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Research article

Soft prime and semiprime int-ideals of a ring

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Abstract: In this paper, some properties of soft radical of a soft int-ideal have been developed and soft prime int-ideal, soft semiprime int-ideal of a ring are defined. Several characterizations of soft prime (soft semiprime) int-ideals are investigated. Also it is shown that the direct and inverse images of soft prime (soft semiprime) int-ideals under homomorphism remains invariant.

Keywords: Soft int-ideal; Soft prime int-ideal; Soft semiprime int-ideal; Soft radical
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1. Introduction

In the real world, always we are facing several situations which are not exact. Classical mathematics is not enough to deal with such ambiguous situation. As a result different mathematical tools like probability theory, fuzzy set theory [1], rough set theory [2] have been introduced in different time but all these theories have inherent difficulties. To minimize such difficulties, in 1999 Molodtsov [3] introduced a new notion called soft set as a parameterized family of subsets of the universal set. Thereafter a lot of works have been done to enrich the theory of soft set as well as its applications in real life problems.

In 2007, Aktas et al. defined soft group [4] as a parameterized family of subgroups of the given group and studied some of its properties. Extending this notion of soft group, the notions like soft ring, soft module, soft ideal are presented in [5–9]. Hence the concepts of hybrid structures like fuzzy soft groups (rings, ideals) are presented in [10–14]. Considering these notions of soft groups (rings, ideals etc.) we can not derive all the results of classical algebra in the settings of soft set theory.

In 2012, Cagman et al. introduced the group structures on a soft set using set inclusion operators, known as soft int-group [15]. After that the notions of soft int-ring [16], soft int-ideals [17, 18] etc are introduced. This new notions are appropriate to study different properties of classical group theory, ring theory etc. in the settings of soft set theory.

In Section 3, some properties of soft radical of a soft int-ideal under homomorphism are studied. In Section 4, the notion of soft prime int-ideal of a ring is presented and different characterizations of it are discussed. In Section 5, the notion of soft semiprime int-ideal of a ring is presented and various characterizations of it are discussed. Also it is established that the direct and inverse images of soft prime (soft semiprime) int-ideals under homomorphism are again soft prime (soft semiprime) int-ideals.

2. Preliminaries

This section is devoted to presenting some basic definitions and results from soft set theory to use in the sequel. Throughout this paper unless otherwise stated, let U refers to an initial universe, E the set of parameters, $P(U)$ the power set of U and $\mathbb{N}$ the set of all natural numbers.

Definition 2.1. [3] Let $A \subseteq E$. A pair (F, A) is called a soft set of A over U when F is a mapping given by $F : A \to P(U)$.
Sometimes we call (F, A) as a soft set of A when no confusion will arise regarding the universal set U and sometimes the soft set (F, A) is simply denoted by F, when no confusions regarding the parameter set A and the universal set U arise. The collection of all soft sets with parameter set A over U will be denoted by $S(A, U)$.

Definition 2.2. [3] Let $F, G \in S(A, U)$. Then F is called a soft subset of G, denoted by $F \widetilde{\subseteq} G$, if $F(t) \subseteq G(t)$ for all $t \in A$.

Definition 2.3. [19] Let $F, G \in S(A, U)$. Then the soft intersection of F, G is denoted by $F \widetilde{\cap} G$ and defined by $(F \widetilde{\cap} G)(t) = F(t) \cap G(t)$ for all $t \in A$.

Definition 2.4. Let $'.'$ be a binary composition in a set A and $F, G \in S(A, U)$. Then product of F and G is defined for all $x \in A$ as follows:

$$(FG)(x) = \bigcup_{x=y.z} [F(y) \cap G(z)] \text{ where } y, z \in A$$
$$= \phi \text{ if } x \text{ is not expressible as } x = y.z \text{ for } y, z \in A.$$

Definition 2.5. [18] Let U be the universal set, E be the set of parameters and $A \subseteq E$. Then the soft characteristic function of A, denoted by χ_A, is defined by the soft set $\chi_A : E \to P(U)$, where

$$\chi_A(x) = U, \quad \text{if } x \in A,$$
$$= \phi, \quad \text{if } x \notin A.$$

Definition 2.6. [15] Let $F \in S(A, U)$ and $K \subseteq U$. Then the set $F_K = \{x \in A : F(x) \supseteq K\}$ is called K-inclusion subset of the soft set F.

Proposition 2.7. Let $F, G \in S(A, U)$ and $K \subseteq U$. Then $(F \widetilde{\cap} G)_K = F_K \cap G_K$.

Definition 2.8. [15] Let $F \in S(A, U)$ and $f : A \to A'$ be any mapping, where A' be a parameter set. Then the image of F under f is denoted by $f(F)$ and defined by

$$f(F)(y) = \bigcup_{x \in f^{-1}(y)} F(x) \quad \text{if } f^{-1}(y) \neq \phi,$$
$$= \phi \qquad \text{otherwise},$$

for all $y \in A'$.

Definition 2.9. [15] Let $A,\ A'$ be any two parameter sets and $f : A \to A'$ be an onto mapping. Let $F \in S(A', U)$. The preimage of F is denoted by $f^{-1}(F)$ and defined by $f^{-1}(F)(x) = F(f(x))$, for every $x \in A$.

Definition 2.10. Let f be any mapping from a set A to a set A'. A soft set F of A is called f-invariant if for $x, y \in A$,

$$f(x) = f(y) \Rightarrow F(x) = F(y).$$

Proposition 2.11. [17] Let f be a mapping from a set A to a set A'. If F is a f-invariant soft set of A, then $f^{-1}(f(F)) = F$.

Definition 2.12. Let $F \in S(A, U)$. Then image of F is denoted by $Im(F)$ and defined by $Im(F) = \{F(x) : x \in A\}$.

Definition 2.13. [16] Let R be a ring. A soft set $F \in S(R, U)$ is called a soft int-ring of R if
(i) $F(x - y) \supseteq F(x) \cap F(y)$, and
(ii) $F(xy) \supseteq F(x) \cap F(y)$ for all $x, y \in R$.

Definition 2.14. [16] Let R be a ring. A soft set $F \in S(R, U)$ is called a soft int-ideal of R if
(i) $F(x - y) \supseteq F(x) \cap F(y)$, and
(ii) $F(xy) \supseteq F(x) \cup F(y)$ for all $x, y \in R$.

Proposition 2.15. [16] Let R be a ring with unity 1 and F a soft int-ideal of R. Then
(i) $F(0) \supseteq F(r) \supseteq F(1)$ for all $r \in R$.
(ii) $F(r_1 - r_2) = F(0) \Rightarrow F(r_1) = F(r_2)$, where $r_1, r_2 \in R$.

Theorem 2.16. [16] Let R, R' be two ordinary rings and F, F', soft int-rings (soft int-ideals) of R, R' respectively. If f is a homomorphism from R onto R', then
(i) $f(F)$ is a soft int-ring (soft int-ideal) of R';
(ii) $f^{-1}(F')$ is a soft int-ring (soft int-ideal) of R.

Definition 2.17. [17] Let F be a soft int-ideal of a ring R over U with $Im(F) = \{K_1, K_2, ..., K_n\}$ and F_{K_i}, the K_i-inclusion ideals of F, where $i = 1, 2, ..., n$. Then the soft radical of F, denoted by $\sqrt{F}$, is the soft set of R defined by

$$\sqrt{F}(r) = \bigcup_{r \in \sqrt{F_{K_i}}} K_i$$

i.e., union is taken over all those K_i such that $r \in \sqrt{F_{K_i}}$ and $\sqrt{F_{K_1}}, \sqrt{F_{K_2}}, ..., \sqrt{F_{K_n}}$ are the radicals of $F_{K_1}, F_{K_2}, ..., F_{K_n}$ respectively.

Proposition 2.18. [17] Let F, G be two soft int-ideals of a ring R and $\sqrt{F}, \sqrt{G}$ be the soft radical of F, G respectively. Then the following properties hold:
(i) $\sqrt{F}(0) = F(0)$;
(ii) $\sqrt{F}$ is a soft int-ideal of R containing F;
(iii) $\sqrt{\sqrt{F}} = \sqrt{F}$;
(iv) for each $r \in R$, $\sqrt{F}(r) \supseteq F(r^n)$ for all $n \in \mathbb{N}$;
(v) for each $r \in R$, there exists some $n \in \mathbb{N}$ such that $\sqrt{F}(r) = F(r^n)$;
(vi) $\sqrt{F \widetilde{\cap} G} = \sqrt{F} \widetilde{\cap} \sqrt{G}$.

Note 2.19. In the proof of the Proposition 2.18(ii), we have proved that $\sqrt{F_K} = (\sqrt{F})_K$ for each soft int-ideal F of R and $K \subseteq U$.

Definition 2.20. [17] A soft int-ideal F of a ring R over U is called soft semiprimary int-ideal if for all $x, y \in R$ either $F(xy) \subseteq F(x^n)$ for some $n \in \mathbb{N}$, or $F(xy) \subseteq F(y^m)$ for some $m \in \mathbb{N}$.

Definition 2.21. [17] A soft int-ideal F of a ring R over U is called soft primary int-ideal if for all $x, y \in R$ either $F(xy) = F(x)$ or $F(xy) \subseteq F(y^m)$ for some $m \in \mathbb{N}$.

Theorem 2.22. [20] A commutative ring R is Von Neumann regular if and only if every ideal of R is semiprime.

3. Soft radical of a soft int-ideal

From this section unless otherwise stated, let R be a commutative ring with unity 1 and 0 is the zero element of R. All soft sets are to be considered with parameter set R over the universal set U. In this section, some properties of soft radical of a soft int-ideal are investigated.

Proposition 3.1. Let F be a soft int-ideal of R and $\sqrt{F}$ be the soft radical of F. Then $\sqrt{F}(r) = \bigcup_{n\in\mathbb{N}} F(r^n)$ for all $r \in R$.

Proof. Let $r \in R$. By the Proposition 2.18, we have $\sqrt{F}(r) \supseteq F(r^n)$ for all $n \in \mathbb{N}$. Then $\sqrt{F}(r) \supseteq \bigcup_{n\in\mathbb{N}} F(r^n)$. Again by the Proposition 2.18, $\sqrt{F}(r) = F(r^n)$ for some $n \in \mathbb{N}$. Hence $\bigcup_{n\in\mathbb{N}} F(r^n) \supseteq \sqrt{F}(r)$. Therefore $\sqrt{F}(r) = \bigcup_{n\in\mathbb{N}} F(r^n)$ for all $r \in R$. □

Proposition 3.2. Let F, G be soft int-ideals of R such that $F \tilde{\subseteq} G$. Then $\sqrt{F} \tilde{\subseteq} \sqrt{G}$.

Proof. It follows from the Proposition 3.1. □

Theorem 3.3. Let F, G be soft int-ideals of R. Then $\sqrt{FG} = \sqrt{F \tilde{\cap} G} = \sqrt{F} \tilde{\cap} \sqrt{G}$.

Proof. Let $r \in R$. Then $\sqrt{FG}(r) = \bigcup_{n\in\mathbb{N}} (FG)(r^n)$
$\supseteq (FG)(r^{2m})$ where $m \in \mathbb{N}$
$\supseteq F(r^m) \cap G(r^m) = (F \tilde{\cap} G)(r^m)$, for all $m \in \mathbb{N}$.
Hence $\sqrt{FG}(r) \supseteq \bigcup_{m\in\mathbb{N}} (F \tilde{\cap} G)(r^m) = \sqrt{F \tilde{\cap} G}(r)$ for all $r \in R$.
Therefore $\sqrt{FG} \tilde{\supseteq} \sqrt{F \tilde{\cap} G}$.
Again $(FG)(r) = \bigcup_{r=st} \{F(s) \cap G(t)\}$.
Now $F(s) \cap G(t) \subseteq F(s) \subseteq F(st) = F(r)$. Then $(FG)(r) \subseteq F(r)$.
Similarly, we have $(FG)(r) \subseteq G(r)$. Hence $(FG)(r) \subseteq (F \tilde{\cap} G)(r)$.
Therefore $FG \tilde{\subseteq} F \tilde{\cap} G$. Hence $\sqrt{FG} \tilde{\subseteq} \sqrt{F \tilde{\cap} G}$. So, $\sqrt{FG} = \sqrt{F \tilde{\cap} G}$. Again by the Proposition 2.18, $\sqrt{F \tilde{\cap} G} = \sqrt{F} \tilde{\cap} \sqrt{G}$. □

Corollary 3.4. For any soft int-ideal F of R, $\sqrt{F^n} = \sqrt{F}$ for all $n \in \mathbb{N}$, where $F^n = F \cdot F \cdots F$ (n times).

Theorem 3.5. Let $f : R \to R'$ be an epimorphism, where R, R' be two rings and F be a soft int-ideal of R. Then $f(\sqrt{F}) \widetilde{\subseteq} \sqrt{f(F)}$. Moreover, if F is f-invariant soft int-ideal of R, then $f(\sqrt{F}) = \sqrt{f(F)}$.

Proof. Suppose F is a soft int-ideal of R. Then by the Proposition 2.18, $\sqrt{F}$ is a soft int-ideal of R and by the Theorem 2.16, $f(F)$ is a soft int-ideal of R'. Let $x' \in R'$. Then there exists $x \in R$ such that $f(x) = x'$ and hence $f(x^n) = x'^n$ for all $n \in \mathbb{N}$.
Now $f(\sqrt{F})(x') = \bigcup_{r \in f^{-1}(x')} \sqrt{F}(r)$
$= \bigcup_{r \in f^{-1}(x')} \bigcup_{n \in \mathbb{N}} F(r^n)$, by the Proposition 3.1
$= \bigcup_{n \in \mathbb{N}} \bigcup_{r \in f^{-1}(x')} F(r^n)$
$\subseteq \bigcup_{n \in \mathbb{N}} \bigcup_{y \in f^{-1}(x'^n)} F(y)$
$= \bigcup_{n \in \mathbb{N}} f(F)(x'^n)$
$= \sqrt{f(F)}(x')$. Therefore $f(\sqrt{F}) \widetilde{\subseteq} \sqrt{f(F)}$.
If F is a f-invariant soft int-ideal of R, then by the Definition 2.10, we have $\bigcup_{y \in f^{-1}(x'^n)} F(y) = F(x^n)$, since $f(x^n) = x'^n$.
Again $F(x^n) \subseteq \bigcup_{r \in f^{-1}(x')} F(r^n) \subseteq \bigcup_{y \in f^{-1}(x'^n)} F(y) = F(x^n)$, since $f(x) = x'$.
So, $\bigcup_{r \in f^{-1}(x')} F(r^n) = \bigcup_{y \in f^{-1}(x'^n)} F(y) = F(x^n)$.
Therefore, if F is a f-invariant soft int-ideal of R, then we can put equality sign instead of set inclusion in the proof of $f(\sqrt{F}) \widetilde{\subseteq} \sqrt{f(F)}$. Hence $f(\sqrt{F}) = \sqrt{f(F)}$. □

Theorem 3.6. Let $f : R \to R'$ be a homomorphism, where R, R' be two rings. If G is a soft int-ideal of R', then $f^{-1}(\sqrt{G}) = \sqrt{f^{-1}(G)}$.

Proof. By the Proposition 2.18, $\sqrt{G}$ is a soft int-ideal of R' and by the Theorem 2.16, $f^{-1}(G)$ is a soft int-ideal of R. Let $x \in R$.
Then $f^{-1}(\sqrt{G})(x) = \sqrt{G}(f(x)) = \bigcup_{n \in \mathbb{N}} G((f(x))^n) = \bigcup_{n \in \mathbb{N}} G(f(x^n))$
$= \bigcup_{n \in \mathbb{N}} f^{-1}(G)(x^n) = \sqrt{f^{-1}(G)}(x)$.
Therefore $f^{-1}(\sqrt{G}) = \sqrt{f^{-1}(G)}$. □

4. Soft prime int-ideals of a ring

In this section, at first we recall some theorems related to soft cosets and soft int-ideals. Then the notion of soft prime int-ideal of a ring is given and some properties are studied.

Theorem 4.1. [17] A soft set F of a ring R is a soft int-ring (soft int-ideal) of R if and only if K-inclusion subsets F_K, $K \subseteq F(0)$, are subrings (ideals) of R.

Definition 4.2. [17] Let F be a soft int-ideal of a ring R and $a \in R$. Then the soft set F_a of R, defined by $F_a(r) = F(r - a)$ for all $r \in R$, is called the soft coset of F in R determined by $a \in R$.

Theorem 4.3. [17] Let F be a soft int-ideal of a ring R. Then R/F, the set of all soft cosets of a soft int-ideal F in R forms a ring with respect to binary compositions $F_a + F_b = F_{a+b}$ and $F_a F_b = F_{ab}$ for all $a, b \in R$.

Lemma 4.4. [17] Let F be a soft int-ideal of a ring R. Then $F_a = F_0 \Leftrightarrow F(a) = F(0)$, where $a \in R$.

Definition 4.5. A soft int-ideal F of R is called soft prime int-ideal if for all $x, y \in R$, either $F(xy) = F(x)$ or $F(xy) = F(y)$.

Note 4.6. From the definition of soft prime int-ideal, soft primary int-ideal and soft semiprimary int-ideal, it follows that every soft prime int-ideal of R is a soft primary int-ideal and also every soft primary int-ideal of R is a soft semiprimary int-ideal.

Theorem 4.7. If A is any prime ideal of R such that $A \neq R$, then the soft set F of R over U, defined by
$F(x) = K_1$, if $x \in A$
$= K_2$, if $x \in R \setminus A$ where $K_1, K_2 \subseteq U$ and $K_1 \supset K_2$,
is a soft prime int-ideal of R.

Proof. Here $Im(F) = \{K_1, K_2\}$. Then all the K-inclusion subsets of F, $K \subseteq F(0)$ coincide with $F_{K_1} = A$, and $F_{K_2} = R$. Hence by the Theorem 4.1, F is a soft int-ideal of R. Now let $x, y \in R$ and $F(xy) \supset F(x)$. Then by definition of F, we have $F(xy) = K_1$ and $F(x) = K_2$. Hence $xy \in A$, but $x \notin A$. This implies $y \in A$, since A is a prime ideal of a commutative ring R. Therefore $F(y) = K_1 = F(xy)$. So, F is a soft prime int-ideal of R. □

Theorem 4.8. A soft ideal F of R is soft prime int-ideal if and only if F_K are prime ideals of R for all $K \subseteq F(0)$.

Proof. Suppose F is a soft prime int-ideal of R. Then by the Theorem 4.1, F_K are ideals of R for all $K \subseteq F(0)$. Let $x, y \in R$ and $xy \in F_K$, where $K \subseteq F(0)$. If $x \notin F_K$, then $F(x) \subset K \subseteq F(xy)$. Since F is a soft prime int-ideal, $F(xy) = F(y)$. Hence $y \in F_K$. So, F_K is a prime ideal of R.
Conversely, let F_K are prime ideals of R for all $K \subseteq F(0)$. Then by the Theorem 4.1, F is a soft int-ideal of R. Let $x, y \in R$ and $F(xy) = K$. Then $xy \in F_K$. If $F(xy) \supset F(x)$ then $x \notin F_K$. This implies $y \in F_K$, since F_K is prime ideal of R. Hence $F(y) \supseteq K = F(xy)$. This implies $F(xy) = F(y)$, as F is a soft int-ideal. Therefore F is soft prime int-ideal of R. □

Example 4.9. Let Z_6 be the ring of integers modulo 6 and $U = S_3$, the set of all permutations on the set $\{1, 2, 3\}$. We define a soft set F of Z_6 over S_3 by
$F(x) = \{(1), (12), (123), (132)\}$ if $x \in <\overline{3}>$
$= \{(1), (12), (123)\}$ if $x \in Z_6 \setminus <\overline{3}>$.
Suppose $\{(1), (12), (123), (132)\} = K_1$ and $\{(1), (12), (123)\} = K_2$. Then $F_{K_1} = <\overline{3}>$ and $F_{K_2} = Z_6$. Then by the Theorem 4.8, F is a soft prime int-ideal of Z_6 over S_3.

Theorem 4.10. An ideal A of R is prime if and only if its soft characteristic function χ_A is soft prime int-ideal of R.

Proof. Here $(\chi_A)_U = A$, $(\chi_A)_\phi = R$. Hence the result follows from the Theorem 4.8. □

Theorem 4.11. A soft int-ideal F of R is soft prime if and only if $F(xy) = F(x) \cup F(y)$ for all $x, y \in R$.

Proof. Suppose F is a soft prime int-ideal of R. By the definition of soft int-ideal, it follows for all $x, y \in R,\ F(xy) \supseteq F(x) \cup F(y) \ldots (1)$
Since F is a soft prime int-ideal of R , either $F(xy) = F(x)$ or, $F(xy) = F(y)$. Now
$F(xy) = F(x) \Rightarrow F(y) \subseteq F(x)$, by condition(1) $\Rightarrow F(xy) = F(x) \cup F(y)$.
Similarly, $F(xy) = F(y) \Rightarrow F(xy) = F(x) \cup F(y)$.
Conversely, we assume that $F(xy) = F(x) \cup F(y), \forall\, x, y \in R$. Let $x, y \in R$, $F(xy) \supset F(x)$ and $F(xy) \supset F(y)$. Then we have $F(xy) \supset F(x) \cup F(y)$, which contradicts our assumption. Hence either $F(xy) = F(x)$ or $F(xy) = F(y)$. Therefore F is a soft prime int-ideal. □

Proposition 4.12. Let $f : R \to R'$ be an epimorphism, where R, R' be two commutative rings with unity. If F is a f-invariant soft prime int-ideal of R then $f(F)$ is soft prime int-ideal of R'.

Proof. Suppose F is a f-invariant soft prime int-ideal of R. By the Theorem 2.16, $f(F)$ is a soft int-ideal of R'. Let $x', y' \in R'$. Since f is a epimorphism, there exists $x, y \in R$, such that $f(x) = x'$, $f(y) = y'$ and $f(xy) = x'y'$. Now we assume that
$f(F)(x'y') \supset f(F)(x')$. This implies $f(F)(f(xy)) \supset f(F)(f(x))$
$\Rightarrow (f^{-1}(f(F)))(xy) \supset (f^{-1}(f(F)))(x)$
$\Rightarrow F(xy) \supset F(x)$, by the Proposition 2.11
$\Rightarrow F(xy) = F(y)$, since F is a soft prime int-ideal
$\Rightarrow f(F)(f(xy)) = f(F)(f(y))$
$\Rightarrow f(F)(x'y') = f(F)(y')$.
This shows that $f(F)$ is a soft prime int-ideal of R'. □

Proposition 4.13. Let $f : R \to R'$ be a homomorphism, where R, R' be two commutative rings with unity. If F is a soft prime int-ideal of R' then $f^{-1}(F)$ is soft prime int-ideal of R.

Proof. It follows directly from the definition of soft prime int-ideal and the Definition 2.9. □

Theorem 4.14. Let $f : R \to R'$ be an epimorphism, where R, R' be two commutative rings with unity.
(i) Let F be a f-invariant soft set of R. Then F is soft prime int-ideal of R if and only if $f(F)$ is a soft prime int-ideal of R'.
(ii) Let F' be a soft set of R'. Then F' is soft prime int-ideal of R' if and only if $f^{-1}(F')$ is a soft prime int-ideal of R.

Proof. Since F is f-invariant soft set then by the Proposition 2.11, we have $f^{-1}(f(F)) = F$. Now let $x' \in R'$. Then
$f(f^{-1}(F'))(x') = \bigcup_{x \in f^{-1}(x')} (f^{-1}(F'))(x) = \bigcup_{x \in f^{-1}(x')} F'(f(x)) = F'(x')$. Since $x' \in R'$ is arbitrary, then $f(f^{-1}(F')) = F'$.
Hence by the Proposition 4.12 and 4.13, the theorem follows. □

Theorem 4.15. If F is a soft prime int-ideal of R, then R/F is an integral domain.

Proof. Suppose F is a soft prime int-ideal of R. Then by the Theorem 4.3, R/F forms a ring. Since R is commutative ring with unity 1, R/F is commutative ring with unity F_1. Now we prove that R/F has no divisor of zero. Let $F_a, F_b \in R/F$ such that $F_aF_b = F_0$. Again $F_aF_b = F_0 \Rightarrow F_{ab} = F_0 \Rightarrow F(ab) = F(0)$, (by Lemma 4.4) $\Rightarrow ab \in F_K$, where $K = F(0)$. Since F is a soft prime int-ideal, F_K is also a prime ideal. Hence $ab \in F_K \Rightarrow$ either $a \in F_K$ or $b \in F_K$. Now $a \in F_K \Rightarrow F(a) = F(0) \Rightarrow F_a = F_0$ and $b \in F_K \Rightarrow F(b) = F(0) \Rightarrow F_b = F_0$. Thus either $F_a = F_0$ or $F_b = F_0$. So, R/F have no divisor of zero. This implies R/F is an integral domain. □

Theorem 4.16. Let F be a soft int-ideal of R such that $Im(F) = \{K_1, K_2\}$, where $K_1 \subset K_2$. If R/F is an integral domain then F is a soft prime int-ideal.

Proof. Let $x, y \in R$ and $F(xy) \supset F(x)$. Then $F(xy) = K_2$ and $F(x) = K_1$. Since F has only two images, by Proposition 2.15, $F(xy) = F(0)$. Hence by the Lemma 4.4, $F(xy) = F(0) \Rightarrow F_{xy} = F_0 \Rightarrow F_xF_y = F_0 \Rightarrow F_x = F_0$ or $F_y = F_0$, since R/F is an integral domain. So, this implies either $F(x) = F(0)$ or $F(y) = F(0)$. If $F(x) = F(0)$ then $F(xy) = F(x)$, which contradicts our assumption $F(xy) \supset F(x)$. Therefore $F(y) = F(0) = F(xy)$. So, F is a soft prime int-ideal of R. □

Theorem 4.17. A soft int-ideal F of R is soft semiprimary int-ideal if and only if $\sqrt{F}$ is a soft prime int-ideal of R.

Proof. Suppose F is a soft semiprimary int-ideal of R. Let $x, y \in R$. By the Proposition 2.18, we have $\sqrt{F}$ is a soft int-ideal of R and
$\sqrt{F}(xy) = F((xy)^n)$ for some $n \in \mathbb{N}$
$= F(x^ny^n)$, since R is commutative ring
$\subseteq F(x^{nl})$ or, $\subseteq F(y^{nm})$ for some $l, m \in \mathbb{N}$, by the Definition 2.21
$\subseteq \sqrt{F}(x)$ or, $\subseteq \sqrt{F}(y)$, by the Proposition 2.18.
Again $\sqrt{F}(xy) \supseteq \sqrt{F}(x)$ and also $\sqrt{F}(xy) \supseteq \sqrt{F}(x)$, since $\sqrt{F}$ is a soft int-ideal. Hence either $\sqrt{F}(xy) = \sqrt{F}(x)$ or, $\sqrt{F}(xy) = \sqrt{F}(y)$. Therefore $\sqrt{F}$ is a soft prime int-ideal.
Conversely, let $\sqrt{F}$ be a soft prime int-ideal and $x, y \in R$. Then either $\sqrt{F}(xy) = \sqrt{F}(x)$ or, $\sqrt{F}(xy) = \sqrt{F}(y)$. Suppose $\sqrt{F}(xy) = \sqrt{F}(x)$. Then $F(xy) \subseteq \sqrt{F}(xy) = \sqrt{F}(x) \subseteq F(x^m)$ for some $m \in \mathbb{N}$. Similarly, when $\sqrt{F}(xy) = \sqrt{F}(y)$, then $F(xy) \subseteq F(y^n)$ for some $n \in \mathbb{N}$. Therefore F is a soft semiprimary int-ideal of R. □

5. Soft semiprime int-ideals of a ring

In this section, the notion of soft semiprime int-ideal of a ring is defined, and some properties of this notion are studied.

Definition 5.1. A soft int-ideal F of R is called soft semiprime int-ideal if for all $x \in R$, $F(x^2) = F(x)$.

Proposition 5.2. (i) Every soft prime int-ideal of R is soft semiprime int-ideal of R.
(ii) Finite intersection of soft semiprime (soft prime) int-ideals of R is a soft semiprime int-ideal of R.
(iii) For every soft int-ideal F of R, $\sqrt{F}$ is the smallest soft semiprime int-ideal of R containing F.
(iv) Let R be a Boolean ring. Then every soft int-ideal of R is a soft semiprime.

Proof. (i) It directly follows from the Definition of soft prime int-ideal.
(ii) Let $F_1, F_2, \ldots, F_n$ be the finite collection of soft semiprime int-ideals. Then for each $x \in R$, we have $(F_1 \widetilde{\cap} F_2 \widetilde{\cap} \ldots \widetilde{\cap} F_n)(x^2)$
$= F_1(x^2) \cap F_2(x^2) \cap \cdots \cap F_n(x^2)$
$= F_1(x) \cap F_2(x) \cap \cdots \cap F_n(x) = (F_1 \widetilde{\cap} F_2 \widetilde{\cap} \ldots \widetilde{\cap} F_n)(x)$.
Therefore $F_1 \widetilde{\cap} F_2 \widetilde{\cap} \ldots \widetilde{\cap} F_n$ is a soft semiprime int-ideal of R.
Since every soft prime int-ideal of R is soft semiprime int-ideal, then intersection of soft prime int-ideals of R is a soft semiprime int-ideal.
(iii) By the Proposition 2.18, $\sqrt{F}$ is a soft int-ideal of R containing F. Let $x \in R$ and $\sqrt{F}(x^2) = K$, where $K \subseteq U$. Also by the Note 2.19, we have $\sqrt{F_K} = (\sqrt{F})_K$.
Now $\sqrt{F}(x^2) = K \Rightarrow x^2 \in (\sqrt{F})_K \Rightarrow x^2 \in \sqrt{F_K}$
$\Rightarrow (x^2)^n \in F_K$ for some $n \in \mathbb{N} \Rightarrow x^{2n} \in F_K$
$\Rightarrow x \in \sqrt{F_K} = (\sqrt{F})_K$
$\Rightarrow \sqrt{F}(x) \supseteq K$
$\Rightarrow \sqrt{F}(x) \supseteq \sqrt{F}(x^2)$.
Since $\sqrt{F}$ is a soft int-ideal of R, $\sqrt{F}(x^2) \supseteq \sqrt{F}(x)$.
Hence $\sqrt{F}(x^2) = \sqrt{F}(x)$. Since $x \in R$ is arbitray, $\sqrt{F}$ is a soft semiprime int-ideal. Now we shall prove that $\sqrt{F}$ is the smallest soft semiprime int-ideal of R containing F. Let I be any soft semiprime int-ideal of R containing F such that $I \widetilde{\subseteq} \sqrt{F}$. Let $x \in R$ and $\sqrt{F}(x) = K$, where $K \subseteq U$. This implies $x \in (\sqrt{F})_K = \sqrt{F_K} \Rightarrow x^n \in F_K$ for some $n \in \mathbb{N}$
$\Rightarrow I(x^n) \supseteq F(x^n) \supseteq K$
$\Rightarrow I(x) = I(x^n) \supseteq K = \sqrt{F}(x)$.
Since $x \in R$ is arbitrary, $I \widetilde{\supseteq} \sqrt{F}$. Hence $I = \sqrt{F}$. Therefore $\sqrt{F}$ is the smallest soft semiprime int-ideal of R containing F.
(iv) In a Boolean ring R, $x^2 = x$, for all $x \in R$. Then for any soft int-ideal F of R, we have $F(x^2) = F(x)$, $\forall\, x \in R$. Therefore F is a soft semiprime int-ideal. □

In the following theorem, some characterizations of soft semiprime int-ideals are given.

Theorem 5.3. Let F be a soft int-ideal of R. Then the following are equivalent.
(i) F is soft semiprime int-ideal;
(ii) K-inclusion sets F_K of R, $K \subseteq F(0)$, are semiprime int-ideals;
(iii) $F(x^n) = F(x)$ for all $n \in \mathbb{N}$ and $x \in R$;
(iv) For all soft int-ideals I of R, $I^2 \widetilde{\subseteq} F \Rightarrow I \widetilde{\subseteq} F$;
(v) For all soft int-ideals I of R and $n \in \mathbb{N}$, $I^n \widetilde{\subseteq} F \Rightarrow I \widetilde{\subseteq} F$;
(vi) $F = \sqrt{F}$, where $\sqrt{F}$ is the soft radical of F.

Proof. $(i) \Leftrightarrow (ii)$: Proof is similar as Theorem 4.8.
$(i) \Rightarrow (iii)$: At first we claim that $F(x^{2^m}) = F(x)$ for all $m \in \mathbb{N}, x \in R$. Now $F(x^{2^m}) = F(x^{2^{m-1}}.x^{2^{m-1}}) = F(x^{2^{m-1}})$, since F is a soft semiprime int-ideal and $x^{2^{m-1}} \in R$. Proceeding in this way, we have $F(x^{2^{m-1}}) = F(x^{2^{m-2}}) = ... = F(x^2) = F(x)$. So, $F(x^{2^m}) = F(x)$ for all $m \in \mathbb{N}$, $x \in R$. Let $n \in \mathbb{N}$. Then there exists $k \in \mathbb{N}$ such that $F(x^n) \subseteq F(x^{2^k})$. Hence $F(x^n) \subseteq F(x)$. Again $F(x^n) \supseteq F(x)$, since F is a soft int-ideal. Therefore $F(x^n) = F(x)$ for all $n \in \mathbb{N}$ and $x \in R$.
$(iii) \Rightarrow (i)$: Obvious.
$(i) \Rightarrow (iv)$: Let $x \in R$. Since F is a soft semiprime int-ideal, $I^2 \widetilde{\subseteq} F \Rightarrow I^2(x^2) \subseteq F(x^2) = F(x)$. Again

by the Definition 2.4,
$I^2(x^2) = \bigcup_{x^2=y.z} [I(y) \cap I(z)]$ where $y, z \in R$
$\supseteq I(x) \cap I(x) = I(x)$.
Hence $I(x) \subseteq F(x)$ for all $x \in R$. So, $I \widetilde{\subseteq} F$.
$(iv) \Rightarrow (i)$: Let $x \in R$ and $F(x^2) = K$. Then $x^2 \in F_K$. Hence $< x^2 > \subseteq F_K$. Define a soft int-ideal $I : R \to P(U)$ as follows:
$I(a) = K$ if $a \in < x >$
$= \phi$ otherwise.
Then by Definition 2.4, we have $I^2(z) = \bigcup_{z=a.b} [I(a) \cap I(b)]$ where $z, a, b \in R$.
Hence $I^2(z) = K$ if $z \in < x^2 >$
$= \phi$ otherwise.
Therefore $I^2_K =< x^2 > \subseteq F_K \Rightarrow I^2 \widetilde{\subseteq} F \Rightarrow I \widetilde{\subseteq} F$, by hypothesis.
Then $K = I(x) \subseteq F(x) \subseteq F(x^2) = K$. Therefore $F(x^2) = F(x)$.
$(iii) \Rightarrow (v)$: Let $x \in R$. Then $I^n \widetilde{\subseteq} F \Rightarrow I^n(x^n) \subseteq F(x^n) = F(x)$, by using (iii). Again by the Definition 2.4,
$I^n(x^n) = \bigcup_{x^n=uv} [I(u) \cap I^{n-1}(v)] \supseteq I(x) \cap I^{n-1}(x^{n-1})$. Proceeding in this way we have $I^n(x^n) \supseteq I(x) \cap I(x) \cap ... \cap I(x) = I(x)$. Hence $I(x) \subseteq I^n(x^n) \subseteq F(x)$ for all $x \in R$. So, $I \widetilde{\subseteq} F$.
$(v) \Rightarrow (iv)$: Obvious.
$(iii) \Rightarrow (vi)$: Let $x \in R$. By the Proposition 2.18(v), there exists $n \in \mathbb{N}$ such that $\sqrt{F}(x) = F(x^n) = F(x)$. So, $\sqrt{F} = F$.
$(vi) \Rightarrow (iii)$: Let $x \in R$. Then by the Proposition 2.18(iv), we have $F(x) = \sqrt{F}(x) \supseteq F(x^n)$ for all $n \in \mathbb{N}$. Again $F(x^n) \supseteq F(x)$, since F is a soft int-ideal of R. Therefore $F(x^n) = F(x)$ for all $n \in \mathbb{N}$ and $x \in R$. □

Similar to the Theorems 4.7 and 4.10 on soft prime int-ideals, we get the following theorems:

Theorem 5.4. If A is any semiprime ideal of R such that $A \neq R$, then the soft set F of R over U, defined by
$F(x) = K_1$, if $x \in A$
$= K_2$, if $x \in R \setminus A$ where $K_1, K_2 \subseteq U$ and $K_1 \supset K_2$,
is a soft semiprime int-ideal of R.

Theorem 5.5. An ideal A of R is semiprime if and only if its soft characteristic function χ_A is a soft semiprime int-ideal of R.

Example 5.6. Let Z be the ring of integers and U be the universal set. We define a soft set F of Z over U as
$F(x) = K_1$ if $x \in 6Z$
$= K_2$ if $x \in Z \setminus 6Z$, where $K_1, K_2 \subseteq U$ and $K_1 \supset K_2$.
Then $F_{K_1} = 6Z$ and $F_{K_2} = Z$. Since $6Z$ is not a prime ideal of Z but a semiprime ideal of Z, then by the Theorem 4.8 and 5.3, F is not a soft prime int-ideal but a soft semiprime int-ideal of Z.

Example 5.7. Let Z be the ring of integers and U be the universal set. Let F, G be two soft sets of Z over U, given by

$F(x) = K_1$ if $x \in <2>$
$= K_2$ if $x \in Z \setminus <2>$, where $K_1 \supset K_2$ and $K_1, K_2 \subseteq U$.
$G(x) = K_3$ if $x \in <3>$
$= K_2$ if $x \in Z \setminus <3>$ where $K_3 \supset K_2$ and $K_3, K_2 \subseteq U$.
Then F, G are two soft prime int-ideals of Z over U and
$(F \widetilde{\cap} G)(x) = K_1 \cap K_3$ if $x \in <6>$
$= K_1 \cap K_2 = K_2$ if $x \in <2> \setminus <6>$
$= K_2 \cap K_3 = K_2$ if $x \in <3> \setminus <6>$
$= K_2$ if $x \in Z \setminus (<2> \cup <3>)$.
Since $K_1 \cap K_3 \supset K_2$, we have $F_{K_1 \cap K_3} =<6>$ and $F_{K_2} = Z$.
Since $<6>$ is a semiprime int-ideal but not a prime int-ideal of Z, then $F \widetilde{\cap} G$ is a soft semiprime int-ideal of Z but not a soft prime int-ideal of Z.

Example 5.8. Let Z be the ring of integers and U be the universal set. Let F, G be two soft sets of Z over U, given by
$F(x) = K_1$ if $x \in <2>$
$= K_2$ if $x \in Z \setminus <2>$, where $K_1 \supset K_2$ and $K_1, K_2 \subseteq U$.
$G(x) = K_3$ if $x \in <2>$
$= K_2$ if $x \in Z \setminus <2>$ where $K_3 \supset K_2$ and $K_3, K_2 \subseteq U$.
Then F, G are two soft prime int-ideals of Z over U and
$(F \widetilde{\cap} G)(x) = K_1 \cap K_3$ if $x \in <2>$
$= K_2$ if $x \in Z \setminus <2>$.
Hence $F \widetilde{\cap} G$ is a soft prime int-ideal of Z.

Theorem 5.9. If $F_1, F_2, ..., F_n$ are soft prime (soft semiprime) int-ideals of R such that $F_1 \widetilde{\subseteq} F_2 \widetilde{\subseteq} ... \widetilde{\subseteq} F_n$, then $\widetilde{\bigcap}_{i=1}^{n} F_i$ and $\widetilde{\bigcup}_{i=1}^{n} F_i$ are soft prime (soft semiprime) int-ideals of R.

Corollary 5.10. If F is a soft prime int-ideal of R then the following conditions hold:
(i) $F(x^n) = F(x)$ for all $n \in \mathbb{N}$ and $x \in R$;
(ii) For all soft int-ideals I of R, $I^2 \widetilde{\subseteq} F \Rightarrow I \widetilde{\subseteq} F$;
(iii) For all soft int-ideals I of R and $n \in \mathbb{N}$, $I^n \widetilde{\subseteq} F \Rightarrow I \widetilde{\subseteq} F$;
(iv) $F = \sqrt{F}$, where $\sqrt{F}$ is the soft radical of F.

Proof. By the Proposition 5.2(i), every soft prime int-ideal of R is a soft semiprime int-ideal. Hence the corollary directly follows from the Theorem 5.3. □

Corollary 5.11. Let F be any soft int-ideal of R. Then F and $\sqrt{F}$ are contained in the same soft prime int-ideal of R.

Proof. Suppose P be a soft prime int-ideal of R such that $F \widetilde{\subseteq} P$. Let $x \in R$. Then by the Proposition 2.18, $\sqrt{F}(x) = F(x^n)$ for some $n \in \mathbb{N}$,
$\subseteq P(x^n) = P(x)$ by the Corollary 5.10. Therefore $\sqrt{F} \widetilde{\subseteq} P$. □

Corollary 5.12. Let F be a soft prime (soft semiprime) int-ideal of R. Then $\sqrt{F^n} = F$ for all $n \in \mathbb{N}$.

Proof. Let F be a soft semiprime int-ideal of R. Then by the Corollary 3.4, $\sqrt{F^n} = \sqrt{F}$ for all $n \in \mathbb{N}$. Again by the Theorem 5.3, $\sqrt{F} = F$. Therefore $\sqrt{F^n} = F$ for all $n \in \mathbb{N}$. If F is a soft prime int-ideal of R, we get the similar result. □

Theorem 5.13. If F is a soft semiprimary int-ideal of R, then $\sqrt{F}$ is the smallest soft prime int-ideal of R containing F.

Proof. By the Proposition 2.18 and Theorem 4.17, we have $\sqrt{F}$ is a soft prime int-ideal of R containing F. Our next task is to prove $\sqrt{F}$ is the smallest soft prime int-ideal of R containing F. On the contrary, let I be any soft prime int-ideal of R containing F such that $I \widetilde{\subseteq} \sqrt{F}$.
Let $\sqrt{F}(x) = K$ where $K \subseteq U$. This implies $x \in (\sqrt{F})_K = \sqrt{F_K}$
$\Rightarrow x^n \in F_K$ for some $n \in \mathbb{N} \Rightarrow I(x^n) \supseteq F(x^n) \supseteq K \Rightarrow I(x^n) \supseteq \sqrt{F}(x)$.
Again by the Corollary 5.10, $I(x^n) = I(x)$.
Hence $I(x^n) \supseteq \sqrt{F}(x) \Rightarrow I(x) \supseteq \sqrt{F}(x)$. This holds for arbitrary $x \in R$. So, $I \widetilde{\supseteq} \sqrt{F}$. Hence $I = \sqrt{F}$. Therefore $\sqrt{F}$ is the smallest soft prime int-ideal of R containing F. □

Following the proof of the Propositions 4.12, 4.13 and Theorem 4.14 , the next theorem can be obtained.

Theorem 5.14. Let $f : R \rightarrow R'$ be an epimorphism, where R, R' be two commutative rings with unity.
(i) Let F be a f-invariant soft set of R. Then F is soft semiprime int-ideal of R if and only if $f(F)$ is a soft semiprime int-ideal of R'.
(ii) Let F' be a soft set of R'. Then F' is soft semiprime int-ideal of R' if and only if $f^{-1}(F')$ is a soft semiprime int-ideal of R.

Theorem 5.15. If F is a soft semiprime int-ideal of R, then R/F has no nonzero nilpotent element.

Proof. Suppose F is a soft semiprime int-ideal of R. Then by the Theorem 4.3, R/F forms a ring. Let F_a be a nilpotent element of R/F. Then $(F_a)^k = F_0$ for some $k \in \mathbb{N}$. This implies $F_{a^k} = F_0$ and hence by the Lemma 4.4, we have $F(a^k) = F(0)$. Since F is a soft semiprime int-ideal, by the Theorem 5.3, we have $F(a) = F(a^k) = F(0)$. This implies $F_a = F_0$. So, R/F has no nonzero nilpotent element. □

Theorem 5.16. Let F be a soft int-ideal of R such that $Im(F) = \{K_1, K_2\}$, where $K_1 \subset K_2$. If R/F has no nonzero nilpotent element then F is a soft semiprime int-ideal.

Proof. Let $x \in R$ and $F(x^2) \supset F(x)$. Since F has only two images, then by the Proposition 2.15, $F(x^2) = F(0)$. Hence by the Lemma 4.4, $F(x^2) = F(0) \Rightarrow F_{x^2} = F_0 \Rightarrow (F_x)^2 = F_0$. Since R/F has no nonzero nilpotent element then $F_x = F_0$. This implies $F(x) = F(0)$, which contradicts our assumption. Hence $F(x^2) = F(x)$. Therefore F is a soft semiprime int-ideal. □

Definition 5.17. Let F be a soft int-ideal of R. Then the soft int-ideal
$r(F) = \widetilde{\bigcap}\{I : F \widetilde{\subseteq} I, I \text{ is a soft prime int-ideal of } R\}$ is called a soft prime radical of F.

Proposition 5.18. Let F be any soft int-ideal of R. Then
(i) $r(F)$ is a soft semiprime int-ideal of R containing F;
(ii) $\sqrt{F} \widetilde{\subseteq} r(F)$;
(iii) If F is a soft prime int-ideal of R then $F = \sqrt{F} = r(F)$.

Proof. (i) It follows from the Proposition 5.2(ii).
(ii) By the Proposition 5.2(iii), $\sqrt{F}$ is the smallest soft semiprime int-ideal of R containing F. Since $r(F)$ is a soft semiprime int-ideal of R containing F, then $\sqrt{F} \,\widetilde{\subseteq}\, r(F)$.
(iii) Since F is a soft prime int-ideal of R, F is also a soft semiprimary int-ideal. Hence by the Theorem 4.17, $\sqrt{F}$ is a soft prime int-ideal of R containing F. Again by the Definition 5.17, $r(F)$ is the intersection of all soft prime int-ideals of R containing F. Hence $r(F) \,\widetilde{\subseteq}\, \sqrt{F}$. Then by part (ii), we have $\sqrt{F} = r(F)$. By the Corollary 5.10, $\sqrt{F} = F$. Therefore $F = \sqrt{F} = r(F)$. □

Here some characterizations of Von Neumann regular with respect to soft int-ideals are presented.

Theorem 5.19. Let R be a commutative ring. Then the following conditions are equivalent:
(i) R is Von Neumann regular;
(ii) $FG = F \,\widetilde{\cap}\, G$ where F, G are any two soft int-ideals of R;
(iii) every soft int-ideals of R is idempotent;
(iv) every soft int-ideals of R is soft semiprime.

Proof. $(i) \Rightarrow (ii)$: Let R be a and $x \in R$. Then there exists $r \in R$ such that $x = xrx$. Now $(FG)(x) = \bigcup_{x=yz} \{F(y) \cap G(z)\}$.
Here $F(y) \cap G(z) \subseteq F(y) \subseteq F(yz) = F(x)$. Then $(FG)(x) \subseteq F(x)$.
Similarly, we get $(FG)(x) \subseteq G(x)$. Hence $(FG)(x) \subseteq (F \,\widetilde{\cap}\, G)(x)$.
Again $(FG)(x) = \bigcup_{x=yz} \{F(y) \cap G(z)\} \supseteq F(x) \cap G(rx) \supseteq F(x) \cap G(x) = (F \,\widetilde{\cap}\, G)(x)$. Therefore $(FG)(x) = (F \,\widetilde{\cap}\, G)(x)$. Since $x \in R$ is arbitrary, $FG = F \,\widetilde{\cap}\, G$.
$(ii) \Rightarrow (iii)$: Let F be any soft int-ideal of R. Then by (ii) $F^2 = FF = F \,\widetilde{\cap}\, F = F$. So, F is idempotent.
$(iii) \Rightarrow (iv)$: Let F be any soft int-ideal of R and S be another soft int-ideal of R such that $S^2 \,\widetilde{\subseteq}\, F$. By hypothesis $S^2 = S$. Hence $S \,\widetilde{\subseteq}\, F$. Then by the Theorem 5.3, F is soft semiprime.
$(iv) \Rightarrow (i)$: Suppose that every soft int-ideal of R is soft semiprime . Let I be any ideal of R. Then the soft characteristic function χ_I is a soft int-ideal of R. By hypothesis χ_I is soft semiprime. Then by the Theorem 5.5 I is a semiprime int-ideal of R. Hence by the Theorem 2.22, R is Von Neumann regular. □

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Conflict of interest

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