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Stability of dust ion acoustic solitary waves in a collisionless unmagnetized nonthermal plasma in presence of isothermal positrons

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A three-dimensional KP (Kadomtsev Petviashvili) equation is derived here describing the propagation of weakly nonlinear and weakly dispersive dust ion acoustic wave in a collisionless unmagnetized plasma consisting of warm adiabatic ions, static negatively charged dust grains, nonthermal electrons, and isothermal positrons. When the coefficient of the nonlinear term of the KP-equation vanishes an appropriate modified KP (MKP) equation describing the propagation of dust ion acoustic wave is derived. Again when the coefficient of the nonlinear term of this MKP equation vanishes, a further modified KP equation is derived. Finally, the stability of the solitary wave solutions of the KP and the different modified KP equations are investigated by the small-k perturbation expansion method of Rowlands and Infeld [J. Plasma Phys. **3**, 567 (1969); **8**, 105 (1972); **10**, 293 (1973); **33**, 171 (1985); **41**, 139 (1989); Sov. Phys. - JETP **38**, 494 (1974)] at the lowest order of k, where k is the wave number of a long-wavelength plane-wave perturbation. The solitary wave solutions of the different evolution equations are found to be stable at this order. *Published by AIP Publishing.* [http://dx.doi.org/10.1063/1.4956462]

I. INTRODUCTION

It is believed that four component electron-positron-ion-dust plasma (e-p-i-d plasma) may exist in various astrophysical environments, such as galactic centre,[1] intersteller medium,[1–3] in the remnants of supernova explosions,[4,5] near the surface of the neutron stars,[5–7] interior regions of accretion disks near neutron stars and magnetars,[8] dusty cosmological environments such as milky way,[3] and in the magnetosphere and ionosphere of the Earth.[4] Beside these cosmological sites, e-p-i-d plasma can also be found in laboratory experiments.[7,8] Several authors[8–12] investigated ion acoustic (IA)/dust ion acoustic (DIA) solitary structures in e-p-i-d plasma considering different velocity distribution function of electrons.

In the present paper, we have investigated the existence and stability of small amplitude DIA solitary waves in a collisionless unmagnetized e-p-i-d plasma consisting of warm adiabatic ions, static negatively charged dust grains, isothermal positrons, and nonthermal electrons due to Cairns *et al.*[13] Nonthermal electrons are observed in a number of heliospheric environments.[14–17] Specifically, nonthermal electrons are observed in Earth's Bow shock and Fore shock,[14,15] in lower part of magnetosphere.[16] Observation of Voyager 2 indicates the presence of non-Maxwellian electrons in the magnetosphere of Saturn[17] and also in the atmosphere of Uranus.[17]

Kadomtsev and Petviashvili[18] have made an attempt to investigate the stability of Korteweg-de Vries (KdV) solitons in a collisionless unmagnetized plasma. Kako and Rowlands[19] derived the Kadomtsev Petviashvili (KP) equation for ion acoustic waves in a collisionless unmagnetized plasma. Infeld *et al.*[20] used this KP equation to study the stability of ion acoustic KdV solitons and reported that ion acoustic solitary wave solutions are stable with respect to the transverse long-wavelength plane-wave perturbation. Using the small-k perturbation expansion method of Rowlands and Infeld,[21–26] Chakraborty and Das[27] investigated the stability of ion acoustic solitary wave solutions of a modified KP (MKP) equation having the nonlinear term of the form $\frac{\partial}{\partial \xi}\left[\sqrt{\phi^{(1)}}\frac{\partial \phi^{(1)}}{\partial \xi}\right]$ in a multi-species collisionless unmagnetized plasma consisting of non-isothermal electrons having Schamel distribution. In a later paper, Chakraborty and Das[28] investigated higher order stability of the same modified KP equation with the help of multiple-scale perturbation expansion method of Allen and Rowlands.[29,30]

In the present paper, we have derived KP and different modified KP equations describing the nonlinear behaviour of DIA waves in different regions of parameter space when the weak dependence of the spatial coordinates perpendicular to the direction of propagation of the wave is taken into account. We have also investigated the stabilities of solitary wave solutions of a more general evolution equation having a nonlinear term of the form $\frac{\partial}{\partial \xi}\left[\left(\phi^{(1)}\right)^r\frac{\partial \phi^{(1)}}{\partial \xi}\right]$ for any real positive value of r with the help of the small-k perturbation expansion method of Rowlands and Infeld,[20–25] and finally, we have considered the stabilities of DIA solitary wave solutions of KP and different modified KP equations for the present plasma system. The problem of existence and stability of the DIA solitary wave solutions of KP and different modified KP equations in the present plasma system has not been considered by any author.

II. BASIC EQUATIONS

We consider a collisionless unmagnetized dusty plasma consisting of warm adiabatic ions, static negatively charged

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dust grains, nonthermal electrons, and isothermal positrons. The nonlinear behaviour of DIA waves in this plasma system can be described by the equation of continuity, the equation of motion, and the pressure equation for ion fluid together with the Poisson equation

$$n_t + \nabla.(n\mathbf{u}) = 0, \tag{1}$$

$$M_s^2[\mathbf{u}_t + (\mathbf{u}.\nabla)\mathbf{u}] + \frac{(1-p)\sigma_{ie}}{n}\nabla P = -\nabla\phi, \tag{2}$$

$$P_t + (\mathbf{u}.\nabla)P + \gamma P(\nabla.\mathbf{u}) = 0, \tag{3}$$

$$C\nabla^2\phi = n_e - n - n_p + \frac{Z_d n_{d0}}{N}, \tag{4}$$

where

$$C = \frac{1-p}{M_s^2 - \gamma\sigma_{ie}}, \tag{5}$$

and we have used the following notations:

$$\frac{\partial\Psi}{\partial t} = \Psi_t, \nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}, \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2},$$

$\psi = \psi(x, y, z, t, \ldots) \iff$ the quantity ψ is a function of $x, y, z, t, \ldots$.

Here, n, n_e, n_p, $\mathbf{u} = (u, v, w)$, P, ϕ, (x, y, z), and t are, respectively, the ion number density, the electron number density, the positron number density, the ion fluid velocity, the ion fluid pressure, the electrostatic potential, the spatial variables, and time, and they have been normalized by $N(=n_0 + n_{p0})$, N, N, C_D (linearized velocity of the DIA wave in the present plasma system for long-wavelength plane-wave perturbation), $n_0 K_B T_i$, $\frac{K_B T_e}{e}$, λ_D (Debye length of the present plasma system) and λ_D/C_D respectively, where n_0, n_{p0}, K_B, and T_e are, respectively, the unperturbed ion number density, the unperturbed positron number density, the Boltzmann constant, the average temperature of nonthermal electrons, T_i $(=T_{i0})$ is the average unperturbed temperature of ions with m is the mass of an ion, e is the positive unit charge, $\gamma(=3)$ is the adiabatic index, $p = \frac{n_{p0}}{N}$, $\sigma_{ie} = \frac{T_i}{T_e}$, n_{d0} is the constant dust number density with Z_d being the number of electrons residing on the dust grain surface. We have assumed that the electric charge of an ion is e. The expressions of C_D, λ_D, and M_s and the four basic parameters p, μ, σ_{ie}, and σ_{pe} are given by the following equations:

$$C_D = C_s M_s, \quad C_s = \sqrt{\frac{K_B T_e}{m}},$$
$$M_s = \sqrt{\gamma\sigma_{ie} + \frac{(1-p)\sigma_{pe}}{p + \mu(1-\beta_e)\sigma_{pe}}}, \tag{6}$$

$$\frac{1}{\lambda_D^2} = \frac{1-\beta_e}{\lambda_{De}^2} + \frac{1}{\lambda_{Dp}^2}, \quad \lambda_{De}^2 = \frac{K_B T_e}{4\pi e^2 n_{e0}},$$
$$\lambda_{Dp}^2 = \frac{K_B T_p}{4\pi e^2 n_{p0}}, \tag{7}$$

$$\sigma_{ie} = \frac{T_i}{T_e}, \quad \sigma_{pe} = \frac{T_p}{T_e}, \quad p = \frac{n_{p0}}{N}, \quad \mu = \frac{n_{e0}}{N}. \tag{8}$$

In fact, the linear dispersion relation of the DIA wave for the present dusty plasma system can be written as

$$\frac{\omega}{k} = C_D\sqrt{\frac{1 + \frac{\gamma\sigma_{ie}}{M_s^2}k^2\lambda_D^2}{1 + k^2\lambda_D^2}}, \tag{9}$$

where ω and k are, respectively, the wave frequency and wave number of the plane wave perturbation. For long-wavelength plane-wave perturbation, i.e., for $k \to 0$, from linear dispersion relation (9), we have

$$\lim_{k\to 0}\frac{\omega}{k} = C_D \quad \text{and} \quad \lim_{k\to 0}\frac{d\omega}{dk} = C_D, \tag{10}$$

and consequently, the dispersion relation (9) shows that the linearized velocity of the DIA wave for long-wavelength plane-wave perturbation is C_D with λ_D as the Debye length. So, according to the prescription of Dubinov,[31] here each spatial coordinate is normalized by λ_D and the time is normalized by λ_D/C_D.

Based on the above normalization of the dependent variables, the expressions of the number densities of the nonthermal electrons (n_e) and isothermal positrons (n_p) can be written as

$$n_e = \mu(1 - \beta_e\phi + \beta_e\phi^2)e^{\phi}, \; n_p = pe^{-\phi/\sigma_{pe}}, \tag{11}$$

where β_e is the nonthermal parameter associated with the Cairns distributed electrons[13] that determines the proportion of fast energetic electrons. It can be easily checked that β_e is restricted by $0 \le \beta_e < 4/3$. However, we cannot take the whole interval of β_e. In fact, for increasing β_e, the velocity distribution function of nonthermal electrons develops wings, which become stronger as β_e increases, and at the same time, the center density in phase space drops; consequently, we should not take values of $\beta_e > 4/7$ as that stage might stretch the credibility of the Cairns model too far.[32] So, we take $0 \le \beta_e \le 4/7 = 0.6$ (approximately).

The charge neutrality condition can be written as

$$\frac{n_0}{N} = 1 - p \quad \text{and} \quad \frac{Z_d n_{d0}}{N} = 1 - \mu. \tag{12}$$

Expanding n_e and n_p up to ϕ^4, the Poisson equation (4) can be written as

$$C\nabla^2\phi = 1 - p + \sum_{i=1}^{4} Q_i\phi^i - n, \tag{13}$$

where Q_1, Q_2, Q_3, and Q_4 are given in Appendix A.

Equations (1), (2), (3), and (13) are the basic equations.

III. DERIVATION OF DIFFERENT EVOLUTION EQUATIONS

To derive the different evolution equations, we assume that the DIA wave is propagating mainly along the x direction whereas it has weak y and z dependence in the wave number, and consequently, we use the following stretching of space coordinates and time:

$$\xi = \varepsilon(x - Vt), \eta = \varepsilon^2 y, \zeta = \varepsilon^2 z, \tau = \varepsilon^3 t, \tag{14}$$

where ε is a small expansion parameter measuring the weakness of dispersion and weakness of nonlinearity and V is a constant.

A. KP equation in three dimensions

We take the following perturbation expansions of the dependent variables:

$$f = f^{(0)} + \sum_{i=1}^{\infty} \varepsilon^{2i} f^{(i)}, \; g = g^{(0)} + \sum_{i=1}^{\infty} \varepsilon^{2i+1} g^{(i)}, \tag{15}$$

where $f = n, P, \phi, u$ with $n^{(0)} = 1 - p, P^{(0)} = 1, \phi^{(0)} = 0$, $u^{(0)} = 0$, and $g = v, w$ with $v^{(0)} = w^{(0)} = 0$. Substituting the stretching (14) and perturbation expansions (15) in the Equations (1), (2), (3), and (13) and finally equating the coefficient of different powers of ε on each side of every equation, we get a sequence of equations. From these sequences of equations, we get the following Kadomtsev Petviashvili (KP) equation:

$$\frac{\partial}{\partial \xi}\left[\phi_\tau^{(1)} + AB_1 \phi^{(1)} \phi_\xi^{(1)} + \frac{1}{2} AC \phi_{\xi\xi\xi}^{(1)}\right] + \frac{1}{2} AD\left(\phi_{\eta\eta}^{(1)} + \phi_{\zeta\zeta}^{(1)}\right) = 0, \tag{16}$$

where the coefficients A, B_1, and D are given by

$$A = \frac{1}{1-p} \frac{\left(M_s^2 V^2 - \gamma\sigma_{ie}\right)^2}{V M_s^2}, \tag{17}$$

$$B_1 = \frac{1}{2}\left[(1-p)\frac{3M_s^2 V^2 + \gamma(\gamma-2)\sigma_{ie}}{\left(M_s^2 V^2 - \gamma\sigma_{ie}\right)^3} - \left(\mu - \frac{p}{\sigma_{pe}^2}\right)\right], \tag{18}$$

$$D = (1-p)\frac{M_s^2 V^2}{\left(M_s^2 V^2 - \gamma\sigma_{ie}\right)^2}, \tag{19}$$

and the constant V is determined by

$$V^2 = 1. \tag{20}$$

Equation (16) describes the propagation of weakly nonlinear and weakly dispersive DIA waves in a nonthermal e-p-i-d plasma, where the parameters p, μ, β_e, γ, σ_{ie}, and σ_{pe} have some fixed values. For different sets of values of those parameters, we arrive at a different e-p-i-d plasma systems. As Equation (16) cannot describe the nonlinear dynamics of DIA waves when the coefficient AB_1 of the nonlinear term of (16) vanishes, it is of considerable importance to find for what values of the above mentioned parameters AB_1 becomes equal to zero. This consideration has led us to find a relationship between the parameters when $AB_1 = 0$. As $A \neq 0$ for any set of physically admissible values of the parameters of the system, $AB_1 = 0$ implies $B_1 = 0$. Now, it is simple to check that B_1 is a function of p, μ, and β_e, i.e., $B_1 = B_1(p, \mu, \beta_e)$ for fixed values of γ, σ_{ie}, and σ_{pe}. Therefore, $B_1 = B_1(p, \mu, \beta_e) = 0$ gives a functional relationship between μ and p for any fixed value of β_e within the physically admissible interval of β_e, i.e., $0 \leq \beta_e \leq 0.6$. In Figure 1, this functional relationship $(B_1(p, \mu, \beta_e) = 0)$ between μ and p is plotted for different values of β_e with $\gamma = 3$, $\sigma_{ie} = 0.9$ and $\sigma_{pe} = 0.9$. This figure shows the existence of a region in the parameter space where $B_1 = 0$, and in this region of parameter space, it is not possible to describe the nonlinear behaviour of DIA waves by the KP equation (16). So, a modification of the KP equation (16) is necessary. In Subsection III B, we have derived a Modified KP (MKP) equation in three dimensions when $B_1 = 0$.

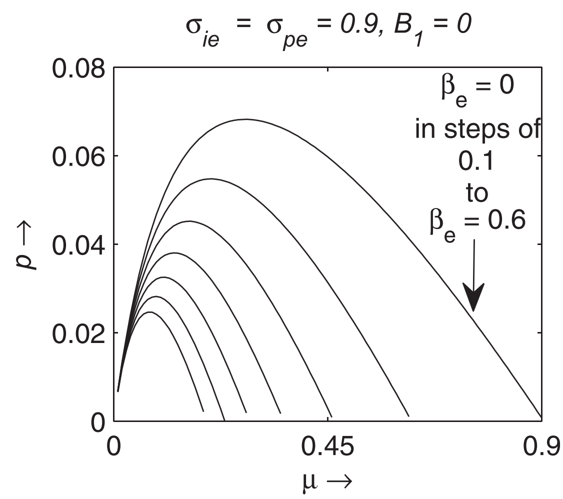

FIG. 1. p is plotted against μ for different values of β_e with $\gamma = 3$, $\sigma_{ie} = \sigma_{pe} = 0.9$ when $B_1 = 0$.

Figure 1 shows that for $p > 0.07$ (approximately) and for all $0 \leq \beta_e \leq 0.6$, B_1 is not equal to zero ($B_1 \neq 0$) for any physically admissible value of μ whereas for $p = 0$, i.e., when there is no positron in the system, for any physically admissible value of β_e, there exists a value $\mu = \mu_c$ such that $B_1 = 0$ at $\mu = \mu_c$. In fact, in Figure 2(a), B_1 is plotted against μ for $p = 0.08$ and for different values of β_e whereas in Figure 2(b), B_1 is plotted against μ for $p = 0$ and for different values of β_e. Figure 2(b) clearly shows that there exists a value μ_c of μ such that $B_1 = 0$ at $\mu = \mu_c$ whereas $B_1 > 0$ or $B_1 < 0$ according to whether $\mu > \mu_c$ or $\mu < \mu_c$. Figure 2(a) clearly indicates that B_1 is strictly positive for any physically admissible value of μ. So, there exists a region in parameter space where $B_1 = 0$.

B. Modified KP (MKP) equation in three dimensions

When $B_1 = 0$, we take the following perturbation expansions of the dependent variables:

$$f = f^{(0)} + \sum_{i=1}^{\infty} \varepsilon^{i} f^{(i)}, \quad g = g^{(0)} + \sum_{i=1}^{\infty} \varepsilon^{i+1} g^{(i)}, \tag{21}$$

where $f = n, P, \phi, u$ with $n^{(0)} = 1 - p, P^{(0)} = 1, \phi^{(0)} = 0$, $u^{(0)} = 0$, and $g = v, w$ with $v^{(0)} = w^{(0)} = 0$. Substituting the stretching (14) and perturbation expansions (21) in Equations (1), (2), (3), and (13) and finally equating the coefficient of different powers of ε on each side of every equation, we get a sequence of equations. From this sequence

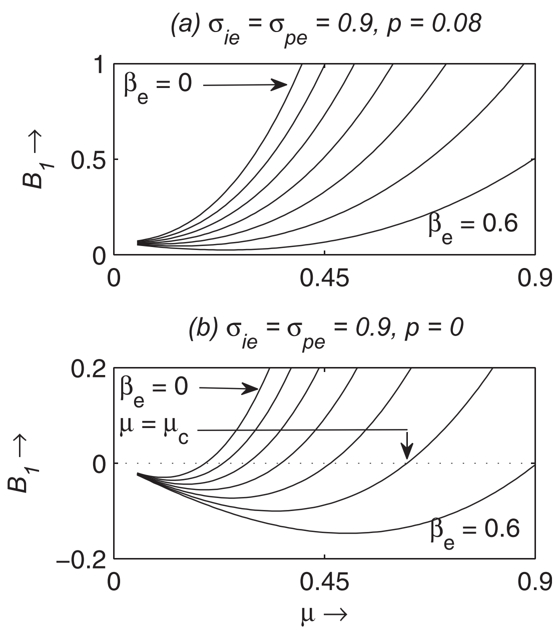

FIG. 2. In this figure, B_1 is plotted against μ for different values of β_e with $\gamma=3$, $\sigma_{ie}=\sigma_{pe}=0.9$ and in (a) $p=0.08$, in (b) $p=0$. (a) indicates that B_1 is strictly positive for any physically admissible value of μ. (b) clearly shows that there exists a value μ_c of μ such that $B_1=0$ at $\mu=\mu_c$ whereas $B_1>0$ or $B_1<0$ according to whether $\mu>\mu_c$ or $\mu<\mu_c$.

of equations, we get the following modified Kadomtsev Petviashvili (MKP) equation:

$$\frac{\partial}{\partial\xi}\left[\phi_{\tau}^{(1)}+AB_2\left(\phi^{(1)}\right)^2\phi_{\xi}^{(1)}+\frac{1}{2}AC\phi_{\xi\xi\xi}^{(1)}\right]+\frac{1}{2}AD\left(\phi_{\eta\eta}^{(1)}+\phi_{\zeta\zeta}^{(1)}\right)=0, \tag{22}$$

where

$$B_2=\frac{1-p}{4\left(M_s^2V^2-\gamma\sigma_{ie}\right)^5}\left[15M_s^4V^4+\gamma\left(\gamma^2+13\gamma-18\right)\times M_s^2V^2\sigma_{ie}+\gamma^2(\gamma-2)(2\gamma-3)\sigma_{ie}^2\right]-\frac{3}{2}Q_3. \tag{23}$$

A and D are given by (17) and (19), respectively, and V is determined by (20).

We have used the condition $B_1=0$ to eliminate the term $AB_1\frac{\partial^2}{\partial\xi^2}(\phi^{(1)}\phi^{(2)})$ from the final form of (22). It is also important to note that the Poisson equation at the order ϵ^2 is identically satisfied under the condition $B_1=0$ and dispersion relation (20). In fact, the MKP equation (22) is obtained from the equation of continuity, x-component of equation of motion and pressure equation at the order ϵ^4 together with the Poisson equation at the order ϵ^3. Therefore, if the Poisson equation at the order ϵ^2 is not identically satisfied, we cannot use the Poisson equation at the order ϵ^3 to get the MKP equation (22).

Equation (22) describes the nonlinear dynamics of DIA waves when $B_1=0$ and $B_2\neq 0$. But Equation (22) loses its nonlinear character when $B_1=B_2=0$. So, it is of considerable importance to find a relation(s) between the parameters of the present plasma system when B_1 and B_2 are simultaneously equal to zero. This consideration has led us to show a variation of μ against β_e in β_e–μ parameter plane when $B_1=B_2=0$. It is simple to check that B_1 and B_2 are the functions of p, μ, and β_e, i.e., $B_1=B_1(p,\mu,\beta_e)$ and $B_2=B_2(p,\mu,\beta_e)$ for prescribed values of γ, σ_{ie}, and σ_{pe}. Now $B_1(p,\mu,\beta_e)=0$ gives a cubic equation for the unknown p and consequently, for the given values of μ and β_e, the equation $B_1(p,\mu,\beta_e)=0$ gives a real solution for p. Let $p=p(\mu,\beta_e)$ be the physically admissible real solution of the equation $B_1(p,\mu,\beta_e)=0$, i.e., the physically admissible real solution p of the equation $B_1(p,\mu,\beta_e)=0$ can be considered as a function of μ and β_e. If we put this value of p $(=p(\mu,\beta_e))$ in the expression of B_2, then B_2 is a function of μ and β_e only, i.e., $B_2=B_2(p(\mu,\beta_e),\mu,\beta_e)$, and consequently, for given value of β_e, B_2 is a function of μ only and the solution of the equation $B_2(p(\mu,\beta_e),\mu,\beta_e)=0$ may give a physically admissible value of μ. Therefore, corresponding to each β_e within the interval $0\leq\beta_e\leq 0.6$, one can find a μ such that $p(=p(\mu,\beta_e))$, μ, β_e solve the equations $B_1=0$ and $B_2=0$ simultaneously. So, one can generate a set of values of μ by taking a variation of β_e within the interval $0\leq\beta_e\leq 0.6$ for a given step length. Using these two sets of data (one for β_e and other for μ), we get a curve in β_e–μ parameter plane and the values of B_1 and B_2 are simultaneously equal to zero at each point on the curve. Figure 3 shows a variation of μ against β_e in β_e–μ parameter plane when $B_1=B_2=0$ for $\gamma=3$ and $\sigma_{ie}=\sigma_{pe}=0.9$. This figure shows the existence of a curve in the β_e–μ parameter plane along which $B_1=0$ and $B_2=0$. Different values of σ_{ie} and/or σ_{pe} will give different curves in the β_e–μ parameter plane along which $B_1=0$ and $B_2=0$. Therefore, there exists a region in the parameter space where $B_1=B_2=0$, and consequently, in this region of parameter space, it is not possible to describe the nonlinear dynamics of DIA waves either by the KP equation (16) or by the MKP equation (22). So, a further modification of the MKP equation (22) is necessary. In Subsection III C, we have derived a Further Modified KP (FMKP) equation in three dimensions when $B_1=B_2=0$.

C. Further Modified KP (FMKP) equation in three dimensions

When $B_1=B_2=0$, we take the following perturbation expansions of the dependent variables:

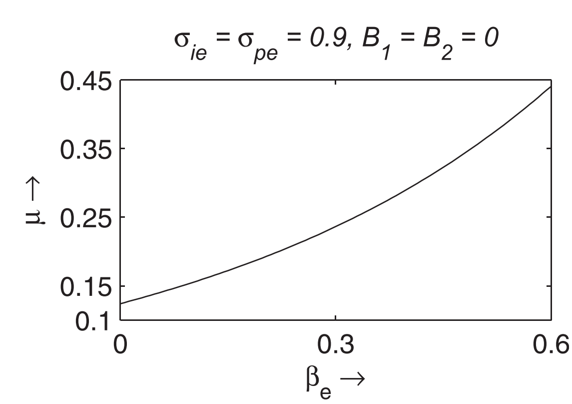

FIG. 3. μ is plotted against β_e for $\gamma=3$, $\sigma_{ie}=\sigma_{pe}=0.9$ when $B_1=B_2=0$.

$$f = f^{(0)} + \sum_{i=1}^{\infty} \varepsilon^{\frac{2}{3}i} f^{(i)}, \; g = g^{(0)} + \sum_{i=1}^{\infty} \varepsilon^{\frac{2}{3}i+1} g^{(i)}, \tag{24}$$

where $f = n, P, \phi, u$ with $n^{(0)} = 1 - p, P^{(0)} = 1, \phi^{(0)} = 0$, $u^{(0)} = 0$, and $g = v, w$ with $v^{(0)} = w^{(0)} = 0$. Substituting the stretching (14) and perturbation expansions (24) in Equations (1), (2), (3), and (13) and finally equating the coefficient of different powers of ε on each side of every equation, we get a sequence of equations. From this sequence of equations, we get the following further modified Kadomtsev Petviashvili (FMKP) equation:

$$\frac{\partial}{\partial \xi}\left[\phi_\tau^{(1)} + AB_3\left(\phi^{(1)}\right)^3 \phi_\xi^{(1)} + \frac{1}{2}AC\phi_{\xi\xi\xi}^{(1)}\right] + \frac{1}{2}AD\left(\phi_{\eta\eta}^{(1)} + \phi_{\zeta\zeta}^{(1)}\right) = 0, \tag{25}$$

where

$$B_3 = \frac{1-p}{12\left(M_s^2 V^2 - \gamma\sigma_{ie}\right)^7}[105 M_s^6 V^6 + \gamma(\gamma^3 + 21\gamma^2 + 161\gamma - 174) M_s^4 V^4 \sigma_{ie} + \gamma^2\left(8\gamma^3 + 53\gamma^2 - 162\gamma + 108\right) M_s^2 V^2 \sigma_{ie}^2 + \gamma^3(\gamma-2)(2\gamma-3)(3\gamma-4)\sigma_{ie}^3] - 2Q_4, \tag{26}$$

A, D, and V are given by (17), (19), and (20), respectively. Here, we have used the conditions $B_1 = 0$ and $B_2 = 0$ to eliminate the terms $AB_1 \frac{\partial^2}{\partial \xi^2}\left[\frac{1}{2}\left(\phi^{(2)}\right)^2 + \phi^{(1)}\phi^{(3)}\right]$ and $AB_2 \frac{\partial^2}{\partial \xi^2}\left[\left(\phi^{(1)}\right)^2 \phi^{(2)}\right]$ from the final form of Equation (25). It is also important to note that the Poisson equations at the order $\epsilon^{4/3}$ and at the order ϵ^2 are identically satisfied under the conditions $B_1 = 0$, $B_2 = 0$, and the dispersion relation (20). In fact, the FMKP equation (25) is obtained from the equation of continuity, x-component of equation of motion and pressure equation at the order $\epsilon^{11/3}$ together with the Poisson equation at the order $\epsilon^{8/3}$. Therefore, if the Poisson equations at the order $\epsilon^{4/3}$ and at the order ϵ^2 are not identically satisfied, we cannot use the Poisson equation at the order $\epsilon^{8/3}$ to obtain the FMKP equation (25).

Equation (25) describes the nonlinear dynamics of DIA waves in the present e-p-i-d plasma system when $B_1 = B_2 = 0$ but $B_3 \neq 0$. If $B_1 = B_2 = B_3 = 0$, the nonlinear dynamics of the DIA wave cannot be described by the evolution Equation (25) but it can be easily shown that when $B_1 = B_2 = 0$ then $B_3 > 0$. In fact, in Figure 4, B_3 is plotted against β_e when $B_1 = B_2 = 0$ for fixed values of σ_{ie} and σ_{pe}. This figure clearly indicates that $B_3 > 0$ when $B_1 = B_2 = 0$ for fixed values of σ_{ie} and σ_{pe}, and consequently, the FMKP equation is sufficient to describe nonlinear behaviour of DIA waves when $B_1 = B_2 = 0$.

Equations (16), (22), and (25) can be written in a more compact form as

$$\frac{\partial}{\partial \xi}\left[\phi_\tau^{(1)} + AB_r\left(\phi^{(1)}\right)^r \phi_\xi^{(1)} + \frac{1}{2}AC\phi_{\xi\xi\xi}^{(1)}\right] + \frac{1}{2}AD\left(\phi_{\eta\eta}^{(1)} + \phi_{\zeta\zeta}^{(1)}\right) = 0, \tag{27}$$

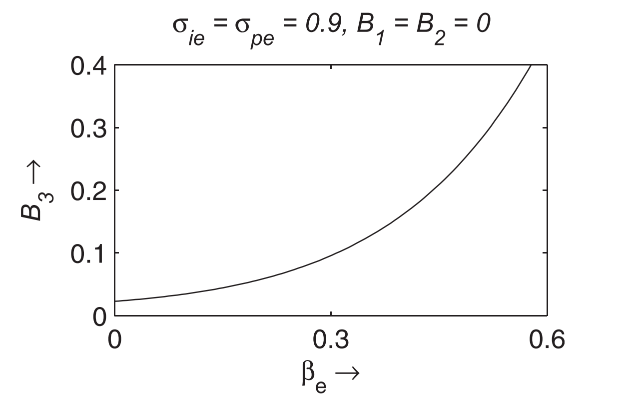

FIG. 4. B_3 is plotted against β_e for $\gamma = 3$, $\sigma_{ie} = \sigma_{pe} = 0.9$ when $B_1 = 0$ and $B_2 = 0$. This figure clearly indicates $B_3 > 0$ when $B_1 = B_2 = 0$.

where $r = 1, 2, 3$. But instead of considering $r = 1, 2, 3$, we consider the solitary wave solution and its stability analysis of Equation (27) for any fixed $r > 0$.

IV. SOLITARY-WAVE SOLUTION

For a solitary wave solution of (27), we take the following transformation of the independent variables:

$$X = \xi - U\tau, \quad \eta' = \eta, \quad \zeta' = \zeta, \quad \tau' = \tau. \tag{28}$$

Under the above change of the independent variables, Equation (27) assumes the following form (in which we drop the primes on the independent variables η, ζ, and τ to simplify the notations):

$$\frac{\partial}{\partial X}\left[-U\phi_X^{(1)} + \phi_\tau^{(1)} + AB_r\left(\phi^{(1)}\right)^r \phi_X^{(1)} + \frac{1}{2}AC\phi_{XXX}^{(1)}\right] + \frac{1}{2}AD\left(\phi_{\eta\eta}^{(1)} + \phi_{\zeta\zeta}^{(1)}\right) = 0. \tag{29}$$

Now, for the travelling wave solitons of (29), we set

$$\phi^{(1)} = \phi_0(X). \tag{30}$$

Substituting (30) in (29), using the boundary conditions $\phi_0, \frac{d\phi_0}{dX}, \frac{d^2\phi_0}{dX^2}, \frac{d^3\phi_0}{dX^3} \to 0$ as $|X| \to \infty$, the solitary wave solution of (29) propagating along X-axis can be written as

$$\phi_0 = a\left[\operatorname{sech}\left(\frac{X}{W}\right)\right]^{2/r}, \tag{31}$$

where a and W are, respectively, given as

$$a^r = \frac{(r+1)(r+2)U}{2AB_r} \quad \text{and } W^2 = \frac{2AC}{r^2 U}. \tag{32}$$

From the first equation of (32), we see that the amplitude a of the solitary wave solution (31) is not well defined when $B_r = 0$. This is the reason for the derivation of the different modified KP equations.

V. STABILITY ANALYSIS

In this section, we have investigated the stability of the solitary wave solution (31) of Equation (29) for long-wavelength plane-wave perturbation by the method of Rowlands and Infeld.[20–26] For this purpose, we decompose $\phi^{(1)}$ as

$$\phi^{(1)} = \phi_0(X) + q(X, \eta, \zeta, \tau). \tag{33}$$

Substituting (33) into (29) and then linearizing it with respect to q, we get the following equation:

$$\frac{\partial}{\partial X}\left[-U\frac{\partial q}{\partial X} + \frac{\partial q}{\partial \tau} + AB_r\frac{\partial}{\partial X}(\phi_0^r q) + \frac{1}{2}AC\frac{\partial^3 q}{\partial X^3}\right] + \frac{1}{2}AD\left[\phi^{(1)}_{\eta\eta} + \phi^{(1)}_{\zeta\zeta}\right] = 0. \tag{34}$$

Now, for a long-wavelength plane-wave perturbation along a direction having direction cosines (l, m, n), we set

$$q(X, \eta, \zeta, \tau) = \bar{q}(X)\exp[i\theta], \quad \theta = k(lX + m\eta + n\zeta) - \omega\tau, \tag{35}$$

where k is small and $l^2 + m^2 + n^2 = 1$.

Again, according to small-k perturbation expansion method of Rowlands and Infeld,[20–26] $\bar{q}(X)$ and ω can be expanded as follows:

$$\bar{q}(X) = \sum_{j=0}^{\infty} k^j q^{(j)}(X), \quad \omega = \sum_{j=0}^{\infty} k^j \omega^{(j)}, \tag{36}$$

with $\omega^{(0)} = 0$, $j = 0, 1, 2, \ldots$. Substituting (35) and (36) into (34), equating different powers of k on the both sides of Equation (34), we get the following sequence of equations:

$$U\frac{d}{dX}\left(L_r q^{(j)}\right) = M_r^{(j)}, \quad j = 0,\ 1,\ 2,\ \ldots, \tag{37}$$

where

$$L_r = -1 + \frac{1}{2}(r+1)(r+2)\text{sech}^2\left(\frac{X}{W}\right) + \frac{r^2W^2}{4}\frac{d^2}{dX^2}, \tag{38}$$

$$M_r^{(j)} = \int_{-\infty}^{X} Q_r^{(j)} dX, \tag{39}$$

and $Q_r^{(0)}$, $Q_r^{(1)}$, $Q_r^{(2)}$ are given in Appendix B.

To solve (37), we have assumed that each $q^{(j)}$ and its derivative up to third order vanish as $|X| \to \infty$. We have also assumed that each $q^{(j)}$ is bounded. Under these assumptions, the solution of Equation (37) for $j = 0$ and $j = 1$ can be written as

$$q^{(0)} = h_0\frac{d\phi_0}{dX}, \tag{40}$$

$$q^{(1)} = h_1\frac{d\phi_0}{dX} + ih_0\left(s_1\phi_0 + s_2X\frac{d\phi_0}{dX}\right), \tag{41}$$

where h_0, h_1 are constants and s_1, s_2 are given by

$$s_1 = \frac{rW^2}{2AC}\left\{u_1 - \frac{2}{(r+1)(r+2)}u_2 a^r\right\}, \tag{42}$$

$$s_2 = \frac{r^2W^2}{4AC}\left\{u_1 - \frac{4}{r^2W^2}u_3\right\}, \tag{43}$$

$$u_1 = \omega^{(1)} + 2lU,\ u_2 = 2lAB_r,\ u_3 = 2lAC. \tag{44}$$

Now, for the solution of Equation (37) to exist, the right hand side of Equation (37) must be perpendicular to the kernel of the operator adjoint to the operator $\frac{d}{dX}L_r$; this kernel, which must tend to zero, is ϕ_0. Thus, we get the following consistency condition for the existence of the solution of Equation (37):

$$\int_{-\infty}^{\infty} \phi_0 M_r^{(j)} dX = 0. \tag{45}$$

It can be easily checked that the consistency condition (45) is trivially satisfied for $j = 0$ and $j = 1$. Using (40) and (41), computing all the integrals appearing on the left hand side of Equation (45) for $j = 2$, it can be easily checked that the consistency condition (45) for $j = 2$ reduces to the following quadratic equation for $\omega^{(1)}$:

$$(r-4)(\omega^{(1)})^2 + 2rUV(m^2 + n^2) = 0. \tag{46}$$

For $r = \frac{1}{2}$, Equation (46) is exactly same as Equation (5.23) of Chakraborty and Das[27] and also Equation (5.23) of Chakraborty and Das[28] if we put $n = 0$.

For $r \neq 4$, the quadratic equation (46) can be written as

$$(\omega^{(1)})^2 = \frac{2r}{4-r}UV(m^2 + n^2). \tag{47}$$

Equation (47) shows that $\omega^{(1)}$ is real if $r < 4$, and consequently, the solitary wave solution (31) is stable for $r < 4$ whereas $\omega^{(1)}$ is imaginary if $r > 4$, and consequently, the solitary wave solution (31) is unstable for $r > 4$. So, the imaginary part $\Gamma(= Im(\omega^{(1)}))$ of $\omega^{(1)}$ is given by

$$\Gamma^2 = \frac{2r}{r-4}UV(m^2 + n^2). \tag{48}$$

This equation gives a positive value of Γ for any $r > 4$, which gives the growth rate of instability of the solitary wave solution (31) for $r > 4$.

The present method fails to analyse the stability of a modified KdV soliton if $r = 4$. For $r = 4$, from Equation (46), we get $8UV(m^2 + n^2) = 0$, and this relation holds good only when $m = n = 0$, i.e., when there is no transverse perturbation. But this is not possible. On the other hand, for $r = 4$, there does not exist any value of $\omega^{(1)}$ which solves Equation (46) for $m^2 + n^2 \neq 0$.

Islam *et al.*[33] investigated the stabilities of the obliquely propagating (δ = angle of propagation of the solitary wave with z-axis $\neq 0$) solitary wave solutions of the Korteweg-de Vries-Zakharov-Kuznetsov (KdV-ZK) and different modified KdV-ZK equations of the form

$$\phi^{(1)}_\tau + AB_r\left(\phi^{(1)}\right)^r\phi^{(1)}_\zeta + \frac{1}{2}A\phi^{(1)}_{\zeta\zeta\zeta} + \frac{1}{2}AD\frac{\partial}{\partial\zeta}\left(\phi^{(1)}_{\xi\xi} + \phi^{(1)}_{\eta\eta}\right) = 0, \tag{49}$$

where $r > 0$. They reported that solitary wave solutions are stable for $r \geq 4$, and under certain conditions, solitary wave solutions are unstable for $r < 4$. In the investigation of Islam *et al.*,[33] it is important to note that there does not exist any value of $\omega^{(1)}$ which solves Equation (3.33) of Islam *et al.*[33] when $\delta = 0$ and $r = 4$, and for this case, i.e., for $\delta = 0$, the solitary wave solutions of KdV-ZK different modified KdV-ZK equations are stable for $r > 4$ and unstable for $r < 4$. In the present paper, we see that the solitary wave solutions of KP and different modified KP equations are stable for $r < 4$ and unstable for $r > 4$.

VI. CONCLUSION

In the present problem, r takes the values 1, 2, and 3 only, and consequently, DIA solitary waves are stable at the lowest order of wave number.

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APPENDIX A: COEFFICIENTS OF DIFFERENT POWERS OF φ IN EQ. (13)

$$Q_1 = \mu(1-\beta_e) + p/\sigma_{pe}, \tag{A1}$$

$$Q_2 = \frac{1}{2}\left[\mu - p/\sigma_{pe}^2\right], \tag{A2}$$

$$Q_3 = \frac{1}{6}\left[\mu(1+3\beta_e) + p/\sigma_{pe}^3\right], \tag{A3}$$

$$Q_4 = \frac{1}{24}\left[\mu(1+8\beta_e) - p/\sigma_{pe}^4\right]. \tag{A4}$$

APPENDIX B: INTEGRAND OF THE INTEGRATION IN EQ. (39) FOR $J = 0$, 1 AND 2

$$Q_r^{(0)} = 0, \tag{B1}$$

$$Q_r^{(1)} = \frac{d}{dX}\left[-2il\left\{-Uq^{(0)} + AB_r\phi_0^r q^{(0)} + AC\frac{d^2q^{(0)}}{dX^2}\right\} + i\omega^{(1)}q^{(0)}\right], \tag{B2}$$

$$\begin{aligned} Q_r^{(2)} = &-2il\frac{d}{dX}\left[-Uq^{(1)} + AB_r\phi_0^r q^{(1)} + AC\frac{d^2q^{(1)}}{dX^2}\right] \\ &+ l^2\left[-Uq^{(0)} + AB_r\phi_0^r q^{(0)} + 3AC\frac{d^2q^{(0)}}{dX^2}\right] \\ &+ \frac{1}{2}AD(m^2+n^2)q^{(0)} \\ &+ i\frac{d}{dX}\left[\omega^{(1)}q^{(1)} + \omega^{(2)}q^{(0)}\right] - l\omega^{(1)}q^{(0)}. \end{aligned} \tag{B3}$$

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