*Indian J. Phys.* **75B (3)**, 235–237 (2001)

**I J P B**
an international journal

# Single-mode graded index fiber directional coupler : analysis by a simple and accurate method

S Sanyal, S Gangopadhyay[+] and S N Sarkar*
Fiber Optics Research Group, Department of Electronic Science, University of Calcutta, 92, A P C Road, Kolkata-700 009, India
[+]Department of Physics, Surendranath College, 24 2 M G Road, Kolkata-700 009, India
E-mail sns@cucc ernet in

**Abstract** We present simple and accurate analysis of coupling characteristics of identical single-mode graded index fiber directional couplers Based on the recently developed simple series expression for the fundamental mode of such a fiber involving Chebyshev technique Analytical expressions of the relevant parameters are presented and the concerned calculations are executable with much less computations With examples of step and parabolic index fiber directional couplers we show that our estimations match excellently with the exact results

**Keywords** Single-mode fibers, directional couplers.

**PACS No.** 42 81 Qb

## 1. Introduction

Single-mode fiber directional couplers have emerged as the most prospective device in optical-fiber sensors [1–3], wavelength filters [4] and nonlinear optical fiber based devices [5] *etc.*. One should, therefore, know the coupling parameters accurately in order to prescribe its appropriate design in each field of use. Coupled-mode theory [6] has already been found to estimate accurately the coupling characteristics of step index fiber directional coupler over the practical range of parameters [7]. Further, during fabrication of single-mode step index fiber directional coupler, it becomes very difficult to maintain the step nature of the refractive index profile and therefore, the study of graded index fiber directional coupler is of tremendous importance.

The application of coupled-mode theory requires the knowledge of modal field distribution of each single-mode graded index fiber when they are non-interacting. In order to find the fundamental modal field for a single-mode graded index fiber one has to employ either numerical techniques or appropriate methods excepting the case of step index fiber where analytical expressions are easily available. The Gaussian approximation provides the simplest method in this context but as it fails to predict the cladding field accurately, it is not judicious to apply it for the analysis of fiber coupling device. The variational technique involving two parameter Gaussian approximation [8] has been shown to estimate excellently the coupling features of step index fiber directional coupler. Again the variational technique involving Gaussian-exponential-Hankel function [9, 10] has been employed to predict accurately the coupling characteristics of single-mode parabolic core fiber directional coupler. These analyses, however, involve extensive computations. Therefore, prescription of a simple but accurate method in this connection is very necessary. Based on simple power series expression for the fundamental mode of graded index fiber [11] we quantify graded index fiber directional coupler over a wide and practical single-mode range in a simple but accurate manner.

## 2. Theory

For a directional coupler formed between a pair of identical graded index fibers, the refractive index corresponding to the coupled structure can be expressed as :

$$\begin{aligned} n^2(R) &= n_1^2\left[1 - \delta R_1^q\right] && 0 < R_1 < 1, \\ &= n_1^2\left[1 - \delta R_2^q\right] && 0 < R_2 < 1, \\ &= n_1^2\left[1 - \delta\right] = n_2^2, && \text{otherwise}, \end{aligned} \tag{1}$$

* Corresponding Author

where $R_1 = r_1/a$ and $R_2 = r_2/a$ with $r_1$ and $r_2$ being the radial distance measured from the centres of the two fibers. $q$ is the profile exponent, which defines the shape of the profile. Under the condition of weak coupling, the coupled mode theory leads to the following expression for coupling coefficient ($C$) [6,10] given by :

$$C = \frac{k_0^2}{2\beta_1} \frac{\int_0^{\infty}\int_0^{2\pi}\left[n^2(R) - n_2^2(R)\right]\psi_1^*\psi_2 R_2 dR_2 d\phi_2}{\int_0^{\infty}\int_0^{2\pi}\psi_1^*\psi_1 R_1 dR_1 d\phi_1} \tag{2}$$

where $k_0$ is the free space wave number. $\psi_1$ and $\psi_2$ are the fundamental modes for respective fiber in isolation with $\beta_i$ being the propagation constant in the each fiber.

Based on Chebyshev technique, it has been recently shown [11] that the transverse field of fundamental mode of graded index fiber can be accurately given :

$$(R) = \begin{cases} a_0\left(1 + A_2 R^2 + A_4 R^4 + A_6 R^6\right), & R \le 1 \\ a_0\left(1 + A_2 + A_4 + A_6\right)\dfrac{K_0(W_c R)}{K_0(W_c)}, & R > 1 \end{cases} \tag{3}$$

where $W_c$ is the value of $W$ found by the said Chebyshev formalism. If is relevant to mention that for a given value of $V$ one can easily find $W$ and the corresponding constant $A_{2j}$ ($j$ = 1,2,3) with very little computations.

Using eq. (3) in eq. (2), we evaluate $C$ in case of step index fiber ($q = \infty$) as follows :

$$C = \frac{k_0^2\left(n_1^2 - n_2^2\right)}{\beta_1} \times \frac{K_0(W_c d/a)}{K_0(W_c)} \frac{S_2\left[S_9 I_1(W_c) - S_{10} I_0(W_c)\right]}{\left[S_1 + S_2^2\left\{\dfrac{K_1^2(W_c)}{K_0^2(W_c)} - 1\right\}\right]}, \tag{4}$$

where $d$ represents the separation between the cores of the fibers and

$$\begin{aligned}
S_1 &= 1 + A_2^2/3 + A_4^2/5 + A_6^2/7 + A_2 + 2A_4/3 + A_6/2 \\
&\quad + A_2 A_4/2 + 2A_2 A_6/5 + A_4 A_6/3, \\
S_2 &= 1 + A_2 + A_4 + A_6, \\
S_3 &= 1/W_c + 4/W_c^2, \\
S_4 &= 2/W_c^2, \\
S_5 &= 1/W_c + 16/W_c^3 + 64/W_c^5, \\
S_6 &= 4/W_c^2 + 32/W_c^4, \\
S_7 &= 1/W_c + 36/W_c^3 + 576/W_c^5 + 2304/W_c^7, \\
S_8 &= 6/W_c^2 + 144/W_c^4 + 1152/W_c^6, \\
S_9 &= 1/W_c + A_2 S_3 + A_4 S_5 + A_6 S_7, \\
S_{10} &= A_2 S_4 + A_4 S_6 + A_6 S_8.
\end{aligned} \tag{5}$$

Further, in case of parabolic index fiber ($q = 2$), the coupling coefficient is found by employing eq. (3) in eq. (2)

$$C = \frac{k_0^2\left(n_1^2 - n_2^2\right)}{\beta_1} \frac{K_0(W_c d/a)}{K_0(W_c)} \frac{S_2\left[S_{13} I_1(W_c) - S_{14} I_0(W_c)\right]}{\left[S_1 + S_2^2\left\{\dfrac{K_1^2(W_c)}{K_0^2(W_c)} - 1\right\}\right]}, \tag{6}$$

$$\begin{aligned}
\text{where } S_{11} &= 1/W_c + 64/W_c^3 + 2304/W_c^5 \\
&\quad + 36864/W_c^7 + 147456/W_c^9, \\
S_{12} &= 8/W_c^2 + 384/W_c^4 + 9216/W_c^6 + 73728/W_c^8, \\
S_{13} &= 1/W_c + A_2 S_3 + A_4 S_5 + A_6 S_7 - A_2 S_5 - A_4 S_7 - A_6 S_{11}, \\
S_{14} &= A_2 S_4 + A_4 S_6 + A_6 S_8 - S_4 - A_2 S_6 - A_4 S_8 - A_6 S_{12}
\end{aligned} \tag{7}$$

## 3. Results and discussion

In order to show the accuracy of our formulations for coupling coefficient we compare our results with the exact results in case of step as well as parabolic index fiber directional coupler. In Figure 1, we have plotted coupling coefficient $C$ (on a $\log_{10}$ scale) against the normalised centre-to-centre core separation $D$ (= $d/a$) for a step index fiber directional coupler for $V = 2.2$. It is seen that our predictions virtually remain indistinguishable from the exact ones In this way, the superiority of our formalism over Gaussian and modified Gaussian methods [8] is established.

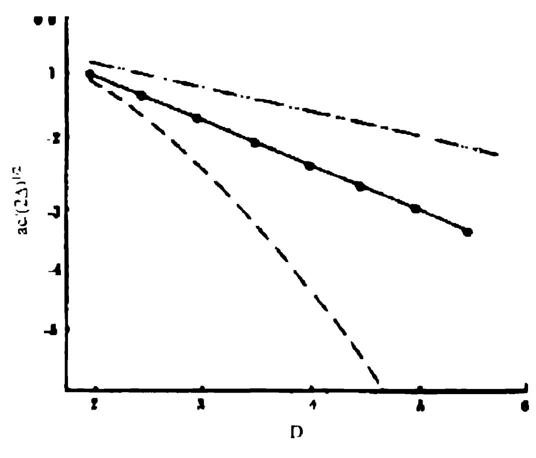


**Figure 1.** Variation of normalised scalar coupling coefficient $C$ with normalised separation $D$ for step core fiber Fiber directional coupler having $V$ = 2 2 (o o o our results, ---- Gaussian approximations, ·· — exact results, —··— modified Gaussian approximation).

It is relevant to mention that though the modified Gaussian method can quantify the step index fiber directional coupler quite accurately but it involved extensive computations while our formalism in this context require very little computations. Further, following Thyagarajan and Tewari [10] we also depict in Figure 2, the variation of normalised coupling length ($\tilde{L}_c = L_c\delta^{1/2}/a$) with normalised separation ($D$) for a parabolic index directional coupler for four different $V$ values namely $V = 2.0$, $V = 2.5$, $V = 3.0$ and $V = 3.5$. It is seen that our estimations agree excellently for all said $V$ values in the entire range of $D$ values. Application

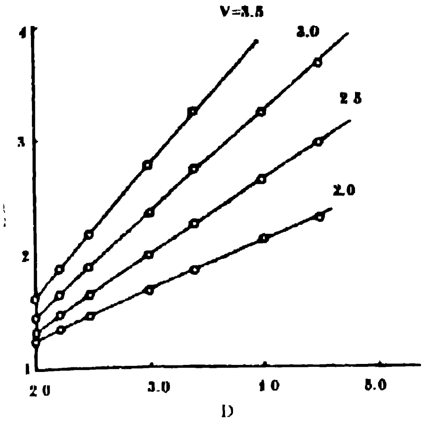


Figure 2. Variation of normalised coupling length with normalised separation $D$ for parabolic core fiber directional coupler (o o o our results, — exact results)

of Gaussian-exponential-Hankal function also produces accurate results for the present problem but it requires quite involved calculations of exponential integral functions. On the other hand, our simple formalism quantifies the single-mode parabolic index fiber directional coupler excellently with very little computations. Accordingly, our formalism should invite attention of system users because of its simplicity.

## 4. Conclusions

We present simple analytical formulations for the analysis of coupling coefficient of fiber directional coupler made of two identical single-mode graded index fibers. With examples of step and parabolic index fiber directional couplers, we show that predictions by our simple formalism agree excellently with the exact ones. The concerned calculations require very little computations.

**Acknowledgment**

The financial assistance of All India Council of Technical Education, India is gratefully acknowledged.

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