Solitary waves in two-dimensional dusty plasma crystal: Effects of weak magnetic field

Samiran Ghosh[1,a] and M. R. Gupta[2]
[1]*Department of Applied Mathematics, University of Calcutta, 92, Acharya Prafulla Chandra Road, Kolkata-700 009, India*
[2]*Department of Instrumentation Science, Jadavpur University, Kolkata-700 032, India*

(Received 8 October 2009; accepted 22 February 2010; published online 29 March 2010)

It is shown that in the presence of weak magnetic field, the dust lattice solitary wave in two-dimensional (2D) hexagonal dusty plasma crystal is governed by a gyration-modified 2D Korteweg-de Vries equation due to the action of Lorentz force on the dust particles. Numerical solutions reveal that only for weak magnetic field an apparently single hump solitary wave solution exist. But, for strong magnetic field dust lattice solitary wave becomes unstable showing repetitive solitary hump of increasing magnitude with time. © *2010 American Institute of Physics*. [doi:10.1063/1.3361162]

The Coulomb complex (dusty) plasma consists of a finite number of charged microparticles, i.e., charged dust grains interacting via a shielded strong Coulomb potential and confined by external forces (e.g., of electrostatic nature). The interaction potential is believed to be of the Yukawa type. In thermodynamic equilibrium, this complex (dusty) plasma is characterized by the Coulomb coupling parameter Γ which is defined as the ratio of the electrostatic potential energy to the thermal energy between two neighboring dust particles. The system is called weakly coupled when $\Gamma \ll 1$, whereas the system is called strongly coupled when $\Gamma > 1$. The interesting feature is that $\Gamma > \Gamma_{\text{cr}}$ (critical value) $\gg 1$, the system is in solid state and form ordered crystalline structures which is known as dusty plasma crystal.[1–3] This dusty plasma crystal supports wave modes such as longitudinal and transverse dust lattice wave (DLW).[4–6] In the absence of an external magnetic field, the linear wave propagation characteristics as well as nonlinear structures such as Mach cones,[7,8] solitons[9–13] and also shock[14–16] were extensively studied experimentally and theoretically in one-dimensional (1D) as well as in two-dimensional (2D) dusty plasma crystals.

However, the magnetic field plays an important role for many complex (dusty) plasma environments. The influences of an external magnetic field were investigated for oscillations in 1D particle chain in a crystals.[17] Recent theoretical investigations[18,19] predict the existence of "upper-hybrid dust lattice wave," "lower-hybrid dust lattice wave," and the coupling of these two modes in presence of a constant external magnetic field in a 2D hexagonal dusty plasma crystal due to the Lorentz force acting on the dust particles. In this brief communication, the effects of an external weak magnetic field on quasilongitudinal dust lattice solitary wave in 2D dusty plasma crystal are reported.

The dusty plasma consists of electrons, ions, and small particles of solid matter i.e., the dust grain with (constant) negative charge Q and (constant) mass m_d. A single layer 2D hexagonal lattice structure is considered (Fig. 1). In the unperturbed situation, the particle coordinate is $(x_s=\Delta(n+m/2),\quad y_s=m\sqrt{3}\Delta/2)$, where Δ is the interdust distance, i.e., lattice spacing and $s=(n,m)$ denotes a pair of integers such that $s=\{(0,\pm 1),(\pm 1,0),(1,-1),(-1,1)\}$. The constant external magnetic field $\vec{B}(=B_0\hat{z})$ of field strength B_0 is pointing perpendicular to the particle layer.

The following assumptions are made to formulate the problem and to find the explicit final results:

(i) In the presence of strong magnetic field, charged dust grains, electrons, and ions are magnetized. However, for a very low frequency (ω), long wavelength, longitudinal motion with $\omega \ll k_\perp V_{te(i)}$ and $k_\perp \rho_{e(i)} \ll 1$ [where $k_\perp$ is the longitudinal wave number, i.e., wave number perpendicular to the direction of magnetic field, $V_{te(i)}$ and $\rho_{e(i)}$ are the electron (ion) thermal velocity and electron (ion) gyroradius], the electron (ion) polarization effect is insignificant and the electrons (ions) are rapidly thermalized along the Z axis to establish a Boltzmann response. Therefore, it is assumed that the interaction potential between each particle is the usual Debye–Hückel interaction potential (Yukawa potential). The corresponding interdust force between the sth and $\acute{s}$th particle is $\vec{F}_{\acute{s},s}(=-\vec{\nabla}_{r_s}[Q^2\exp(-r_{\acute{s},s}/\lambda_D)/4\pi\varepsilon_0 r_{\acute{s},s}])$, where ϵ_0 is the permittivity of free space, $r_{\acute{s},s}=|\vec{r}_{\acute{s},s}|=|\vec{r}_{\acute{s}}-\vec{r}_s|$, and λ_D is the plasma Debye Length, which is assumed to be constant.

(ii) In strongly coupled complex plasma, the charged and magnetized dust grains also interact with each other due to induced magnetic dipoles via a dipole magnetic force.[20,21] However, it is found[21] that in the presence of a weak magnetic field, there exist experimental situations, where electrostatic Yukawa force dominates over the magnetic force. Therefore, dust-dust interactions due to magnetic force are neglected in comparison with the Yukawa force.

(iii) Experimental observation and 2D molecular dynamics simulation[7] reveal that the nearest neighbor approximation is well justified for 2D monolayer hexagonal dusty plasma crystals for lattice parameter

[a]Electronic mail: sran_g@yahoo.com.

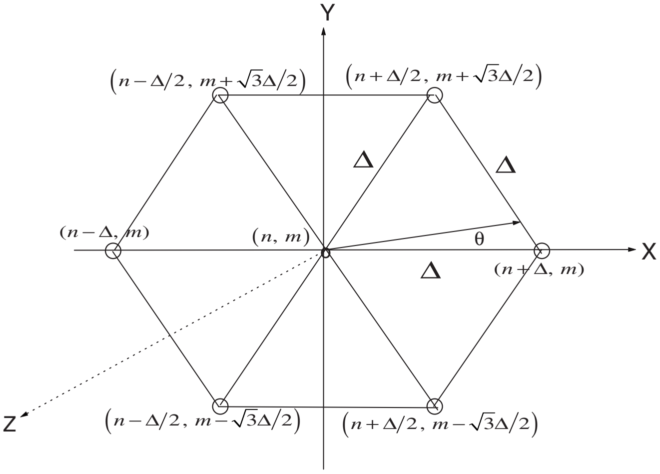

FIG. 1. Elementary single layer hexagonal 2D crystal centered at (n,m) with lattice spacing Δ. The quasilongitudinal wave propagation vector makes an angle θ with the x-axis.

$\kappa(=\Delta/\lambda_D)\geq 2.1$. Therefore, it is assumed that $\kappa \geq 2.1$, so that the dust particles in this monolayer 2D hexagonal crystal (Fig. 1) interact with their six nearest neighbor.

(iv) To incorporate the effects of a weak magnetic field on nonlinear quasilongitudinal DLW, it is assumed that the dust cyclotron frequency $\omega_{cd}(=QB_0/m_d)$ is small compared to the dust lattice frequency $\omega_L(=C_{DL}/\Delta)$ i.e., $\omega_{cd}/\omega_L \ll 1$, but $\neq 0$, where $C_{DL} = \sqrt{Q^2 e^{-\kappa}(\kappa^2+2\kappa+2)/4\pi\epsilon_0 m_d \kappa \lambda_D}$ is the dust lattice speed in the nearest neighbor approximation.[9,11]

To write the equation of motion under the action of weak external force, let us assume that $\vec{r}_{\acute{s},s}=\vec{r}_{\acute{s},s}(0)+\vec{D}(\acute{s},s)$, where $\vec{r}_{\acute{s},s}(0)$ is the relative equilibrium position and $\vec{D}(\acute{s},s)$ is the relative displacement of the sth lattice under the action of this weak force. If $d_l(s)$ and $d_t(s)$ are, respectively, the $\hat{x}$-direction (longitudinal) and $\hat{y}$-direction (transverse) displacement components, then $\vec{D}(\acute{s},s)\{=[d_l(\acute{s})-d_l(s)]\hat{x}+[d_t(\acute{s})-d_t(s)]\hat{y}\}$. Therefore, due to the above assumptions (i)–(iii), the equation of motion of the sth particle in the $\hat{x}$-direction and $\hat{y}$-direction are as follows:

$$\frac{\partial^2 d_l}{\partial t^2} = \frac{(\Sigma_{s\neq \acute{s}} F_{s,\acute{s}})_x}{m_d} + \omega_{cd}\frac{\partial d_t}{\partial t},$$
$$\frac{\partial^2 d_t}{\partial t^2} = \frac{(\Sigma_{s\neq \acute{s}} F_{s,\acute{s}})_y}{m_d} - \omega_{cd}\frac{\partial d_l}{\partial t}, \tag{1}$$

where $s,\acute{s}\in\{(0,\pm 1),(\pm 1,0),(1,-1),(-1,1)\}$. The first term on the right-hand side (RHS) of both equations in Eq. (1) are, respectively, the X and Y components of the Yukawa force, whereas the second term on RHS of both equations arises as a result of the Lorentz force acting on the dust grains due to the magnetic field $(=B_0\hat{z})$.

To study the propagation characteristics of small amplitude nonlinear quasilongitudinal DLW, the reductive perturbation technique is adopted and the following stretched coordinates are introduced:

$$\xi = \epsilon\frac{(x-\Lambda t)}{\Delta}, \quad \eta = \epsilon^2\frac{y}{\Delta}, \quad \tau = \epsilon^3 \omega_L t, \tag{2}$$

where ϵ is a small parameter that indicates the magnitude of the rate change and Λ is the wave velocity. It is to be noted that for such a quasilongitudinal wave, the transverse displacements have a higher-order smallness than the amplitude of longitudinal displacements $(d_t=\epsilon d_l)$ and thus the dependent variables (d_l,d_t) are expanded in powers of ϵ in the following way:

$$d_l = \epsilon d_l^{(1)} + \epsilon^2 d_l^{(2)} + \cdots; \quad d_t = \epsilon^2 d_t^{(1)} + \epsilon^3 d_t^{(2)} + \cdots. \tag{3}$$

Also, because of the assumption (iv) and for the perturbation consistent with Eqs. (2) and (3), the following scaling is assumed:

$$\frac{\omega_{cd}}{\omega_L} = \Omega\epsilon^2, \tag{4}$$

where Ω is of the order of unity, i.e., $\Omega \sim O(1)$. To consider the continuum approximation, it is assumed that the characteristic scale length L (the typical scale length of the wave form) is much larger than the lattice spacing Δ so that $s=(n,m)$ can be considered as quasicontinuous variable (coordinate). Expansion of $D(\acute{s},s)$ in Taylor's series retaining the terms $O(\Delta/L)^4$ [12,13] in the nearest neighbor approximation and substitution of Eqs. (2)–(4) in Eq. (1) yields the following relations in the lowest order of :

$$\Lambda^2 = \frac{9}{8}C_{DL}^2; \quad \frac{\partial^2 d_t^{(1)}}{\partial \xi^2} = \frac{\partial^2 d_l^{(1)}}{\partial \xi\,\partial \eta} + \sqrt{2}\Omega\frac{\partial d_l^{(1)}}{\partial \xi}. \tag{5}$$

Finally, the usual perturbation analysis yields [keeping the terms $O(\epsilon^5)$], the following 2D Korteweg-de Vries (KdV) equation with a term due to the gyration of dust particles under the action of Lorentz force,

$$\frac{\partial}{\partial \xi}\left[\frac{\partial u}{\partial \tau} - \alpha u\frac{\partial u}{\partial \xi} + \beta\frac{\partial^3 u}{\partial \xi^3}\right] + \frac{3}{4\sqrt{2}}\frac{\partial^2 u}{\partial \eta^2} = \frac{\Omega^2}{\sqrt{2}}u, \tag{6}$$

where $\partial d_l^{(1)}/\partial \xi = \Delta u$ and

$$\alpha = \frac{3}{16\sqrt{2}}\left[\frac{2\kappa^3+5\kappa^2+10\kappa+10}{\kappa^2+2\kappa+2}\right], \quad \beta = \frac{11}{192\sqrt{2}}. \tag{7}$$

The above Eq. (6) shows that in the absence of cyclotron frequency, i.e., for $\Omega=0$, the usual 2D KdV equation is recovered which represents a completely integrable Hamiltonian system and thus possesses the following traveling wave solution in the form of a single planar soliton,

$$u(\xi,\eta,\tau) = -N\,\mathrm{sech}^2\sqrt{N}(fN\tau - k_\xi \xi - k_\eta \eta), \tag{8}$$

where f is the frequency, k_ξ and k_η are the wave numbers such that $k_\eta = k_\xi \tan\theta$, θ is the angle that the wave makes with the X-axis and $k_\xi^2=\alpha/4\beta$. Also k_ξ, k_η, and f are related by the relation

$$N(f-4\beta k_\xi^3) - \frac{3k_\eta^2}{4k_\xi\sqrt{2}} = 0, \tag{9}$$

where N is the maximum amplitude of the soliton and width of the soliton is proportional to $(1/\sqrt{N})$. For $\Omega\neq 0$, Eq. (6) is

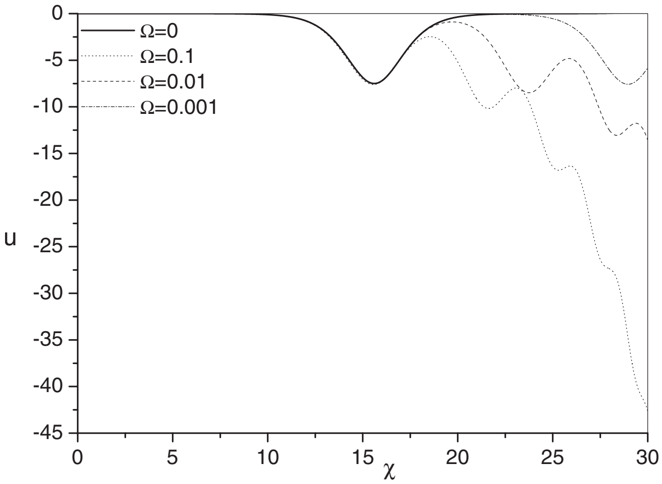

FIG. 2. Numerical solution of Eq. (6) in the wave frame χ for $\theta=10°$. The curves are $\Omega=0$ (solid), $\Omega=0.001$ (dot-dashed), $\Omega=0.01$ (dash), and $\Omega=0.1$ (dot).

not a completely integrable system. However, a traveling wave solution of the gyration-modified 2D KdV Eq. (6) can be obtained by numerical method. First, Eq. (6) is transformed to the wave frame $\chi=fN\tau-k_\xi\xi-k_\eta\eta$, reducing the nonlinear partial differential equation Eq. (6) to the following pair of coupled nonlinear ordinary differential equation:

$$\frac{d^2u}{d\chi^2}=\frac{(fNk_\xi-3k_\eta^2/4\sqrt{2})}{\beta k_\xi^4}u+\frac{\alpha u^2}{2\beta k_\xi^2}+\frac{\Omega^2}{\beta k_\xi^4\sqrt{2}}v;\quad \frac{d^2v}{d\chi^2}=u. \tag{10}$$

Integration of the above pair with respect to χ yielding the traveling wave solution as shown in Fig. 2 is obtained by Runge–Kutta–Fehlberg method subject to the initial conditions $u,du/d\chi,v,dv/d\chi\to 0$ as $\chi\to-\infty$ (for actual numerical computation: $u=0$, $du/d\chi=-10^{-4}$, $dv/d\chi=-10^{-4}$). For $\Omega=0$ (solid curve in Fig. 2), the usual KdV soliton is obtained. For weak magnetic field ($\Omega=0.001$) an apparently quasistable solitary wave structure is observed in the dot-dashed curve; as χ increases the solitary structure becomes repetitive with gradually increasing magnitude, i.e., the dust lattice solitary wave slowly becomes unstable. On the other hand, for strong magnetic field, no stable solitary wave structure is observed in dash ($\Omega=0.01$) and dot ($\Omega=0.1$) curves in Fig. 2 and the solitary wave rapidly becomes unstable.

Finally, to analyze the physical assumptions, the typical laboratory plasma conditions are considered: $T_e=3$ eV (electron temperature), $T_i=0.03$ eV (ion temperature), $T_d=0.03$ eV (dust temperature), $B_0=2$ T, $m_i=6.69\times10^{-26}$ kg (ion mass), $a=0.1$ μm (dust grain radius), $m_d=6.33\times10^{-18}$ kg and $\Delta(=2.1\lambda_D)=100$ μm. Orbital motion limited theory for dust charging equation gives $z_d=500$ (number of charges residing on dust grains; $Q=-z_de$) for dusty plasma parameter $\bar{z}(=z_de^2/4\pi\epsilon_0 aT_e)=2.4$. These yield $\Gamma=120(\gg 1)$ so that the dust grains are in ordered, crystalline state.

The other plasma parameters are $\omega_L=3.44\times10^3$ rad/s, $\omega_{cd}=25.31$ rad/s, $\rho_d=1.18\times10^3$ μm (dust gyroradius), $\rho_e=2.07$ μm, and $\rho_i=56$ μm. For long wavelength limit ($k_\perp\Delta\ll1$), it is chosen that $k_\perp\Delta=0.1$ which estimate $k_\perp\rho_d=1.18$, $k_\perp\rho_e=0.002(\ll1)$, and $k_\perp\rho_i=0.06(\ll1)$ implying both electron and ion polarization effects due to magnetic field are insignificant, whereas dust polarization is significant and thus justifies the assumption (i). Moreover, the magnetic field will not significantly affect the charging of the dust grains as $a/\rho_{e(i)}\ll1$, i.e., the curvature effect of the trajectory of an electron (ion) impinging on a dust grains can be neglected.[22] For the specified plasma parameters, one can easily see that between the two neighboring dust grains, the ratio of the magnitude of Yukawa force to the maximum magnetic force[21] is ≈4.1 implying that Yukawa force dominates over the magnetic force. Thus one can neglect the dust-dust interactions due to magnetic force in comparison with the Yukawa force that justifies assumption (ii). The ratio of dust cyclotron frequency to dust lattice frequency $\omega_{cd}/\omega_L=7.4\times10^{-3}$ implying $\omega_{cd}/\omega_L\ll1$, but $\neq0$ that justifies the assumption (iv) as well as the scaling [Eq. (4)].

S.G. would like to thank Professor B. Bagchi, Department of Applied Mathematics, University of Calcutta, India for his stimulating influence during the course of this work.

[1]G. E. Morfill and A. V. Ivlev, Rev. Mod. Phys. **81**, 1353 (2009).
[2]J. H. Chu and L. Li, Phys. Rev. Lett. **72**, 4009 (1994); H. Thomas, G. E. Morfill, V. Demmel, J. Goree, B. Feuerbacher, and D. Mohlmann, *ibid.* **73**, 652 (1994); A. Homann, A. Melzer, S. Peters, R. Madani, and A. Piel, Phys. Lett. A **242**, 173 (1998).
[3]V. E. Fortov, A. V. Ivlev, S. A. Khrapak, A. G. Khrapak, and G. E. Morfill, Phys. Rep. **421**, 1 (2005).
[4]S. Nunomura, D. Samsonov, and J. Goree, Phys. Rev. Lett. **84**, 5141 (2000); S. Nunomura, J. Goree, S. Hu, X. Wang, and A. Bhattacharjee, Phys. Rev. E **65**, 066402 (2002); S. Nunomura, J. Goree, S. Hu, X. Wang, A. Bhattacharjee, and K. Avinash, Phys. Rev. Lett. **89**, 035001 (2002).
[5]P. K. Kaw and A. Sen, Phys. Plasmas **5**, 3552 (1998).
[6]J. Pramanik, G. Prasad, A. Sen, and P. K. Kaw, Phys. Rev. Lett. **88**, 175001 (2002).
[7]D. Samsonov, J. Goree, Z. W. Ma, A. Bhattacharjee, H. M. Thomas, and G. Morfill, Phys. Rev. Lett. **83**, 3649 (1999).
[8]D. H. E. Dubin, Phys. Plasmas **7**, 3895 (2000).
[9]F. Melandso, Phys. Plasmas **3**, 3890 (1996).
[10]S. V. Vladimirov, P. V. Shevchenko, and N. F. Cramer, Phys. Rev. E **56**, R74 (1997).
[11]D. Samsonov, A. V. Ivlev, R. A. Quinn, G. Morfill, and S. Zhdanov, Phys. Rev. Lett. **88**, 095004 (2002).
[12]S. Zhdanov, D. Samsonov, and G. Morfill, Phys. Rev. E **66**, 026411 (2002).
[13]B. Farokhi, I. Kourakis, and P. K. Shukla, Phys. Plasmas **13**, 122304 (2006); W. Duan, J. Parkes, and M. Lin, *ibid.* **12**, 022106 (2005).
[14]D. Samsonov, S. K. Zhdanov, R. A. Quinn, S. I. Popel, and G. Morfill, Phys. Rev. Lett. **92**, 255004 (2004).
[15]S. Ghosh, JETP Lett. **87**, 281 (2008); **88**, 702 (2008); Contrib. Plasma Phys. **48**, 569 (2008).
[16]S. Ghosh, Phys. Plasmas **16**, 103701 (2009).
[17]S. V. Vladimirov, G. E. Morfill, V. V. Yaroshenko, and N. F. Cramer, Phys. Plasmas **10**, 2659 (2003).
[18]G. Uchida, U. Konopka, and G. Morfill, Phys. Rev. Lett. **93**, 155002 (2004).
[19]K. Jiang, Y. Song, and Y. Wang, Phys. Plasmas **14**, 103708 (2007); B. Farokhi, M. Shahmansouri, and P. K. Shukla, *ibid.* **16**, 063703 (2009).
[20]V. V. Yaroshenko, G. E. Morfill, D. Samsonov and S. V. Vladimirov, New J. Phys. **5**, 18 (2003).
[21]V. V. Yaroshenko, G. E. Morfill, D. Samsonov, and S. V. Vladimirov, IEEE Trans. Plasma Sci. **32**, 675 (2004).
[22]V. N. Tsytovich, N. Sato, and G. E. Morfill, New J. Phys. **5**, 43 (2003); J. S. Chang and K. Spariosu, J. Phys. Soc. Jpn. **62**, 97 (1993).