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A Simple Method for Studying Single-mode Graded Index Fibers in the low V Region

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Summary

Based on a simple series expansion method involving Chebyshev technique, we predict the cladding decay parameter (W) and the fundamental modal field inside the core as well as cladding in case of single-mode graded index fiber in the low normalised frequency (V) region. The method involves formulation of linear relationship of $K_1(W)/K_0(W)$ with 1/W for a few intervals of W, where W < 0.6. With examples of step and parabolic index fibers we show that said predictions by our simple formalism agree excellently with the available exact results. Thus the accuracy of our formalism is established.

1 Introduction

Single-mode optical fibers have emerged as the most important broad band transmission media for telecommunication. Such optical communication systems are operated in the wavelength range from 1.3 to 1.6 μm. This is so since optical fibers made of silica are characterised by extremely low attenuation loss (~ 0.2dB/Km) at the wavelength 1.55 μm while the material dispersion vanishes around the wavelength 1.31 μm. Study of single-mode graded index fibers in the low V region is of tremendous importance in the light of its application as a directional coupler which involves evanescent field coupling. For derivation of fundamental mode for a single-mode graded index fiber, one has to employ either numerical techniques or approximate methods excepting the case of step index fiber where analytical solutions are available. The application of variational technique involving single parameter [1] and two parameter [2, 3] Gaussian trial functions for the fundamental made of graded index fiber has shown that only the two parameter approximation can predict the fiber characteristics excellently over a wide single-mode range. Still the two parameter variational technique lacks accuracy in the low normalised frequency (V) region. However, the variational technique involving Gaussian-exponential-Hankel function [4, 5] provides accuracy over the entire single-mode range including the low V region. These analyses, however, involve extensive computation. Hence prescription of simple but accurate expressions of fundamental mode for graded index fiber is still proliferating in literature. An accurate but simple power series form of fundamental mode of graded index fiber based on Chebyshev technique has been reported recently [6]. This method is based on linear formulation of $K_1(W)/K_0(W)$ with 1/W in the range $0.6 \leq W \leq 2.5$ Thus the method has not quantified graded index fiber characteristics in the low V region, namely V< 1.4 for step index and V < 1.9 for parabolic index fibers. However the excellent predictions of graded index fiber characteristics over a wide single-mode range by the said Chebyshev formalism have motivated us to extend the analysis in the low V region. Further, from the study of data and graph given in [7], it is apparent that in order to prescribe a linear relation of $K_1(W)/K_0(W)$ with 1/W where W < 0.6, one has to choose a few small intervals of W.

In this communication, we report formulation of linear relationship of $K_1(W)/K_0(W)$ with 1/W by least square fitting technique for a few small intervals of W where W < 0.6. Using those relations, we predict cladding decay parameter W and field inside the core as well as cladding for graded index fiber by Chebyshev formalism.

2 Analysis

The refractive index profile of a graded index optical fiber can be expressed as

$$\begin{aligned} n^2(R) &= n_1^2\left(1-2\delta f(R)\right), \quad && 0 \leq R \leq 1 \\ &= n_2^2, && R > 1 \end{aligned} \quad (1)$$

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where R=r/a, 'a' being the core radius; $\delta = (n_1^2 - n_2^2)/(2n_1^2)$ with n_1 and n_2 being the refractive indices of the core axis and the cladding respectively. Here, f(R) defines the shape of the refractive index profile and in case of a power-law profile, it is given by

$$f(R) = R^q, \;\; R \le 1 \tag{2}$$

'q' being the profile exponent whose value is ∞ for step index fiber and 2 for parabolic index fiber.

For a weakly guiding fiber, the fundamental modal field ψ(R) inside the core is the solution of the scalar wave equation

$$\frac{d^2\psi}{dR^2} + \frac{1}{R}\frac{d\psi}{dR} + \left[V^2\left(1 - f(R)\right) - W^2\right]\psi = 0, \;\; R \le 1 \tag{3}$$

together with the boundary condition

$$\left(\frac{1}{\psi}\frac{d\psi}{dR}\right)_{R=1} = -\frac{WK_1(W)}{K_0(W)} \tag{4}$$

where $V[= k_0 a(n_1^2 - n_2^2)^{1/2}]$ and $W[= a(\beta^2 - n_2^2 k_0^2)^{1/2}]$ are the normalised frequency and cladding decay parameter respectively with k_0 and β being the free space wave number and propagation constant respectively.

The fundamental modal field in the cladding is given by

$$\psi(R) \sim K_0(WR), \;\; R > 1. \tag{5}$$

By least square fitting over the interval W ≤ 0.6, we formulate $K_1(W)/K_0(W)$ as below

$$\frac{K_1(W)}{K_0(W)} = \alpha + \frac{\beta}{W} \tag{6}$$

where the values of α and β are found for some short intervals by the least square fitting technique and these have been presented in Table 1.

Taking into consideration that the fundamental modal field ψ(R) is an even function of R and ψ(0) is nonzero, we can express ψ(R) in terms of a Chebyshev power series as [8, 9]

$$\psi(R) = \sum_{j=0}^{j=M-1} a_{2j} R^{2j}. \tag{7}$$

The Chebyshev points are given as [9]

$$R_m = \cos\left(\frac{2m-1}{2M-1}\frac{\pi}{2}\right), \;\; m = 1,2,\ldots (M-1). \tag{8}$$

Following [6] we approximate (7) by taking M = 4 when we obtain

$$\psi(R) = \sum_{j=0}^{j=3} a_{2j} R^{2j} \tag{9}$$

and the corresponding Chebyshev points are found from [8] as

$$R_1 = 0.4338, \;\; R_2 = 0.7818 \;\text{ and }\; R_3 = 0.9749. \tag{10}$$

The expression for ψ(R) given by (9) is employed in (3) to obtain the following three equations corresponding to the three Chebyshev points given in (10)

$$\begin{aligned} & a_0\left[V^2\left(1 - f(R_i)\right) - W^2\right] + \\ & + a_2\left[4 + R_i^2\left(V^2\left(1 - f(R_i)\right) - W^2\right)\right] + \\ & + a_4\left[16R_i^2 + R_i^4\left(V^2\left(1 - f(R_i)\right) - W^2\right)\right] + \\ & + a_6\left[36R_i^4 + R_i^6\left(V^2\left(1 - f(R_i)\right) - W^2\right)\right] = 0 \\ & \text{where } i = 1,2 \text{ and } 3. \end{aligned} \tag{11}$$

Further, using (9) and (6) in (4) we get

$$\begin{aligned} & a_0(\alpha W + \beta) + a_2(\alpha W + 2 + \beta) + a_4(\alpha W + 4 + \beta) + \\ & + a_6(\alpha W + 6 + \beta) = 0 \end{aligned} \tag{12}$$

a_0, a_2, a_4 and a_6 given by three equations of (11) and (12) will lead to non-trivial solution provided

$$\begin{vmatrix} A_1 & B_1 & C_1 & D_1 \\ A_2 & B_2 & C_2 & D_2 \\ A_3 & B_3 & C_3 & D_3 \\ A_4 & B_4 & C_4 & D_4 \end{vmatrix} = 0 \tag{13}$$

where

$$\begin{aligned} A_i &= V^2\left[1 - f(R_i)\right] - W^2 \\ B_i &= 4 + R_i^2\left[V^2\left(1 - f(R_i)\right) - W^2\right] \\ C_i &= 16R_i^2 + R_i^4\left[V^2\left(1 - f(R_i)\right) - W^2\right] \\ D_i &= 36R_i^4 + R_i^6\left[V^2\left(1 - f(R_i)\right) - W^2\right] \end{aligned} \tag{14}$$

with i being 1, 2 and 3

and

$$\begin{aligned} A_4 &= \alpha W + \beta; \qquad B_4 = 2 + A_4; \\ C_4 &= 4 + A_4; \qquad D_4 = 6 + A_4. \end{aligned} \tag{15}$$

W for a given value of V can be found by solving (13). Again, any three of the four equations given by (11) and (12) can be employed to evaluate the constants a_{2j} (j = 1, 2, 3) in terms of a_0. Thus the field inside the core and cladding can be expressed as

$$\begin{aligned} \psi(R) &= a_0\left(1 + A_2R^2 + A_4R^4 + A_6R^6\right), && R \le 1 \\ &= a_0\left(1 + A_2 + A_4 + A_6\right)\frac{K_0(W_C R)}{K_0(W_C)}, && R > 1 \end{aligned} \tag{16}$$

where $A_{2j} = a_{2j}/a_0$, j = 1, 2, 3 and W_c is the value of W found by the present formalism.

In case of a step index fiber the above values for field are compared with the exact analytical values given as

$$\psi(R) = J_0(UR), \qquad R \le 1$$
$$= \frac{J_0(U)}{K_0(W)} K_0(WR), \qquad R > 1. \qquad (17)$$

For graded index fiber, the comparison in this context is made with the available exact numerical results. Here U is a normalised propagation constant being equal to $(V^2 - W^2)^{1/2}$

3 Results and discussions

In order to verify the validity of our formalism, we first estimate the values of cladding decay parameter W for different values of normalised frequeny V in the low V region. We compare our results with the exact analytical results [10–13] in case of a step index fiber and also with the exact numerical results [12, 13] for a parabolic index fiber. In Table 1, we present different values of α and β for different short intervals of W when $W < 0.6$. In Table 2, we present the values of W obtained by our method as W_c and compare it with the available analytical values [10–13] for step index fiber. It is found that the values predicted by the present formalism are extremely accurate within an accuracy of 0.560 %. In Table 3, we present the values of W_c and W in case of parabolic index fiber for some typical values of V where $V < 1.9$. It is seen that our simple technique provides accuracy within 1.279 %. Further, one can predict fundamental modal field by employing (16) provided it is normalised in terms of a_0. Accordingly in Figs. 1(a), 1(b) and 1(c) the variation of normalised fundamental mode $\psi(R)$ for step index fiber with normalised radius R is depicted for three typical low values of V. $\psi(R)$ found from (16) is presented by solid line while exact $\psi(R)$ obtained from (17) is presented by crosses. In each case our predictions match excellently with the exact ones. In case of parabolic index fiber, we also present the variation of normalised $\psi(R)$ with R in Figs. 2(a), 2(b) and 2(c) for three different low values of V. Here also the values obtained by the present method and the simulated exact ones are represented by solid lines and crosses respectively. In case of parabolic index fiber, we also find excellent agreement between our formalism and the simulated exact ones in this context. Further, it is relevant to mention in this connection that the Chebyshev technique [8] estimated the first higher order mode cut-off frequency for step and parabolic index fibers within an accuracy of 0.17 % and 1.17 %, respectively. In this regard our simple formalism seems to be quite consistent with the work [8] in predicting cladding decay parameter and fundamental modal field in case of step and parabolic index fibers.

Table 1: Values of α and β for different ranges of cladding decay parameter W

0.000008 – 0.00001	1784.03559000	0.06519083
0.00001– 0.00005	397.14916110	0.08228378
0.0003 – 0.0007	36.29120034	0.11094628
0.001– 0.005	11.89030264	0.13114227
0.01 – 0.10	2.12331170	0.20702490
0.10 – 0.20	1.36850086	0.27248694
0.20 – 0.30	1.22199127	0.30182093
0.30 – 0.40	1.15243299	0.32309927
0.40 – 0.50	1.11884710	0.33673703

Table 2: Values of Cladding decay parameter W for single-mode step index fiber in the low V region

1.3	0.49724300	0.49700000	0.048%
1.2	0.39211600	0.39200000	0.029%
1.1	0.29271700	0.29300000	0.096%
1.0	0.20150500	0.20200000	0.245%
0.9	0.12475000	0.12471050	0.031%
0.8	0.06447995	0.06412047	0.560%
0.7	0.02472997	0.02460669	0.501%
0.6	0.00565997	0.00568281	0.402%
0.5	0.00048997	0.00049019	0.045%
0.4	0.00000997	0.00001002	0.499%

Table 3: Values of Cladding decay parameter W for single-mode parabolic index fiber in the low V region

1.7	0.48855990	0.48270000	1.213%
1.5	0.31538000	0.31763000	0.708%
1.3	0.17120000	0.17340000	1.268%
1.1	0.06615994	0.06617078	0.016%
1.0	0.03351496	0.03394951	1.279%
0.9	0.01279997	0.01286055	0.471%
0.8	0.00349997	0.00346049	1.140%
0.7	0.00051997	0.00052037	0.076%
0.6	0.00002997	0.00003031	1.121%

4 Conclusion

A linear variation of $K_1(W)/K_0(W)$ with $1/W$ is formulated for a few intervals of W, where $W < 0.6$. With this relation as the kingpin, we employ Chebyshev technique in order to estimate accurately the cladding decay parameter as well as field both inside the core and cladding for single-mode graded index fibers. Taking step and parabolic index fibers as examples, we show that the results predicted by our simple theory match excellently with the available exact results. Such excellent predictions in the low V region should favour these simple formulations being employed in the study of such fiber characteristics, particularly for devices involving evanescent field coupling.

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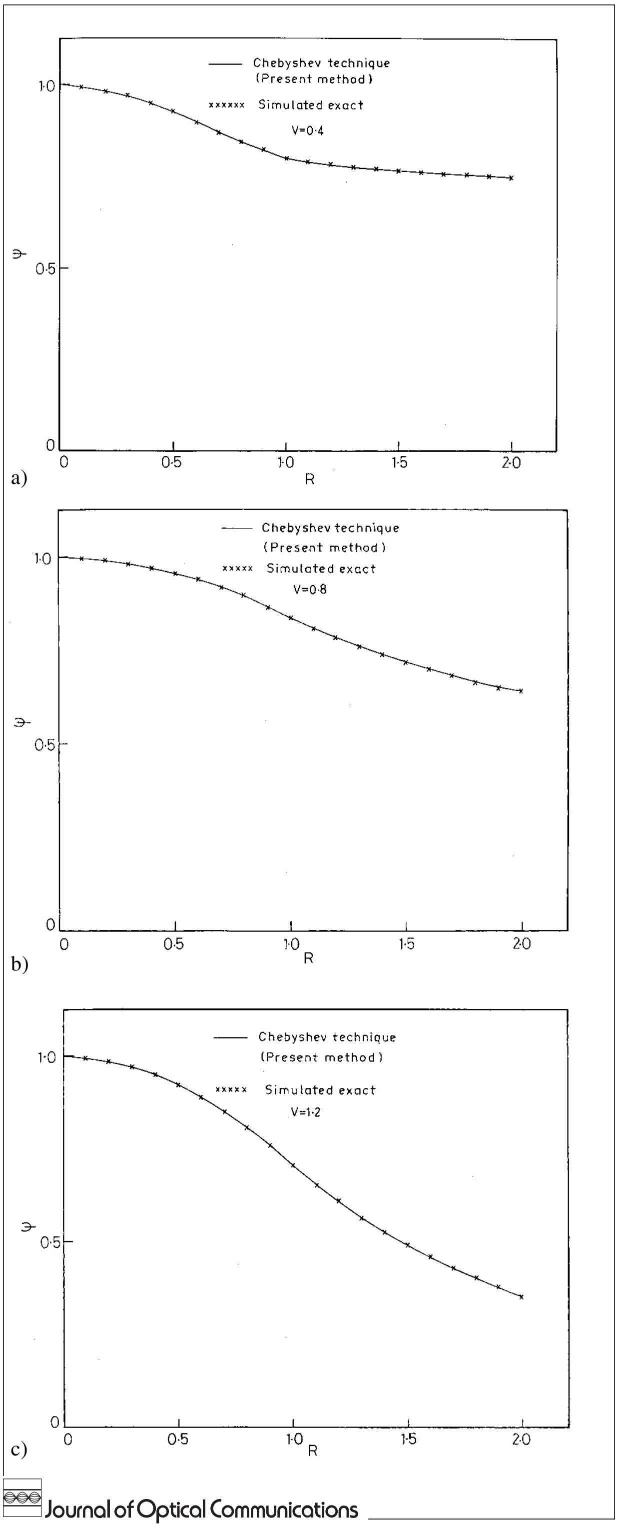

Fig. 1: Variation of normalised field ψ with normalised radius R for single-mode step index fiber having (a) V = 0.4, (b) V = 0.8 and (c) V = 1. 2

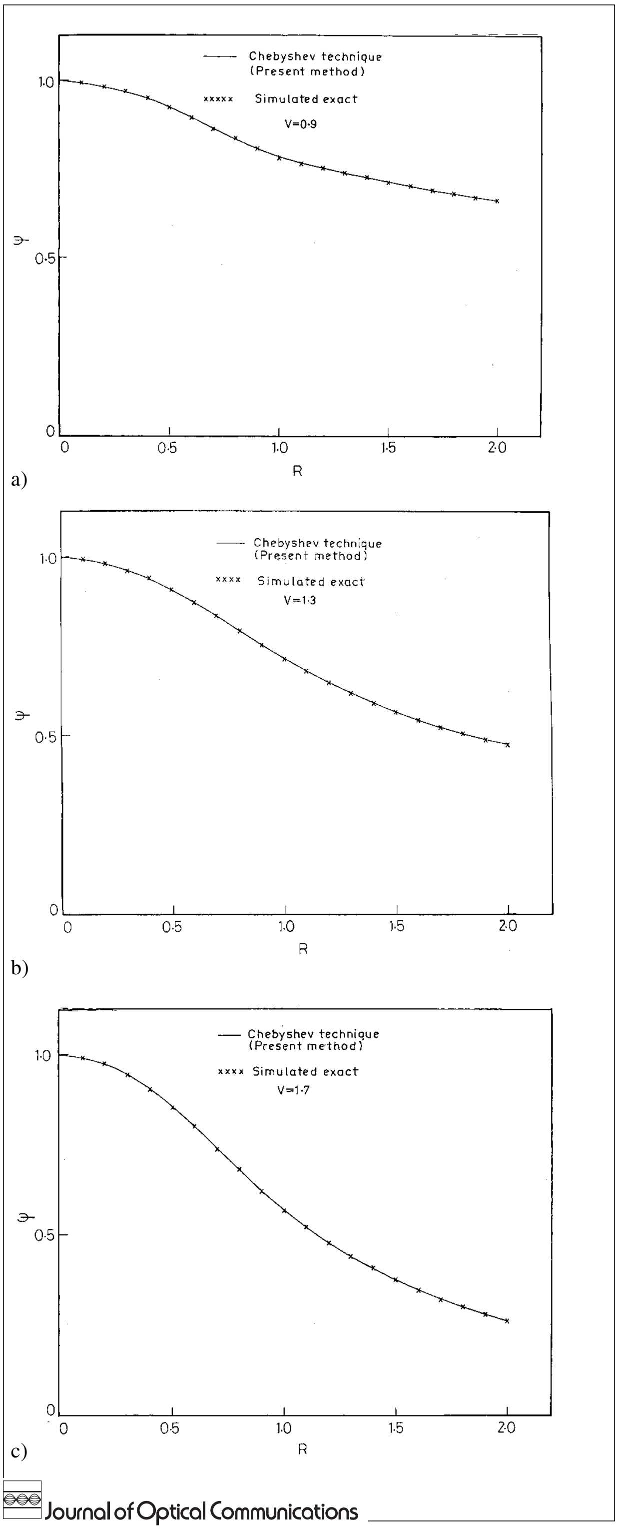

Fig. 2: Variation of normalised field ψ with normalised radius R for single-mode parabolic index fiber having (a) V = 0.9, (b) V = 1.3 and (c) V = 1.7

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