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Small amplitude nonlinear electron acoustic solitary waves in weakly magnetized plasma

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Nonlinear propagation of electron acoustic waves in homogeneous, dispersive plasma medium with two temperature electron species is studied in presence of externally applied magnetic field. The linear dispersion relation is found to be modified by the externally applied magnetic field. Lagrangian transformation technique is applied to carry out nonlinear analysis. For small amplitude limit, a modified KdV equation is obtained, the modification arising due to presence of magnetic field. For weakly magnetized plasma, the modified KdV equation possesses stable solitary solutions with speed and amplitude increasing temporally. The solutions are valid upto some finite time period beyond which the nonlinear wave tends to wave breaking. © *2013 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4776692]

I. INTRODUCTION

An appreciable number of investigations have been carried out on nonlinear propagation of electron acoustic wave (EAW), probably, because of its relevance in interpreting various satellite observations in different parts of magnetosphere. This wave mode has been argued to be responsible for generation of broadband electrostatic noise (BEN) in the terrestrial cusp of the magnetosphere and hiss in polar cusp regions.[1–4] Apart from its wide applications in space plasma, study of electron acoustic wave is crucial in laboratory plasma with two temperature electron species, beam plasma interaction and stimulated electron acoustic scattering in the laser plasma interaction also.[5,6] Plasma comprising energetic hot electrons, diluted cold electrons, and static positively charged ions can support the propagation of this high frequency, electrostatic wave. This mode is basically an analog to ion acoustic wave with cold electrons playing the role of ions providing the inertia and hot electrons providing the pressure for sustaining the wave. In the fast time scale of electrons, ions are taken to be at rest forming positively charged background and maintaining charge neutrality throughout the plasma. As only high frequency electrons are involved in whole dynamics, there is always a chance of wave damping, hence, some criteria for density and temperature of the two electron species should be satisfied for EAW to remain undamped in the plasma. The first is, cold electron temperature (T_c) should be much less than hot electron temperature (T_h), i.e., $T_c \ll T_h$ and second is, hot electron density (n_h) should be much larger than cold electron density (n_c), i.e., $n_h \gg n_c$.[7] The phase velocity of EAW is given by $c_{se} = \sqrt{T_h/m(n_{h0}/n_{c0})}$, where n_{h0}, n_{c0} are equilibrium hot electron and cold electron density. The linear dispersion relation for EAW in homogeneous, collisionless, dispersive plasma medium is given by

$$\omega^2 = \frac{k^2 c_{se}^2}{1 + k^2 \lambda_{Dh}^2}, \tag{1}$$

where λ_{Dh} is the hot electron Debye length. For long wavelength limit, $k\lambda_{Dh} \ll 1$, the above dispersion relation reduces to $\omega = kc_{se}$.

In the nonlinear regime, a good deal of literature is devoted to investigate the propagation of electron acoustic waves in un-magnetized plasma.[8–10] However, as EAW is a common occurrence in earth's magnetosphere, it is worthwhile to examine this mode in a magnetized plasma. In magnetized plasma, analyses are performed to get solitary wave solutions for EAW in weakly nonlinear, dispersive medium.[9,11] Most of the approaches were made by using either reductive perturbation technique (RPT) or Sagdeev's pseudo potential technique. However, Lagrangian formalism has been found to be effective to obtain arbitrary amplitude temporal and spatial variation of cold electron density in inhomogeneous plasma immersed in uniform magnetic field. The above mentioned study has been carried out for long wave length consideration so that spatial variation of electric field is negligibly small throughout the plasma and the plasma can be taken as quasineutral. Though usual dispersion is absent there, a dispersion like term comes due to ion inhomogeneity through collective effect. Results show that the spatial profile of cold electron density is Gaussian in nature and oscillating nonlinearly with time. However, this dispersion like effect cannot stop nonlinearity, and the solution tends to wave breaking. This motivates us to go for a detailed study of EAW in magnetized homogeneous plasma in presence of wave dispersion. In this paper, we include wave dispersion which can balance the nonlinearity and produce some stable structures. Though it is nearly impossible to achieve exact solution analytically by Lagrangian transformation technique for this particular problem, a perturbative analysis can be performed for small amplitude limit. This small amplitude analysis reduces the main evolution equation to KdV with a nonlinear source term. The method of solving the derived equation is described in this paper, and solutions are interpreted in appropriate physical context.

II. GOVERNING EQUATIONS AND LINEAR ANALYSIS

We consider a homogeneous, collisionless plasma medium consisting of highly dense inertia-less hot electrons, diluted inertial cold electrons, and positively charged background of immobile ions in presence of externally applied uniform magnetic field. The magnetic field is directed along $\hat{z}$ direction with a constant magnitude B_0. Ions having higher mass and low energy compared to electrons fail to respond in the fast timescale of electrons. They only form positively charged static background and maintain charge neutrality throughout the plasma. We are considering electrostatic waves moving perpendicular to the applied magnetic field $\vec{B}_0$. However, to describe the actual situation of real plasma, the angle between wave propagation vector and magnetic field should be taken slightly less than $\pi/2$, so that the hot electrons can have a velocity component along the magnetic field to carry out Debye shielding in that direction also. Hence, highly energetic, inertia-less hot electrons are assumed to follow Boltzmann Distribution in the plasma, as for them, electric field is being balanced by the pressure gradient. The slight deviation in the angle has no impact on the less energetic cold electron motion as they fail to move long distances due to their high effective mass or inertia.[12] Now, the governing equations for this plasma system are given by

$$\frac{\partial n_c}{\partial t} + \frac{\partial}{\partial x}(n_c v_{cx}) = 0, \tag{2}$$

$$\left(\frac{\partial}{\partial t} + v_{cx}\frac{\partial}{\partial x}\right)v_{cx} = -\frac{e}{m}E - \frac{eB_0}{mc}v_{cy}, \tag{3}$$

$$\left(\frac{\partial}{\partial t} + v_{cx}\frac{\partial}{\partial x}\right)v_{cy} = \frac{eB_0}{mc}v_{cx}, \tag{4}$$

$$\frac{eE}{m} = -\frac{T_h}{m}\frac{\partial \ln n_h}{\partial x}, \tag{5}$$

$$\frac{\partial E}{\partial x} = 4\pi e(n_0 - n_c - n_h), \tag{6}$$

where n_c, n_h, n_0 are densities of cold electron, hot electron, and ion fluid, respectively, v_{cx}, v_{cy} are the cold electron velocity along $\hat{x}, \hat{y}$ direction, T_h is hot electron temperature, and E is electric field along $\hat{x}$ direction. While deriving these equations, we have considered all dependent variables to have variations along $\hat{x}$ only. Equation (2) is continuity equation for cold electron fluid, Eqs. (3) and (4) are momentum equations in $\hat{x}, \hat{y}$ direction, respectively, for the species. The hot electron pressure $(= n_h T_h)$ being rightly balanced by electric field in Eq. (5) produces Boltzmann distribution for hot electrons. To close the system, Poisson's equation for charge neutrality is included in the last.

We are considering propagation of waves perpendicular to applied magnetic field. So, linearizing the above set of equations, the dispersion relation of the wave is derived as

$$\omega^2 = \Omega_c^2 + \frac{k^2 c_{se}^2}{1 + k^2 \lambda_{Dh}^2}, \tag{7}$$

where, $\Omega_c = eB_0/mc$ is the electron cyclotron frequency. In presence of magnetic field, the usual dispersion relation of EAW is modified by addition of an extra term Ω_c^2. The term Ω_c^2 emerges as a contribution of Lorentz force exerted on electrons apart from the usual acoustic type oscillation. This new mode can be termed as magneto-electron acoustic wave, which is clearly dispersive in nature.

III. NONLINEAR ANALYSIS WITH LAGRANGIAN TRANSFORMATION TECHNIQUE

Nonlinear propagation of this wave can be analyzed by applying Lagrangian Transformation method where the Eulerian coordinates (x, t) are transformed to the new Lagrangian coordinates (ξ, τ) through following relation:[13]

$$\xi = x - \int_0^\tau v_{cx}(\xi, \tau)d\tau; \qquad \tau = t. \tag{8}$$

The advantage of this transformation is that the convective derivative present in the governing equations reduces merely to time derivative in the new Lagrangian coordinates

$$\frac{\partial}{\partial t} + v_{cx}\frac{\partial}{\partial x} = \frac{\partial}{\partial \tau}, \tag{9}$$

whereas relation between old and new space derivatives at a certain time is determined by the initial density profile and instantaneous density profile

$$\frac{\partial}{\partial x} = \frac{n_c(\xi, \tau)}{n_c(\xi, 0)}\frac{\partial}{\partial \xi}. \tag{10}$$

It can be inferred from the above relation that though initially, at $\tau = 0, \xi = x$, as time goes on ξ evolves depending on plasma density. Employing the above conversion relations into governing equations, new set of equations are deduced for Lagrangian description. They are as follows:

$$\frac{\partial}{\partial \tau}\left(\frac{1}{n_c}\right) = \frac{1}{n_c(\xi, 0)}\frac{\partial v_{cx}}{\partial \xi}, \tag{11}$$

$$\frac{\partial v_{cx}}{\partial \tau} = -\frac{eE}{m} - \Omega_c v_{cy}, \tag{12}$$

$$\frac{\partial v_{cy}}{\partial \tau} = \Omega_c v_{cx}, \tag{13}$$

$$\frac{eE}{m} = -\frac{T_h}{m}\frac{n_c}{n_c(\xi, 0)}\frac{\partial \ln n_h}{\partial \xi}, \tag{14}$$

$$\frac{n_c}{n_c(\xi, 0)}\frac{\partial E}{\partial \xi} = 4\pi e(n_0 - n_c - n_h). \tag{15}$$

Now, combining Eqs. (11)–(13) and integrating with respect to τ, we get the following equation for v_{cy}:

$$\frac{1}{n_c(\xi,0)}\frac{\partial v_{cy}}{\partial \xi} = \Omega_c\left(\frac{1}{n_c} - \frac{1}{n_c(\xi,0)}\right), \tag{16}$$

where integration constant is determined from the initial conditions that at $\tau = 0, n_c = n_c(\xi,0)$ and $\partial v_c/\partial\xi = 0$. We proceed eliminating all other variables except hot electron and cold electron densities and finally obtain

$$\left[\frac{\partial^2}{\partial\tau^2} + \Omega_c^2\right]\left(\frac{1}{\hat{n}_c}\right) = \frac{\Omega_c^2}{\hat{n}_c(\xi,0)} + \frac{T_h}{m}\frac{1}{\hat{n}_c(\xi,0)}\frac{\partial}{\partial\xi} \times \left[\frac{\hat{n}_c}{\hat{n}_c(\xi,0)}\frac{\partial \ln\hat{n}_h}{\partial\xi}\right]. \tag{17}$$

Here, $\hat{n}_c, \hat{n}_h$ represents normalized cold electron and hot electron densities, both being normalized by their equilibrium densities n_{c0} and n_{h0}, respectively. Now, as we are considering the case of initially homogeneous plasma, densities of all species at $\tau = 0$ should be at their equilibrium value with no variation of space. Hence, we can replace $\hat{n}_c(\xi,0)$ by 1. Hot electron density n_h should be determined from Poisson's equation, which incorporates spatial variation of electric field. With all these substitutions, we get the following equation for $\hat{n}_c$:

$$\left(\frac{\partial^2}{\partial\tau^2} + \Omega_c^2\right)\left(\frac{1}{\hat{n}_c} - 1\right) = -c_{se}^2\frac{\partial}{\partial\xi}\left[\hat{n}_c\frac{\partial}{\partial\xi}\hat{n}_c\left\{1 - \frac{1}{\omega_{pc}^2} \times \left(\frac{\partial^2}{\partial\tau^2} + \Omega_c^2\right)\left(\frac{1}{\hat{n}_c} - 1\right)\right\}\right], \tag{18}$$

where $\omega_{pc} = \sqrt{4\pi n_{c0}e^2/m}$ is the cold electron plasma frequency. Time and space are now normalized by L/c_{se} and L, respectively, where L is system length. Electron gyrofrequency Ω_c is also normalized by c_{se}/L. We omit hats from normalized variables for notational simplicity. The equation describing spatial and temporal evolution of cold electron density, therefore, becomes in normalized form

$$\left(\frac{\partial^2}{\partial\tau^2} + \Omega_c^2\right)\frac{1}{n_c} - \Omega_c^2 = -\frac{\partial}{\partial\xi}\left[n_c\frac{\partial n_c}{\partial\xi}\right] + \epsilon\frac{\partial}{\partial\xi}\left[n_c\frac{\partial}{\partial\xi}n_c \times \left\{\left(\frac{\partial^2}{\partial\tau^2} + \Omega_c^2\right)\frac{1}{n_c} - \Omega_c^2\right\}\right] \tag{19}$$

ϵ is smallness parameter, defined as λ_{Dh}^2/L^2, where λ_{Dh} is the hot electron Debye length. The term related with ϵ signifies dispersive effects, which come into play for wave length comparable to Debye length. Nonlinearity, apparently disappeared with introduction of Lagrangian variables, was actually embedded in the formalism and reflects through the first term of the right hand side. Interplay between nonlinearity and dispersion, present in the above equation, is expected to produce some stable structure. Further, in absence of magnetic field, i.e., $\Omega_c = 0$, this equation matches exactly with the previous work of EAW in unmagnetized plasma.[14] Equation (19) describes full nonlinear behavior of EAW in homogeneous, magnetized plasma in presence of dispersion. We were ambitious to analyze full nonlinear electron acoustic wave in presence of magnetic field. This is the reason we have formulated the problem with Lagrangian variables. After formulation, we have realized that the full nonlinear problem with dispersion and nonlinearity is mathematically quite involved and needs heavy computing. However, small amplitude limit of this equation is easy to analyze extracting the essential physics of nonlinearity and dispersion adding an extra feature due to the magnetic field. The equation under investigation can also be obtained by reductive perturbation theory. Interestingly, keeping only up to square nonlinearity, we can get back the same equation from arbitrary nonlinear equation formulated in Lagrangian variables. Therefore, we have decided to highlight the physics with small amplitude. If anyone likes to analyze full nonlinear problem, this work would be a stepping stone for it. So, we assume that the cold electron density value, using small amplitude perturbation around its equilibrium, is as follows:

$$n_c = 1 + n, \tag{20}$$

so that $n \ll 1, \epsilon \sim n$. Substituting the value of n_c in the final Eq. (19) and retaining terms upto second order, we get

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right)n = \frac{\partial^2(n^2)}{\partial t^2} + \Omega_c^2 n^2 - \Omega_c^2 n + \frac{1}{2}\frac{\partial^2(n^2)}{\partial x^2} + \epsilon\frac{\partial^2}{\partial x^2}\left[\frac{\partial^2 n}{\partial t^2} + \Omega_c^2 n\right]. \tag{21}$$

For small amplitude nonlinearity, ξ, τ no longer remain Lagrangian variables but become equivalent to x, t. To show that the above equation contains KdV, we write the left hand side of Eq. (21) as

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right)n = \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial x}\right)\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x}\right)n, \tag{22}$$

which indicates wave propagation in both positive and negative $\hat{x}$ directions. If we consider waves propagating in positive $\hat{x}$ direction only then, $\frac{\partial n}{\partial t} \equiv -\frac{\partial n}{\partial x}$. This approximation is valid for weakly nonlinear case. Replacing $\frac{\partial}{\partial t}$ by $-\frac{\partial}{\partial x}$ and applying further transformation, $ax - t = \zeta, t = \tau'$, we get a modified KdV equation, the modification arising due to presence of magnetic field.

$$\frac{\partial n}{\partial \tau'} + \alpha n\frac{\partial n}{\partial\zeta} + \beta\frac{\partial^3 n}{\partial\zeta^3} = \gamma\int(n - n^2)d\zeta, \tag{23}$$

where, $a = 1/\left(1 + \frac{\epsilon\Omega_c^2}{2}\right)$, nonlinearity coefficient $\alpha = 3a/2$, dispersive coefficient $\beta = \epsilon a^3/2$, and coefficient of magnetic field correction term $\gamma = \Omega_c^2/2a$.

In absence of magnetic field, EAW is described by the well known KdV equation. The usual KdV equation is a Hamiltonian system that conserves infinite set of quantities. When $\gamma = 0$ in Eq. (23), energy conservation equation can be obtained by multiplying n both side and integrating with respect to ζ in the interval $(-\infty, \infty)$,

$$\frac{\partial W}{\partial \tau'} = 0 \tag{24}$$

with $W = \int_{-\infty}^{\infty} n^2(\zeta,\tau')d\zeta$ being the total energy of the system. The solution of the equation then given by solitary traveling wave solution,[15–17]

$$n = U\text{sech}^2\left[\sqrt{\frac{\alpha U}{12\beta}}\left(\zeta - \frac{\alpha U}{3}\tau'\right)\right]. \tag{25}$$

U is the normalized speed of the solitary wave. In presence of magnetic field, i.e., nonzero γ, the energy conservation relation becomes

$$\frac{\partial W}{\partial \tau'} = \gamma\int_{-\infty}^{+\infty} n\left[\int_{-\infty}^{\zeta}(n-n^2)d\zeta\right]d\zeta. \tag{26}$$

Exact solution of Eq. (23) is not easily tractable. Nevertheless, there are some approximate methods for small γ perturbation or weak field approximation such that we can observe the effects of applied magnetic field. According to prescribed method, we first let U to be time dependent[18–22]

$$n(\zeta,\tau') = U(\tau')\text{sech}^2\left[\sqrt{\frac{\alpha U(\tau')}{12\beta}}\left(\zeta - \frac{\alpha U(\tau')}{3}\tau'\right)\right]. \tag{27}$$

Replacing the value of n in Eq. (26) with the help of Eq. (27), a time evolution equation for U is obtained. The differential equation is subject to initial condition, at $\tau' = 0$, $U = 0$. Solution of the equation gives explicit functional dependence of U on τ'

$$U(\tau') = \frac{3}{2}\tan^2\left(\sqrt{\frac{2\epsilon}{3}}\Omega_c^2\tau'\right). \tag{28}$$

With the value of $U(\tau')$ substituted in Eq. (27) describes the solution of Eq. (23) for weak field approximation. The expression of time dependent amplitude $U(\tau')$ of the temporally growing solitary waves in Eq. (28) shows that τ_c is the critical time when the value of the argument approaches $\pi/2$ and $U(\tau')$ tends to infinity, which signifies wave breaking. This means that our result is valid up to time, which is less than critical time τ_c. So the above analytical solutions [Eqs. (27) and (28)] are valid only if $\tau' < \tau_c$, a critical value given by

$$\tau_c = \sqrt{\frac{3}{8\epsilon}}\frac{\pi}{\Omega_c^2}. \tag{29}$$

It should be noted that τ_c is inversely proportional to magnitude of applied magnetic field, which is taken to be small to validate our perturbative analysis. So, for weak field, our result sustains for large time. The above solutions [Eqs. (27) and (28)] clearly show that both the amplitude and the speed of the solitary waves increases temporally $(\tau' < \tau_c)$ due to the presence of magnetic field. This is actually the effect of the nonlinear source term present in the KdV equation. However, increase of amplitude does not mean that soliton solution is growing, rather, it is a redistribution of energy in space and time due to nonlinearity. In presence of magnetic field, let us define from our solution Eq. (27), the amplitude $A = U(\tau')$ and spatial width $\Delta = \sqrt{12\beta/(\alpha U(\tau'))}$, where $U(\tau')$ is given by Eq. (28). Interestingly, this time dependent solution retains its well known soliton property $A\Delta^2 = \text{constant}$, which can be expressed as

$$A\Delta^2 = \frac{12\beta}{\alpha},$$

where $12\beta/\alpha$ is constant for the given plasma parameter. Actually, near a critical time τ_c, the amplitude of solitary wave $U(\tau')$ becomes very large; consequently, $\text{sech}^2[\sqrt{U(\tau')}]$ is also very small, and, therefore, the solitary wave solution remains finite. Electron acoustic solitary wave solutions presented in Eq. (27) with amplitude $U(\tau')$ being replaced by Eq. (28) are shown in Fig. 1 for different time.

IV. RESULTS AND DISCUSSIONS

In this paper, we have conducted a study on two electron component, homogeneous plasma embedded in uniform magnetic field in presence of wave dispersion. Linear analysis shows that the dispersion relation of EAW is modified by a contribution (Ω_c^2) from Lorentz force. Nonlinear analysis is carried out by implementing Lagrangian variable method. It has been demonstrated that for weakly nonlinear case, behavior of this mode is depicted by KdV with a nonlinear source term. This nonlinear term corresponds to magnetic field and vanishes as magnetic field goes to zero. The exact solution of the equation seems nearly impossible. However, small amplitude EAW in absence of magnetic field is described well by the KdV equation, which maintains infinite set of conservation laws. Solutions of KdV equation are localized, stable, solitary structures. Exploiting the stability of solitary wave solutions, we can have an approximated solution for weak magnetic field perturbations.[23,24] Fig. 1 shows the solitary wave solutions with its speed and

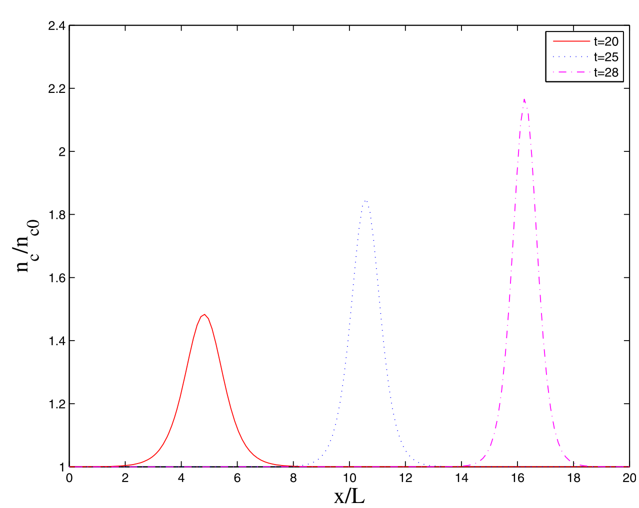

FIG. 1. Electron acoustic solitary wave solutions in presence of external magnetic field and wave dispersion at normalized time $t = 20$, 25, 28 with $\epsilon = 0.1$ and $\Omega_c^2 = 0.1$. Normalized cold electron density (n_c/n_{c0}) is plotted against normalized space variable (x/L).

amplitude increasing with time. The previous work which did not incorporate wave dispersion or electric field variation exhibits nonlinear oscillation with time, tending to wave breaking. Here, successful inclusion of wave dispersion arrests nonlinearity to provide stable, fast moving solitary structures. Satellite observations in earth's magnetosphere report existence of large amplitude, fast moving electron acoustic solitary waves. Fast auroral snapshot (FAST) observations in dayside auroral acceleration region reveal that these high amplitude nonlinear structures travel at a speed far greater than ion acoustic speed and are responsible for strong electron modulation associated with electron acceleration.[4,25–28] Similar type of structures observed by Viking and Polar satellite have been interpreted as electron acoustic waves.[29,30] Therefore, our theory and results may be helpful in interpreting those observations.

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