

Some low energy effects of a light stabilized radion  
in the Randall-Sundrum model

Uma Mahanta, Mehta Research Institute  
Chhatnag Road, Jhusi, Allahabad-211019, India  
and  
Subhendu Rakshit, Department of Physics  
University of Calcutta, Calcutta-700009, India

**Abstract**

In this paper we study some of the low energy effects of a light stabilized radion field in the Randall-Sundrum scenario. We find that the NLC 500 with its projected precision level will be able to probe the radion contribution to  $\kappa_v$  and  $\lambda_v$  for values of  $\langle\phi\rangle$  up to 500 Gev. On the other hand the BNL E821 experiment will be able to test the radion contribution to  $a_\mu$  for  $\langle\phi\rangle=1$  Tev and  $m_\phi \leq m_\mu$ . We have also shown that the higgs-radion mixing induces a 2.6% correction in the WW $\gamma$  coupling. Finally by comparing the radionstrahlung process with the higgsstrahlung process we have found that the LEPI bound of 60 Gev on the higgs mass based on  $Z \rightarrow h\bar{l}l$  decay mode suggests a lower bound of about 35 Gev on the radion mass.

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Recently Goldberger and Wise [1] have shown that the separation between the branes in the Randall-Sundrum (R-S) scenario [2] can be stabilised by a bulk scalar field. They also showed [3] that if the large value of  $kr_c \approx 12$  that is necessary for solving the hierarchy problem arises from a small bulk scalar mass term then the modulus potential is nearly flat near its minimum. The R-S scenario therefore predicts a modulus field which is much lighter than the Kaluza-Klein excitations of the bulk fields which usually lie in the TeV range. The modulus field is therefore expected to be the first experimental signal of R-S model. Its couplings to ordinary fields on the visible brane is suppressed by the TeV scale and completely determined by general covariance. Although the radion vev is more or less well determined in the R-S scenario its mass is an almost free parameter. It turns out to be much lighter than 1 TeV only if the mass of the bulk scalar field that stabilizes the modulus is smaller than 1 TeV. It is the purpose of this paper to investigate what bounds can be derived on the radion vev and particularly its mass from precision tests and direct searches at  $e^+e^-$  collider.

The couplings of the modulus field  $\phi$  to ordinary matter on the visible brane is given by [3]

$$L_I = \frac{T_\mu^\mu}{\langle\phi\rangle} \tilde{\phi}. \quad (1)$$

where  $\phi$  is expanded about its vev as  $\phi = \tilde{\phi} + \langle\phi\rangle$ . In the context of the SM, ignoring the contribution of the higgs sector, we have  $T_\mu^\mu = \sum_f m_f \bar{f}f + 2m_z^2 Z^\mu Z_\mu + 2m_w^2 W^\mu W_\mu$ . The couplings of  $\tilde{\phi}$  to SM fields are therefore suppressed by  $\langle\phi\rangle$  which is in the TeV range. Since conformal invariance is broken explicitly by the mass of the SM fields, the effects of  $\tilde{\phi}$  on low energy phenomenology ( i.e. much below the Kaluza-Klein excitations) can become appreciable only if the SM fields involved are quite heavy and lie in the few hundred GeV range. In discussing the low energy phenomenology of the radion we shall consider particularly those processes that involve the couplings of  $\tilde{\phi}$  to the heavy SM fields e.g. Z, W and t.

### Loop induced virtual effects

Let us first consider the effects of  $\tilde{\phi}$  through loop induced radiative corrections which can be tested through high precision tests. In this category we shall discuss the effect of  $\tilde{\phi}$  on anomalous  $W^+W^-\gamma$ ,  $W^+W^-Z$  couplings and the renormalization of  $Zt_R\bar{t}_R$  coupling. Besides we shall also consider the radion contribution to the magnetic moment anomaly of the muon. Unless stated otherwise we shall assume that  $\langle\phi\rangle=1$  Tev.

i) Anomalous magnetic moment of W boson: The anomalous magnetic moment of W boson is given by the operator [4]

$$L_{WWV} = ie\kappa_v W_\mu^+ W^- \nu V^{\mu\nu}. \quad (2)$$

where  $V^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu$  and  $V=A, Z$ .  $\kappa_v$  is the anomalous magnetic moment of the W boson. The value of  $\kappa_v$  arising from SM interactions at one loop is of the order of  $6 \times 10^{-3}$ . The radion contribution to  $\kappa_v$  arises from only one diagram. After pulling out one momentum corresponding to the incoming  $\gamma$  or Z the remaining scalar integration can be evaluated by naive power counting. For  $m_\phi \leq m_w$  the integral gets most of its contribution from loop momenta of order  $m_W$  and is given by  $I(m_\phi^2, m_w^2) \approx \frac{1}{16\pi^2 m_w^2} f(\frac{m_\phi^2}{m_w^2})$  where  $f(\mu^2) = \int_0^1 x^2 \frac{x^2 - x + 2 + \frac{\mu^2}{2}(1-x)}{x^2 + \mu^2(1-x)}$  and  $\mu = \frac{m_\phi}{m_w}$ . For  $m_\phi \ll m_w$  the radion contribution to  $\kappa_v$  is therefore given by  $\kappa_v^{\tilde{\phi}} \approx \frac{1}{16\pi^2} \frac{m_w^2}{\langle\phi\rangle^2} f(\frac{m_\phi^2}{m_w^2}) \approx 3.7 \times 10^{-4}$ . The anomalous magnetic moment of the W boson arising from radion exchange is therefore roughly one order of magnitude smaller than that from SM processes. The size of the above radiative correction however decreases inversely as  $\langle\phi\rangle^2$ . Thus for  $\langle\phi\rangle = 500$  Gev the value of  $\kappa_v^{\tilde{\phi}}$  increases to  $1.5 \times 10^{-3}$ . However the dependence of  $\kappa_v^{\tilde{\phi}}$  on  $\frac{m_\phi^2}{m_w^2}$  is almost flat over the entire range of the variable and therefore no useful bound on  $m_\phi$  can be obtained.

ii) Electric quadrupole moment of W boson: The electric quadrupole moment of the W boson is given by the operator [4]

$$L_{WWV} = ie \frac{\lambda_v}{m_w^2} W_{\lambda\mu}^+ W_\nu^- \nu V^{\nu\lambda}. \quad (3)$$

In SM  $\lambda_v$  is zero at tree level. As in the case of  $\kappa_v$  the radion contribution to  $\lambda_v$  arises from only one diagram. To evaluate the electric quadrupole moment of the W boson arising from radion exchange we need to extract three momenta corresponding to the three external gauge bosons outside the loop integral. The remaining integral can be shown to converge as  $\frac{1}{m_w^4} g(\frac{m_\phi^2}{m_w^2})$ . The radion contribution to  $\lambda_v$  is therefore given by  $\lambda_v \approx \frac{1}{16\pi^2} \frac{m_w^2}{\langle\phi\rangle^2} g(\frac{m_\phi^2}{m_w^2}) \approx (1.6 - 3.2) \times 10^{-4}$  where we have taken  $g(x)$  to lie between 1 and 2. At one loop order SM processes gives rise to a  $\lambda_v$  that is of order  $5 \times 10^{-3}$ . The modulus contribution to  $\lambda_v$  is therefore suppressed relative to the SM contribution by one order of magnitude.

The sensitivity reach of different colliders for measuring  $\kappa_v$  and  $\lambda_v$  have been estimated by several working groups. These studies [5] indicate that LEP II with a design luminosity of  $500 \text{ pb}^{-1}$  will be able to measure  $\kappa_v$  and  $\lambda_v$  with a precision of a few times  $10^{-2}$ . On the other hand NLC 500 with a design luminosity of  $10 \text{ fb}^{-1}$  will be able to reach precision levels of about  $10^{-3}$  which will enable it to probe the radion contributions for  $\langle\phi\rangle$  up to 500 Gev. If  $\langle\phi\rangle$  lies in the few Tev range then NLC 500 will not be able to probe the radion contribution to  $\kappa_v$  and  $\lambda_v$ . However we have seen that increasing the center of mass energy from 200 Gev to 500 Gev enhances the sensitivity reach for measuring  $\kappa_v$  and  $\lambda_v$  by one order of magnitude. Hence a multi Tev  $e^+e^-$  collider with a very high luminosity may be able to probe the small radion contribution to  $\kappa_v$  and  $\lambda_v$  for  $\langle\phi\rangle = 1$  Tev.

iii) Effect of  $\tilde{\phi}$  on the renormalization of  $Zt\bar{t}$  couplings: The radion contribution on the renormalization of  $Zt\bar{t}$  couplings is important because top quark is by far the heaviest known particle and has the strongest coupling to the radion. The stabilized modulus of Golberger and Wise is expected to be lighter than 1 Tev and therefore it would contribute to the renormalization of  $Zt\bar{t}$  coupling from  $m_t$  to 1 Tev. There are two distinct vertex correction diagrams due to  $\tilde{\phi}$  exchange. The dominant diagram involves the exchange of  $\tilde{\phi}$  between the outgoing  $t_R$  and  $\bar{t}_R$ . The vertex renormalization constant corresponding to this diagram is given by  $Z_v^1 = 1 - \frac{1}{32\pi^2} \ln \frac{q^2 g_t^t}{m_t^2 g_R^t} (\frac{m_t}{\langle\phi\rangle})^2$  where  $q$  is the momentum of the incoming Z boson,  $g_L^t = \frac{1}{2} - \frac{2}{3} \sin^2 \theta_w$  and  $g_R^t = -\frac{2}{3} \sin^2 \theta_w$ . In the second diagram Z goes into virtual  $\tilde{\phi}$  and Z which then exchanges a top quark to

produce a  $t_R, \bar{t}_R$  pair in the final state. This diagram involves a chirality flip through a mass insertion on the internal top quark line. The renormalization constant corresponding to this diagram does not involve a leading log term and therefore it does not contribute to the renormalization of  $Zt\bar{t}$  couplings. The self energy correction of Z due to  $\tilde{\phi}$  produces only a mass renormalization of Z but no wavefunction renormalization. The wavefunction renormalization of  $t_R$  arising from  $\tilde{\phi}$  exchange is given by  $Z_{t_R} = 1 + \frac{1}{32\pi^2} (\frac{m_t}{\langle\phi\rangle})^2 \ln \frac{q^2}{m_t^2}$ . The effect of a light radion on the renormalization of  $g_R^t$  from  $m_t^2$  to  $q^2 = (1000\text{Gev})^2$  is therefore given by

$$\frac{[g_R^t(q)]_{RS}}{[g_R^t(q)]_{SM}} = 1 + (1 - \frac{g_L^t}{g_R^t}) (\frac{m_t}{\langle\phi\rangle})^2 \frac{1}{32\pi^2} \ln \frac{q^2}{m_t^2}. \quad (4)$$

where we have assumed that  $g_R^t(m_t)_{RS} = g_R^t(m_t)_{SM} = g_R^t(m_t)_{expt}$ . The presence of an extra light field ( $\tilde{\phi}$ ) in the RS scenario causes the renormalization of  $g_R^t$  to deviate from that in the SM by .1% for  $\langle\phi\rangle = 1$  Tev. The splitting increases to .004% for  $\langle\phi\rangle = 500$  Gev. This effect is of the same magnitude as the radiative corrections due to ordinary QED processes and it might be possible to detect this effect at a multi Tev scale  $e^+e^-$  collider. Note that the splitting between  $[g_R^t(q)]_{RS}$  and  $[g_R^t(q)]_{SM}$  increases logarithmically with the high energy scale at which they are compared.

iv) The muon magnetic moment anomaly: Although the muon coupling to the radion is small the muon magnetic moment anomaly is an extremely well measured quantity. The present experimental value of  $a_\mu$  is given by [6]

$$a_\mu^{exp} = (116592.30 \pm .8) \times 10^{-8} \quad (5)$$

The extremely high precision with which  $a_\mu$  can be measured compensates for the low radion coupling strength to the muon. The radion contribution to the muon anomaly arises from only one diagram and is given by

$$a_\mu^{\tilde{\phi}} = \frac{1}{4\pi^2} (\frac{m_\mu}{\langle\phi\rangle})^2 \int_0^1 dx \frac{x^2(2-x)}{x^2 + r(1-x)}. \quad (6)$$

where  $r = (\frac{m_{\tilde{\phi}}}{m_\mu})^2$  is a free parameter. An upper bound on  $a_\mu^{\tilde{\phi}}$  can be obtained by setting  $r=0$ . In this limit we get  $a_\mu^{\tilde{\phi}} \approx 4.4 \times 10^{-10}$ . This value should be compared with the present experimental precision of  $10^{-8}$  in measuring  $a_\mu$ . The BNL experiment (E821)[7] hopes to lower the error the error down to  $4 \times 10^{-10}$ . At that level it will be able to probe the small radion contribution to  $a_\mu$ . As  $\frac{m_\phi^2}{m_\mu^2}$  becomes much greater than one the value of  $a_\mu^{\tilde{\phi}}$  decreases from the above estimate. However for all  $m_\phi \leq m_w$  the dependence of  $a_\mu^{\tilde{\phi}}$  on  $m_\phi$  is almost flat. So even with the BNL precision neither a sharp nor a useful bound can be put on  $m_\phi$ . It is worth noting that the bound from muon anomaly experiments are relevant only for very light radion ( $m_\phi \leq m_\mu$ ). For such low radion mass the stabilization of the modulus is very weak.

iv) Radiative mixing between the radion and the higgs: Since the couplings of the radion and the higgs boson to SM particles are similar in structure they can mix through loop corrections. The dominant correction comes from the top loop because of a color factor and its large yukawa coupling. It can be shown that this contribution is given by

$$\delta m_\phi^2 \approx -\frac{3N_c}{4\pi^2} \frac{m_t^2}{v\langle\phi\rangle} m_t^2 \ln \frac{m_t^2}{\mu^2} \quad (7)$$

where  $\mu$  is the renormalization scale. For  $\mu = m_h = m_\phi \approx 100$  Gev the mixing angle is given by  $\theta \approx \frac{\delta m_\phi^2}{m_h^2} \approx .11$ . This mixing between the radion and the higgs boson will shift the coupling of the physical higgs boson to W and Z. For example the WWh coupling is now given by

$$L_{wwh} = [gm_w + 2m_w \frac{m_w}{\langle\phi\rangle} \theta] WW h = gm_w [1 - .026] WW h. \quad (8)$$

. which implies a 2.6% correction to the WWh coupling. The shift in the Z coupling due to  $\phi - h$  mixing is also of the same order. Needless to say that probing this shift is a challenging task but it should be reachable at NLC 500 with a high luminosity.

## Direct production processes

Let us now consider the effects arising from direct production of  $\tilde{\phi}$ . The prominent among them is the radionstrahlung at an  $e^+e^-$  collider. It involves the radiation of  $\tilde{\phi}$  from an on shell (LEP I) or off shell (LEP II or NLC) Z boson. This process is similar to higgsstrahlung which is used for higgs search at  $e^+e^-$  collider. At LEP II higgs boson is produced by the process  $e^+e^- \rightarrow Z^* \rightarrow h + Z$ . It can be shown that the momentum of the outgoing Z boson is given by  $p_h^z = \frac{1}{2\sqrt{s}}[s - (m_h - m_z)^2]^{\frac{1}{2}}[s - (m_h + m_z)^2]^{\frac{1}{2}}$ . Hence the momentum of the outgoing Z boson for a 50 GeV higgs production at  $\sqrt{s}=160$  GeV will be given by  $p_h^z = 37$  GeV. On the other hand for a 20 GeV radion emission its momentum will be given by  $p_\phi^z=51$  GeV. So by imposing the cut  $p_z > 45$  GeV it should be possible to suppress the SM higgs production for  $m_h \geq 50$  GeV while still allowing considerable amount of radion production with a lower mass.

The  $\tilde{\phi}ZZ$  coupling is suppressed relative to  $hZZ$  coupling by a factor of  $(\frac{\sin 2\theta_w}{e} \frac{m_z}{\langle\phi\rangle})$ . Hence for a given  $\sqrt{s}$  the production cross section for  $\tilde{\phi}$  will be comparable to that of higgs(h) only if  $m_{\tilde{\phi}}$  is light but  $m_h$  is close to its kinematic limit so that its production is phase space suppressed. We find that at LEP II for a cm energy of 189 GeV,  $\sigma_{\tilde{\phi}z} \approx .089$  pb for  $m_{\tilde{\phi}} = 20$  GeV and  $\langle\phi\rangle = 1$  TeV, but  $\sigma_{hz} = .078$  pb for  $m_h = 97$  GeV. For  $m_{\tilde{\phi}} = 20$  GeV and  $m_h = 97$  GeV the branching ratio for  $\phi \rightarrow b\bar{b}$  and  $h \rightarrow b\bar{b}$  are both roughly equal to one. Hence in this case the number of  $q\bar{q}b\bar{b}$  events for example arising from  $e^+e^- \rightarrow hz$  will be roughly equal to that arising from  $e^+e^- \rightarrow \tilde{\phi}z$ . Let us consider the extreme scenario where the higgs is quite heavy so that it is kinematically inaccessible at LEP II but  $m_{\tilde{\phi}}$  lies in the 20 GeV range. In this case one can derive a lower bound on  $m_{\tilde{\phi}}$  from the condition  $S \leq 2\sqrt{S+B}$  where S is the signal and B is the background. The L3 collaboration at LEP II have placed a lower limit of 96 GeV on the higgs mass at 95% CL based on  $176 \text{ pb}^{-1}$  of data collected at  $\sqrt{s} = 189$  GeV [8]. If we make the somewhat unrealistic assumption that the cut efficiencies and background estimates are the same for the radion then this would imply a lower bound of about 20 GeV on the radion mass. It should however be noted that the optimal cuts that are necessary to enhance the signal to background ratio for a 20 GeV radion will be quite different from that for a 97 GeV higgs. Therefore to obtain a realistic bound on  $m_{\tilde{\phi}}$  from LEP II the different working groups will have to retune their cuts appropriately so that they correspond to light ( $m_\phi \approx 20$  GeV) radion detection. Finally let us consider the implication of LEPI data on the radion mass. At LEPI the higgs scalar has been searched based on  $Z \rightarrow hZ^*$  with the  $Z^*$  going into  $\bar{l}l$ . LEPI has placed a lower bound of about 60 GeV on  $m_h$  by looking for this decay mode. The branching ratio for the above decay mode for  $m_h = 60$  GeV is around  $1.4 \times 10^{-6}$ . The smaller radion coupling to the Z boson implies that the same branching ratio will be attained for  $Z \rightarrow \tilde{\phi}\bar{l}l$  at  $m_\phi = 35$  GeV. We therefore think that the present collider data on direct higgs searches does not rule out the existence of a light radion above 35 GeV. We would like to point out that these values are indications of the likely bounds on the radion mass and should not be considered as realistic collider bounds. More dedicated searches and detailed analysis are necessary to arrive at realistic collider bounds.

## Conclusions

In this paper we have studied some low energy effects of a light stabilized radion in the R-S scenario. We have found that the radion contribution to the anomalous magnetic moment and electric quadrupole moment of the W boson will fall beyond the projected sensitivity reach of NLC 500 if  $\langle\phi\rangle=1$  TeV. However the planned BNL experiment might be able to reach a sensitivity level just adequate for probing the radion contribution to  $a_\mu$  if the radion mass is around 100 MeV. We have also shown that the higgs-radion mixing through loop corrections induces a shift in the  $WW_h$  coupling of the order of 2.6%. The size of the radiative corrections arising from radion exchange however decreases inversely as  $\langle\phi\rangle^2$ . Therefore with decreasing  $\langle\phi\rangle$  the radiative corrections increase rapidly in size and approach the search limits. The dependence of the radiative corrections on the radion mass however does not produce any useful bound on it. We have also shown that the presence of a light radion field will cause the renormalization of  $Zt_R\bar{t}_R$  coupling to deviate from that in the SM by about .1%. Finally by comparing the radion production cross section at LEP II with that of the higgs boson we find that the lower bound of 96 GeV on the higgs mass suggests a bound of about 20 GeV on the radion mass. The LEPI bound of 60 GeV on  $m_h$  however implies a slightly stronger bound of 35 GeV on  $m_\phi$ .

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#### References

1. W.D. Goldberger and M. B. Wise: Phys. Rev. Lett 83, 4922 (1999); Phys. Rev. D 60, 107505 (1999).
2. L. Randall and R. Sundrum: Phys. Rev. Lett. 83, 3370 (1999); Phys. Rev. Lett. 83, 4690 (1999).
3. W. D. Goldberger and M. B. Wise, hep-ph/9911457. C. Csaki, M. Graesser, L. Randall and J. Terning, hep-ph/9911406.
4. K. Hagiwara, R. Peccei, D. Zeppenfeld and K. Hikasa, Nucl. Phys. B 282, 253 (1987).
5. Tests of alternative models at a 500 GeV NLC, ENSLAPP-A-365/92. Published in “ $e^+e^-$  collisions at 500 GeV, the physics potential” ed. P. Zerwas, DESY, 1992.
6. Caso et al., Particle Data group: Review of Particle Physics, Euro. Phys. J. C3, 1 (1998).
7. By E821 BNL collaboration (B. C. Roberts et al.). Published in the proceedings of XXVIII International Conference on High Energy Physics, Warsaw, Poland, 1996, River Edge, NJ World Scientific, 1997, p.1035.
8. Search for the SM higgs boson in  $e^+e^-$  interactions at  $\sqrt{s}=189$  GeV, L3 collaboration, CERN-EP/99-080.