d_j, and $\bar{d}_j$ are given in Appendix A. The solitary wave solutions of the above-mentioned equations have been obtained in Secs. V A–V C.

A. Solitary wave solution of the kinetic Alfvén wave

Substituting

$$n^{(1)}=N_0(z),\quad \varphi^{(1)}=\varphi_0(z) \tag{35}$$

into Eqs. (30) and (31) we get two coupled equations for N_0 and φ_0. Eliminating φ_0 between these two equations, we get

$$-U\frac{dN_0}{dz}+\kappa N_0\frac{dN_0}{dz}+a_3\frac{d^3N_0}{dz^3}=0, \tag{36}$$

where $\kappa=a_1+(b_1/c_1)$. The solitary wave solution of (36) is

$$N_0=a\,\mathrm{sech}^2\,pz, \tag{37}$$

where

$$a=\frac{3U}{\kappa}\equiv-3U\sqrt{\frac{\beta}{2}}\sec\alpha,$$
$$p=\sqrt{\frac{U}{4a_3}}\equiv\left(\frac{1}{2}\sqrt{\frac{\beta}{2}}U\,\mathrm{cosec}^2\,\alpha\sec\alpha\times\left[\frac{1}{2}\beta\left\{1+\frac{5}{3}\sigma(1-\beta')\right\}-(1-\beta')\right]\right)^{1/2}. \tag{38}$$

Now from the expressions of a and p given by Eq. (38), we find that Eq. (37) describes a compressive soliton, if $U\cos\alpha<0$ and $1-\beta'-(1/2)\beta\{1+(5/3)\sigma(1-\beta')\}>0$; and on the other hand, Eq. (37) admits a rarefactive soliton if $U\cos\alpha>0$ and $1-\beta'-(1/2)\beta\{1+(5/3)\sigma(1-\beta')\}<0$. Figure 2 shows the existence of the regions in the $\beta'-\beta$ plane where $1-\beta'-(1/2)\beta\{1+(5/3)\sigma(1-\beta')\}>0$ and where $1-\beta'-(1/2)\beta\{1+(5/3)\sigma(1-\beta')\}<0$ for $\sigma=0.0001$. This graph further shows the existence of both types of kinetic Alfvén solitary waves.

B. Solitary wave solution of the ion-acoustic wave ($B'\neq 0$)

Substituting

$$n^{(1)}=\bar{N}_0(z) \tag{39}$$

into Eq. (32) we get an equation parallel to Eq. (36), the solitary wave solution of which is given by the following:

$$\bar{N}_0=\bar{a}\,\mathrm{sech}^2\,\bar{p}z, \tag{40}$$

where

$$\bar{a}=[6U\sec\alpha\{(5/3)\sigma+(1-\beta')^{-1}\}^{1/2}]/[(40/9)\sigma-(1-\beta')^{-3}+3(1-\beta')^{-1}],$$
$$\bar{p}=\left[\frac{U\,\mathrm{cosec}^2\,\alpha\sec\alpha\{1-\beta'-(1/2)\beta[1+(5/3)\sigma(1-\beta')]\}}{2\{(5/3)\sigma+(1-\beta')^{-1}\}^{1/2}}\right]^{1/2}. \tag{41}$$

Now $B'\neq 0$ gives $(40/9)\sigma-(1-\beta')^{-3}+3(1-\beta')^{-1}\neq 0$ and, therefore, Fig. 2 again shows the existence of both types of solitons. Again from the expression of p and $\bar{p}$ as given in Eqs. (38) and (41), respectively, we find that if $1-\beta'-(1/2)\beta\{1+(5/3)\sigma(1-\beta')\}>0$, then the solitary Alfvén wave exists if $U\cos\alpha<0$ and the solitary ion-acoustic wave exists if $U\cos\alpha>0$. Therefore, for some fixed value of α, the linear velocity of the Alfvén solitary wave is opposite to that of the ion-acoustic solitons. In particular, if $\cos\alpha>0$, the Alfvén wave moves from right to left, whereas the ion-acoustic soliton moves from left to right, and if $\cos\alpha<0$, the solitary Alfvén wave moves from left to right, whereas the ion-acoustic soliton moves from right to left. A similar analysis holds well for the case when $1-\beta'-(1/2)\beta\{1+(5/3)\sigma(1-\beta')\}<0$.

C. Solitary wave solution of the ion-acoustic wave ($B'=0$)

Substituting

$$n^{(1)}=N_0'(z),\quad \varphi^{(2)}=\varphi_0'(z) \tag{42}$$

into Eqs. (33) and (34) we get, respectively, the following two equations:

$$-U\frac{dN_0'}{dz}+\bar{d}_1(N_0')^2\frac{dN_0'}{dz}+d_3\frac{d^3N_0'}{dz^3}=0, \tag{43}$$

$$\frac{d^3}{dz^3}(d_3N_0'+\bar{d}_3\varphi_0')=0. \tag{44}$$

The solitary wave solution of (45) is given by

$$N_0'(z)=\pm a'\,\mathrm{sech}\,p'z, \tag{45}$$

where

$$a' = \left[\frac{12UV(1-\beta')^4 \sec\alpha}{(1-\beta')^4\{(10/27)\sigma - 6V^2 + (3/2)(1-\beta')[3V^2-(5/9)\sigma]^2\} - (1/2)(1+3\beta')} \right]^{1/2}, \tag{46}$$

$$p' = [(U/V)\{1-\beta'-(1/2)\beta[1+(5/3)\sigma \times(1-\beta')]\}\text{cosec}^2\alpha\sec\alpha]^{1/2}. \tag{47}$$

Now the critical condition $B'=0$ and the linear depression relation (30) give the values of V and σ as a function of β' and these are given by

$$V = \{[8(1-\beta')^3]^{-1}[3-(1-\beta')^2]\}^{1/2}, \quad \sigma = (9/40)(1-\beta')^{-3}[1-3(1-\beta')^2]. \tag{48}$$

Using (48) one can find that the denominator of (46) is positive or negative according to whether $0.347 \leqslant \beta' < 1$ or $0 < \beta' < 0.347$. Again $\sigma \geqslant 0$ imposes an extra restriction on β' and this restriction is $0.423 \leqslant \beta' < 1$. Therefore, the denominator of (46) is always positive and consequently a' is well defined, if $U\cos\alpha > 0$. Again from the expression of p' given by the Eq. (47), we find that if $U\cos\alpha>0$, p' is well defined, provided $1-\beta'-(1/2)\beta\{1+(5/3)\sigma(1-\beta')\}>0$. This condition gives a restriction, $\beta<2/V^2$, on β, where V is given by Eq. (48). Figure 3 shows the variation of $\Gamma = 2/V^2$ against β'. From this graph we see that $\Gamma < 1.152$ (approximately) for any admissible value of $\beta'(0.423 \leqslant \beta' < 1)$, which is not inconsistent with the definition of β.

The solution of Eq. (44) for which φ_0' is bounded and tends to zero as $|z|\to\infty$, is given by

$$d_3 N_0' + \bar{d}_3 \varphi_0' = 0. \tag{49}$$

From Eq. (49), we get the following results:

$$d_j N_0' + \bar{d}_j \varphi_0' = 0 \quad \text{for } j = 3,4,...,8. \tag{50}$$

VI. STABILITY ANALYSIS

A. The kinetic Alfvén wave

To analyze the stability of the kinetic Alfvén wave, for long wavelength plane-wave perturbation given in a direction having direction cosines (l,m,n) by the method of Rowlands and Infeld,[2,3,16–20] we set

$$n^{(1)} = N_0(z) + q(\xi,\eta,z,\tau), \quad \varphi^{(1)} = \varphi_0(z) + r(\xi,\eta,z,\tau), \tag{51}$$

$$(q(\xi,\eta,z,\tau), r(\xi,\eta,z,\tau)) = (\bar{q}(z),\bar{r}(z))\exp[i\{k(l\xi+m\eta+nz)-\omega\tau\}], \tag{52}$$

$$(\bar{q}(z),\bar{r}(z),\omega) = \sum_{j=0}^{\infty} k^j (q^{(j)}(z), r^{(j)}(z), \omega^{(j)}), \tag{53}$$

where k is small, $l^2+m^2+n^2=1$, and $\omega^{(0)}=0$.

Substituting (51) into Eqs. (30) and (31), linearizing these equations with respect to q and r, and then using (52) we get two equations for $\bar{q}$ and $\bar{r}$. Finally, substituting the expansions (53) in these two equations for $\bar{q}$ and $\bar{r}$ and then equating coefficients of different powers of k on both sides of each equation we get a sequence of coupled equations which can be put as follows:

$$U\partial_z L q^{(j)} = Q^{(j)}, \quad c_1 \partial_z^2 r^{(j)} = R^{(j)}, \tag{54}$$

where $\partial_z = d/dz$, $L = -1 + 3\,\text{sech}^2 pz + (1/4p^2)(d^2/dz^2)$, and j varies over the set of natural numbers including zero. The expressions of $Q^{(j)}$ and $R^{(j)}$ for $j=0$, 1, 2 are given in Appendix B. Now following Das and Verheest,[3] the general solution of the first equation of (54) can be written as follows:

$$q^{(j)} = A_1^{(j)} f + A_2^{(j)} g + A_3^{(j)} \left[g\int \frac{f}{(W/4p^2)}dz - f\int \frac{g}{(W/4p^2)}dz \right] + g\int \frac{f\int Q^{(j)}dz}{(W/4p^2)}dz - f\int \frac{g\int Q^{(j)}dz}{(W/4p^2)}dz, \tag{55}$$

where $f = RS^2$ and $g = pzRS^2 + \frac{2}{15}S^{-2} + \frac{1}{3} - S^2$ with $R = \tanh pz$, $S = \text{sech}\,pz$ are the two linearly independent solutions of the equation $ULq^{(j)}=0$; $A_1^{(j)}$, $A_2^{(j)}$, $A_3^{(j)}$ being three arbitrary constants of integration and $W = f(dg/dz) - g(df/dz) = (8/15)p$.

Now the solutions given by (55) of the coupled equation (54) for $j=0$, 1, which remain bounded and tend to 0 as $|z|\to\infty$, can be written as follows:

$$q^{(0)} = A_1^{(0)} RS^2, \quad r^{(0)} = -\frac{A_1^{(0)}}{2c_1 p^2} R,$$
$$q^{(1)} = A_1^{(1)} RS^2 - iA_1^{(0)}(\bar{a}_1+\bar{b}_1)pzRS^2 + \tfrac{1}{3} iA_1^{(0)}(3\bar{a}_1+\bar{b}_1)S^2, \tag{56}$$
$$\partial_z r^{(1)} = -\frac{A_1^{(1)}}{2pc_1}S^2 + \frac{iA_1^{(0)}}{2c_1}(\bar{a}_1+\bar{b}_1)zS^2 + \frac{iA_1^{(0)}}{2pc_1} \times \left\{ \frac{2}{3}(3\bar{a}_1+\bar{b}_1) - (\bar{a}_1+\bar{b}_1) + \frac{u_5}{pc_1} \right\} R,$$

where

$$\bar{a}_1 = \frac{1}{2pU}\left[-u_1 - 2p^2u_2 + \frac{1}{2}\left(\frac{au_4}{c_1}\right) + \frac{1}{2}(au_3) \right], \quad \bar{b}_1 = \frac{1}{2pU}\left[6p^2u_2 - \frac{1}{2}\left(\frac{au_4}{c_1}\right) - \frac{1}{2}(au_3) \right], \tag{57}$$

The expressions for $u^{(j)}$ appearing in the above are given in Appendix B. Now for the existence of the solution of the first equation of (54) for $j=2$, its right-hand side must be perpendicular to the kernel of the operator adjoint to the operator

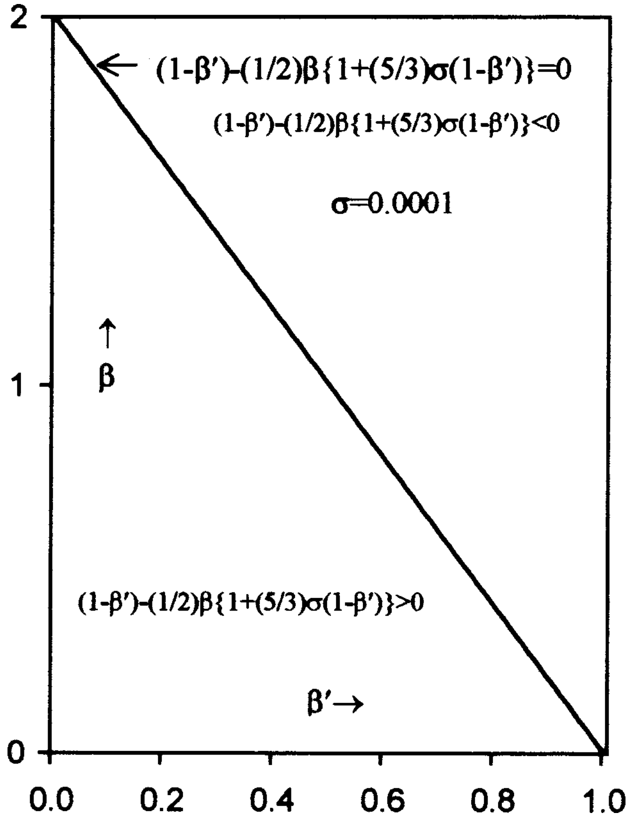

FIG. 2. The regions in the $\beta-\beta'$ plane where $1-\beta'-(1/2)\beta\{1+(5/3)\sigma(1-\beta')\}$ is positive and negative.

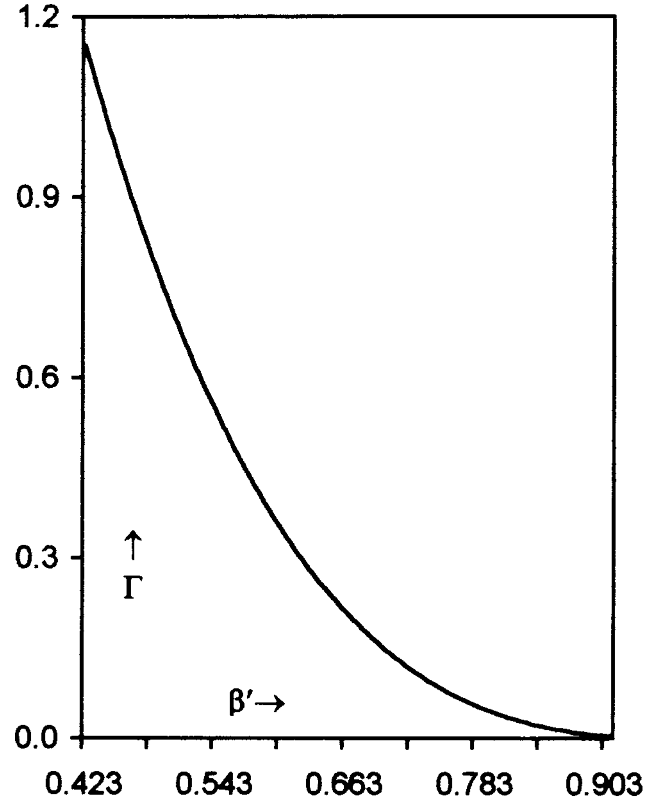

FIG. 3. Γ plotted against β'.

$\partial_z L$. This kernel, which must tend to zero as $|z|\to\infty$, is S^2. Therefore we get the following consistency condition:

$$\int_{-\infty}^{\infty} S^2 Q^{(2)} dz = 0. \tag{58}$$

This equation gives the following quadratic equation for $u_1(=\omega^{(1)}+nU)$:

$$u_1^2+\bar{B}u_1+\bar{C}=0, \tag{59}$$

where

$$\bar{B}=\tfrac{4}{3}\{2(p^2u_2)-(au_3)\},$$
$$\bar{C}=-\frac{8}{45}\left\{6(p^2u_2)^2+6(p^2u_2)(au_3)-2(au_3)^2-(au_3)\times\left(\frac{au_4}{c_1}\right)+\left(\frac{au_4}{c_1}\right)^2+3U\left(\frac{au_4}{c_1}\right)\left(\frac{u_5}{c_1}\right)-6U(p^2u_6)+3U\left(\frac{au_7}{c_1}\right)\right\}. \tag{60}$$

The discriminant Δ of (59) can be simplified as

$$\Delta=\frac{4}{45}\frac{U^2}{\cos^2\alpha\sin^2\alpha}(5l^2+24m^2\cos^2\alpha). \tag{61}$$

Equation (61) shows that the discriminant of the quadratic equation (59) is always positive for all values of l, m, n, α and U. So the kinetic Alfvén waves are stable.

B. The ion-acoustic wave ($B'=0$)

For this case we set

$$n^{(I)}=\bar{N}_0(z)+s(\xi,\eta,z,\tau), \tag{62}$$

$$s(\xi,\eta,z,\tau)=\bar{s}(z)\exp[i\{k(l\xi+m\eta+nz)-\omega\tau\}], \tag{63}$$

$$(\bar{s}(z),\omega)=\sum_{j=0}^{\infty}k^j(s^{(j)}(z),\omega^{(j)}), \tag{64}$$

where $\omega^{(0)}=0$ and $l^2+m^2+n^2=1$.

Now substituting (62) into Eq. (32), linearizing this equation with respect to s, and then using the relation (63), we get an equation for $\bar{s}$. Substituting the expansion (64) in this equation for $\bar{s}$ and finally equating coefficients of different powers of k on both sides of the equation, the following sequence of equations is obtained:

$$U\partial_z\bar{L}s^{(j)}=S^{(j)}, \quad j=0,1,2,..., \tag{65}$$

where $\bar{L}=-1+3\,\mathrm{sech}^2\bar{p}z+(1/4\bar{p}^2)(d^2/dz^2)$ and the expansions for $S^{(j)}$'s for $j=0$, 1, 2 are given in Appendix C. Proceeding exactly in the same way as in Sec. VI A, the solutions of (65) for $s^{(j)}(j=0,1)$, which remain finite and tend to zero as $|z|\to\infty$, can be written as follows:

$$s^{(0)}=\bar{A}_1^{(0)}\bar{R}\bar{S}^2,$$
$$s^{(1)}=\bar{A}_1^{(1)}\bar{R}\bar{S}^2-i\bar{A}_1^{(0)}(\bar{a}_2+\bar{b}_2)\bar{p}z\bar{R}\bar{S}^2+\tfrac{1}{3}i\bar{A}_1^{(0)}(3\bar{a}_2+\bar{b}_2)\bar{S}^2, \tag{66}$$

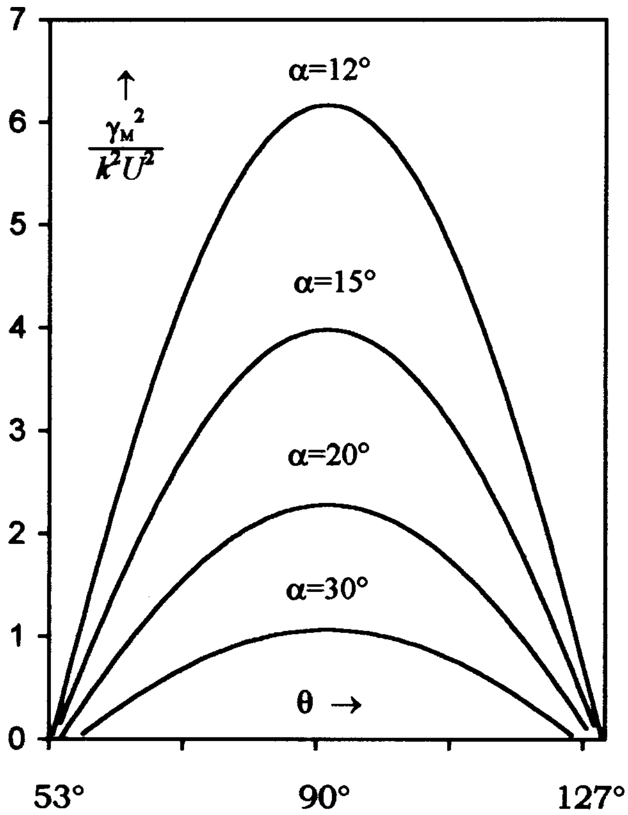

FIG. 4. The square of the maximum growth rate of instability of the ion-acoustic wave ($B'\neq 0$) normalized by k^2U^2 plotted against θ for different values of α.

where $\bar{A}_1^{(0)}$ and $\bar{A}_1^{(1)}$ are two constants, $\bar{R}=\tanh \bar{p}z$, $\bar{S}=\operatorname{sech}\bar{p}z$, and $\bar{a}_2$, $\bar{b}_2$ are given as follows:

$$\bar{a}_2=\frac{1}{2\bar{p}U}\left(-u_1-2\bar{p}^2\bar{u}_2+\frac{1}{2}\bar{a}\bar{u}_3\right),$$
$$\bar{b}_2=\frac{1}{2\bar{p}U}\left(6\bar{p}^2\bar{u}_2-\frac{1}{2}\bar{a}\bar{u}_3\right). \tag{67}$$

The expressions for $\bar{u}_1$, $\bar{u}_2$, etc., appearing in Eq. (67) are given in Appendix C.

Now the solution of Eq. (65) for $j=2$ exists if its right-hand side is perpendicular to the kernel of the operator adjoint to the operator $\partial_z\bar{L}$. This kernel, which tends to zero as $|z|\to\infty$, is $\bar{S}^2$, and consequently we get the following consistency condition:

$$\int_{-\infty}^{\infty}\bar{S}^2S^{(2)}dz=0. \tag{68}$$

Equation (68) gives the following quadratic equation for u_1:

$$u_1^2+\bar{B}_1u_1+\bar{C}_1=0, \tag{69}$$

where

$$\bar{B}_1=\tfrac{4}{3}\{2(\bar{p}^2\bar{u}_2)-(\bar{a}\bar{u}_3)\},$$
$$\bar{C}_1=-\tfrac{16}{45}\{3(\bar{p}^2\bar{u}_2)^2+3(\bar{p}^2\bar{u}_2)(\bar{a}\bar{u}_3)-(\bar{a}\bar{u}_3)^2-3U(\bar{p}^2\bar{u}_6)\}. \tag{70}$$

The discriminant Δ_1 of Eq. (69) is given by

$$\Delta_1=\frac{16}{45}\frac{U^2}{\cos^2\alpha\sin^2\alpha}(5l^2-3m^2\cos^2\alpha). \tag{71}$$

Therefore, for instability we must have $5l^2-3m^2\cos^2\alpha<0$, and in this case the growth rate of instability $\gamma(=k\,\mathrm{Im}\,\omega^{(1)})$ is given by

$$\frac{\gamma^2}{k^2U^2}=\frac{4}{45\cos^2\alpha\sin^2\alpha}(3m^2\cos^2\alpha-5l^2). \tag{72}$$

Now for the perturbation given in a plane through the ζ axis making an angle θ with the $\xi\zeta$ plane, the growth rate of instability is given by

$$\frac{\gamma^2}{k^2U^2}=\frac{4(1-n^2)}{45\cos^2\alpha\sin^2\alpha}(3\cos^2\alpha\sin^2\theta-5\cos^2\theta). \tag{73}$$

This growth rate of instability attains its maximum value γ_M for arbitrary fixed θ when $n=0$. Figure 4 shows the variation of $(\gamma_M/kU)^2$ against θ for some different values of α. This graph shows that for some fixed value of θ, γ_M decreases as α increases and the range of θ for which γ_M^2 is positive decreases as α increases. The maximum value of γ_M is $(4/15)^{1/2}(kU/\sin\alpha)$ and this is attained when $\theta=\pi/2$.

C. The ion-acoustic wave ($B'=0$)

In this case we set

$$n^{(1)}=N_0'(z)+q(\xi,\eta,z,\tau),\quad \varphi^{(2)}=\varphi_0'(z)+r(\xi,\eta,z,\tau), \tag{74}$$

$$(q(\xi,\eta,z,\tau),r(\xi,\eta,z,\tau))=(\bar{q}(z),\bar{r}(z))\exp[i\{k(l\xi+m\eta+nz)-\omega\tau\}], \tag{75}$$

$$(\bar{q}(z),\bar{r}(z),\omega)=\sum_{j=0}^{\infty}k^j(q^{(j)}(z),r^{(j)}(z),\omega^{(j)}). \tag{76}$$

where k is small, $l^2+m^2+n^2=1$, and $\omega^{(0)}=0$.

Substituting (74) into Eqs. (33) and (34), linearizing these equations with respect to q and r, and then using Eq. (75), we get two coupled equations for $\bar{q}$ and $\bar{r}$. Finally, substituting the expansions (76) into these two coupled equations and then equating coefficients of different powers of k on both sides of each equation, we obtain a sequence of coupled equations which can be presented as follows:

$$U\partial_z\bar{\bar{L}}q^{(j)}=\bar{Q}^{(j)}, \tag{77}$$

$$\partial_z^3(d_3q^{(j)}+\bar{d}_3r^{(j)})=\bar{R}^{(j)}, \tag{78}$$

where $\bar{\bar{L}}=(-1+6\operatorname{sech}^2p'z+(1/p'^2)(d^2/dz^2))$ and j varies over the set of natural numbers including zero. The expressions of $\bar{Q}^{(j)}$ and $\bar{R}^{(j)}$ for $j=0,1,2$ are given in Appendix D. The general solution of Eq. (77) can easily be obtained in the form (55).

By the use of these solutions, the solutions of the coupled equations (77) and (78), for $j=0,1$, which remain finite and tend to zero as $|z|\to\infty$, can be written as follows:

$$q^{(0)}=\bar{A}_1^{(0)}R'S', \quad d_3q^{(0)}+\bar{d}_3r^{(0)}=0,$$
$$q^{(1)}=\bar{A}_1^{(1)}R'S'-\tfrac{1}{2}i\bar{A}_1^{(0)}\bar{\bar{a}}_3p'zR'S' + \tfrac{1}{4}i\bar{A}_1^{(0)}(2\bar{\bar{a}}_3+\bar{\bar{b}}_3)S', d_3q^{(1)}+\bar{d}_3r^{(1)} = 0, \tag{79}$$

where

$$\bar{\bar{a}}_3=\frac{1}{p'U}(p'^2\bar{u}_2-u_1), \quad \bar{\bar{b}}_3=\frac{1}{3p'U}(a'^2\bar{\bar{u}}_3-6p'^2\bar{u}_2),$$
$$R'=\tanh p'z, \quad S'=\operatorname{sech} p'z. \tag{80}$$

With the help of the second and fourth equations of (79), we find that

$$d_jq^{(0)}+\bar{d}_jr^{(0)}=0, \quad d_jq^{(1)}+\bar{d}_jr^{(1)}=0 \quad \text{for } j=3,4,...,8. \tag{81}$$

Now for the existence of the solution of Eq. (77) for $j=2$, its right-hand side must be perpendicular to the kernel of the operator adjoint to the operator $\partial_z\bar{\bar{L}}$. This kernel, which must tend to zero as $|z|\to\infty$, is S'. Therefore, we get the following consistency condition:

$$\int_{-\infty}^{\infty} S'\bar{Q}^{(2)}dz=0. \tag{82}$$

Equation (82) gives the following quadratic equation for u_1:

$$u_1^2+\bar{\bar{B}}u_1+\bar{\bar{C}}=0, \tag{83}$$

where

$$\bar{\bar{B}}=\tfrac{2}{3}(3p'^2\bar{u}_2-2a'^2\bar{\bar{u}}_3),$$
$$\bar{\bar{C}}=-\tfrac{1}{9}\{3(p'^2\bar{u}_2)^2+4(p'^2\bar{u}_2)(a'^2\bar{\bar{u}}_3)-2(a'^2\bar{\bar{u}}_3)^2 - 12Up'^2\bar{u}_6\}. \tag{84}$$

The discriminant Δ_2 of the above-mentioned quadratic equation is given by

$$\Delta_2=\frac{16}{3}\frac{U^2}{\sin^2\alpha\cos^2\alpha}(3l^2-m^2\cos^2\alpha). \tag{85}$$

Therefore, for instability we must have $3l^2-m^2\cos^2\alpha<0$, and in this case the growth rate of instability $\gamma(=k\,\mathrm{Im}\,\omega^{(1)})$ is given by

$$\frac{\gamma^2}{k^2U^2}=\frac{4}{3\sin^2\alpha\cos^2\alpha}(m^2\cos^2\alpha-3l^2). \tag{86}$$

For the perturbation given in a plane through the ζ axis making an angle θ with the $\xi\zeta$ plane, the growth rate of instability is given by

$$\frac{\gamma^2}{k^2U^2}=\frac{4(1-n^2)}{3\sin^2\alpha\cos^2\alpha}(\cos^2\alpha\sin^2\theta-3\cos^2\theta). \tag{87}$$

This growth rate of instability (γ) attains its maximum value (γ_M) for an arbitrary fixed value of θ when $n=0$. This maximum value of the growth rate of instability is given by

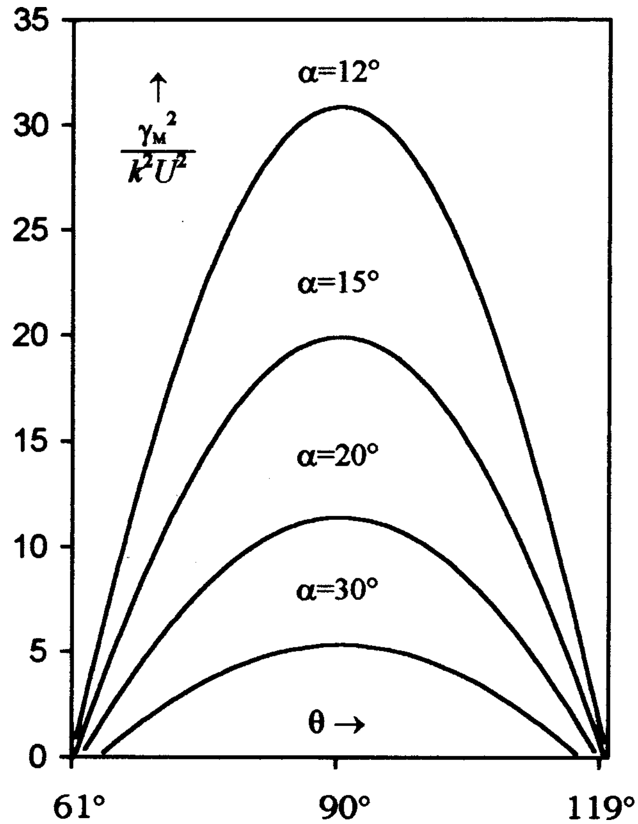

FIG. 5. The square of the maximum growth rate of instability of the ion-acoustic wave $(B'=0)$ normalized by k^2U^2 plotted against θ for different values of α.

$$\frac{\gamma_M^2}{k^2U^2}=\frac{4}{3\sin^2\alpha\cos^2\alpha}(\cos^2\alpha\sin^2\theta-3\cos^2\theta). \tag{88}$$

Figure 5 shows the variation of $(\gamma_M/kU)^2$ against θ. This graph shows that γ_M decreases as α increases and the range of θ for which γ_M^2 is positive decreases as α increases, and the maximum value of γ_M is $(4/3)^{1/2}(kU/\sin\alpha)$, which is attained when $\theta=\pi/2$. Therefore, for any arbitrary fixed α, the growth rate of instability in the case of MKdV solitons is greater than that of the KdV solitons.

VII. CONCLUSIONS

Starting from a set of governing equations, which produce a linear dispersion relation coupling kinetic Alfvén waves and ion-acoustic waves in a magnetized plasma in which the electrons are nonthermal, the three-dimensional KdV equations are derived for both kinetic Alfvén and ion-acoustic waves. The coefficient B' of the nonlinear term of this three-dimensional KdV equation for ion-acoustic waves can vanish along a curve in the $\beta'\sigma$ parameter plane, where β' is a parameter of the nonthermal electrons and σ is the ratio of ion and electron temperatures. In this case, two coupled equations constituting a modified three-dimensional KdV equation describing the nonlinear behavior of ion-acoustic waves are derived. All the nonlinear evolution equations derived are found to have obliquely propagating soliton solutions. The kinetic Alfvén solitons, propagating with a velocity U in the direction making an angle α with the external magnetic field, exist if the two quantities $U\cos\alpha$ and and $1-\beta'$-$(1/2)\{1+(5/3)\sigma(1-\beta')\}$ have opposite signs. But the ion-acoustic solitons, for the case $B'\neq 0$ having the same velocity and propagating in the same direction, exist if the above two quantities have the same sign. Both kinetic Alfvén and ion-acoustic solitons have a sech^2 profile and they can be compressive and also rarefactive. In the case $B'=0$, the ion-acoustic solitons having the above-mentioned velocity and direction of propagation, exist if the two quantities mentioned above are both positive, and the solitons in this case have a sech profile and can be both compressive and rarefactive.

Regarding the stability of these solitons, it is found that the kinetic Alfvén solitons are stable. The ion-acoustic solitons for the case $B'\neq 0$ are unstable if $\cot^2\theta<(3/5)\cos^2\alpha$, while the same for the case $B'=0$ are unstable if $\cot^2\theta<(1/3)\cos^2\alpha$, where θ is the angle between the two planes through the direction of propagation of the solitary wave—one passing through the direction of the external magnetic field and the other passing through the direction of plane wave perturbation. In both cases, the growth rate of instability attains maximum when the perturbation is given in a plane perpendicular to the direction of propagation of the soliton.

APPENDIX A

$$a_1=-AB\cos\alpha,\quad a_2=AB\sin\alpha,$$

$$a_3=-AC\cos\alpha\sin^2\alpha,$$

$$a_4=AC(\sin^3\alpha-2\sin\alpha\cos^2\alpha),$$

$$a_5=-AC(\cos^3\alpha-2\cos\alpha\sin^2\alpha),$$

$$a_6=-AC\cos\alpha,\quad a_7=AC\sin\alpha\cos^2\alpha,$$

$$a_8=AC\sin\alpha,$$

$$b_1=-AD\sin^2\alpha\cos\alpha,\quad b_2=-AD\sin\alpha\cos^2\alpha,$$

$$b_3=-AD\sin\alpha\cos 2\alpha,$$

$$b_4=AD\cos\alpha\sin^2\alpha,\quad b_5=-AD\cos\alpha\cos 2\alpha,$$

$$b_6=-AD\cos\alpha,\quad b_7=AD\cos^2\alpha\sin\alpha,$$

$$b_8=AD\sin\alpha,$$

$$c_1=AE\sin^2\alpha,\quad c_2=AE\sin 2\alpha,$$

$$c_3=AE\cos^2\alpha,\quad c_4=AE,$$

$$d_1=-A'B'\cos\alpha,\quad d_2=A'B'\sin\alpha,$$

$$d_3=-A'C'\cos\alpha\sin^2\alpha,$$

$$d_4=A'C'\sin\alpha(\sin^2\alpha-2\cos^2\alpha),$$

$$d_5=-A'C'(\cos^3\alpha-2\cos\alpha\sin^2\alpha),$$

$$d_6=-A'C'\cos\alpha,\quad d_7=A'C'\sin\alpha\cos^2\alpha,$$

$$d_8=A'C'\sin\alpha,$$

$$\bar{d}_1=-A'B''\cos\alpha,\quad \bar{d}_2=A'B''\sin\alpha,$$

$$\bar{d}_3=-\frac{V}{2}\cos\alpha\sin^2\alpha,\quad \bar{d}_4=\frac{V}{2}\sin\alpha(\sin^2\alpha-2\cos^2\alpha),$$

$$\bar{d}_5=-\frac{V}{2}\cos\alpha(\cos^2\alpha-2\sin^2\alpha),\quad \bar{d}_6=-\frac{V}{2}\cos\alpha,$$

$$\bar{d}_7=\frac{V}{2}\cos^2\alpha\sin\alpha,\quad \bar{d}_3=\frac{V}{2}\sin\alpha.$$

APPENDIX B

$$Q^{(0)}=0,\quad R^0=q^{(0)},$$

$$Q^{(1)}=iu_1q^{(0)}-iu_2\frac{d^2q^{(0)}}{dz^2}-iu_3N_0q^{(0)}-iu_4\frac{dN_0}{dz}\frac{dr^{(0)}}{dz},$$

$$R^{(1)}=q^{(1)}-iu_5\frac{dr^{(0)}}{dz},$$

$$Q^{(2)}=i\omega^{(2)}q^{(0)}+iu_1q^{(1)}-iu_2\frac{d^2q^{(1)}}{dz^2}-iu_3N_0q^{(1)}$$
$$-iu_4\frac{dN_0}{dz}\frac{dr^{(1)}}{dz}-u_7\frac{dN_0}{dz}r^{(0)}+u_6\frac{dq^{(0)}}{dz},$$

$$R^{(2)}=q^{(2)}-iu_5\frac{dr^{(1)}}{dz}+u_8r^{(0)},$$

$$u_1=\omega^{(1)}+nU,\quad u_2=3na_3+la_4,$$

$$u_3=n\left(a_1+\frac{b_1}{c_1}\right)+l\left(a_2+\frac{b_2}{c_1}\right),$$

$$u_4=2nb_1+lb_3-\frac{b_1}{c_1}(2nc_1+lc_2),\quad u_5=2nc_1+lc_2,$$

$$u_6=3n^2a_3+2nla_4+l^2a_5+m^2a_6,$$

$$u_7=\frac{b_1}{c_1}(c_1n^2+c_2nl+c_3l^2+c_4m^2)$$
$$-(b_1n^2+b_3nl+b_4l^2),$$

$$u_8=c_1n^2+c_2nl+c_3l^2+c_4m^2,$$

$$u_9=a_3n^3+a_4n^2l+n(l^2a_5+m^2a_6)+l(l^2a_7+m^2a_8).$$

APPENDIX C

$$S^{(0)}=0,\quad S^{(1)}=iu_1s^{(0)}-i\bar{u}_2\frac{d^2s^{(0)}}{dz^2}-i\bar{u}_3N_0s^{(0)},$$

$$S^{(2)}=i\omega^{(2)}s^{(0)}+\bar{u}_6\frac{ds^{(0)}}{dz}+iu_1s^{(1)}-i\bar{u}_2\frac{d^2s^{(1)}}{dz^2}-i\bar{u}_3\bar{N}_0s^{(1)},$$

$$u_1=\omega^{(1)}+nU,\quad \bar{u}_2=3nd_3+ld_4,\quad \bar{u}_3=nd_1+ld_2,$$

$$\bar{u}_6=3n^2d_3+2nld_4+l^2d_5+m^2d_6.$$

APPENDIX D

$$\bar{Q}^{(0)}=0,$$

$$\bar{Q}^{(1)}=iu_1q^{(0)}-i\bar{u}_2\frac{d^2q^{(0)}}{dz^2}-i\bar{\bar{u}}_3[N_0']^2q^{(0)}-\frac{1}{2}i\frac{m}{\bar{d}_3}\frac{1-D_A(V)}{D_A(V)}(d_3q^{(0)}+\bar{d}_3r^{(0)})\frac{dN_0'}{dz}\sin\alpha,$$

$$\bar{Q}^{(2)}=i\omega^{(2)}q^{(0)}+iu_1q^{(1)}-i\bar{u}_2\frac{d^2q^{(1)}}{dz^2}-i\bar{\bar{u}}_3[N_0']^2q^{(1)}+\bar{u}_6\frac{dq^{(0)}}{dz}-\frac{1}{2}i\frac{m}{\bar{d}_3}\frac{1-D_A(V)}{D_A(V)}(d_3q^{(1)}+\bar{d}_3r^{(1)})\times\frac{dN_0'}{dz}\sin\alpha,$$

$$\bar{R}^{(0)}=0,$$

$$\bar{R}^{(1)}=-i\left\{3n\partial_z^2(d_3q^{(0)}+\bar{d}_3r^{(0)})+l\partial_z^2(d_4q^{(0)}+\bar{d}_4r^{(0)})+\frac{1}{2}\frac{m}{\bar{d}_3}\frac{1+D_A(V)}{D_A(V)}(d_3q^{(0)}+\bar{d}_3r^{(0)})\frac{dN_0'}{dz}\sin\alpha\right\},$$

$$\bar{R}^{(2)}=-i\left\{3n\partial_z^2(d_3q^{(1)}+\bar{d}_3r^{(1)})+l\partial_z^2(d_4q^{(1)}+\bar{d}_4r^{(1)})+\frac{1}{2}\frac{m}{\bar{d}_3}\frac{1+D_A(V)}{D_A(V)}(d_3q^{(1)}+\bar{d}_3r^{(1)})\frac{dN_0'}{dz}\sin\alpha+3n^2\partial_z(d_3q^{(0)}+\bar{d}_3r^{(0)})+2nl\partial_z(d_4q^{(0)}+\bar{d}_4r^{(0)})+l^2\partial_z(d_5q^{(0)}+\bar{d}_5r^{(0)})+m^2\partial_z(d_6q^{(0)}+\bar{d}_6r^{(0)})\right\},$$

$$\bar{\bar{u}}_3=n\bar{d}_1+l\bar{d}_2,$$

$$\bar{\bar{u}}_6=3n^2\bar{d}_3+2nl\bar{d}_4+l^2\bar{d}_5+m^2\bar{d}_6,$$

$$\bar{u}_9=n^2(nd_3+ld_4)+n(l^2d_5+m^2d_6)+l(l^2d_7+m^2d_8),$$

$$\bar{\bar{u}}_9=n^2(n\bar{d}_3+l\bar{d}_4)+n(l^2\bar{d}_5+m^2\bar{d}_6)+l(l^2\bar{d}_7+m^2\bar{d}_8),$$

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