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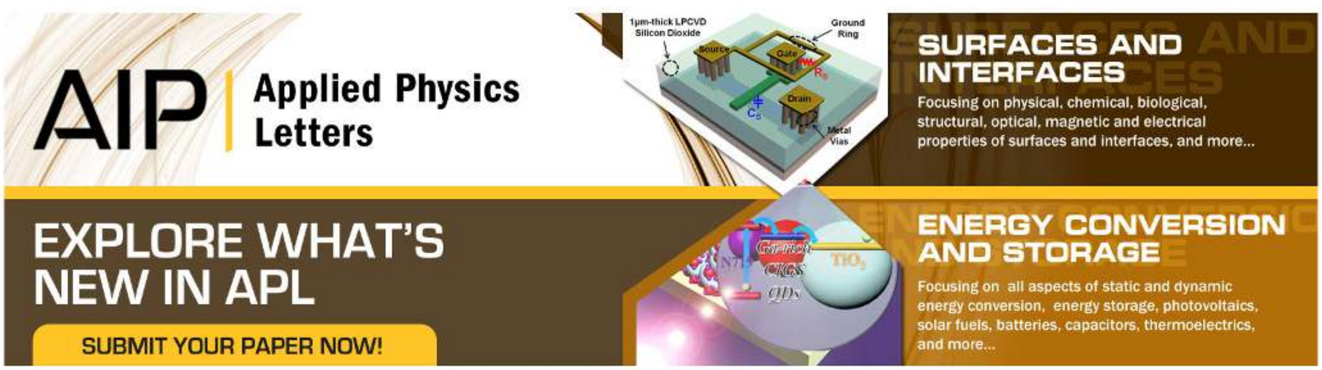

Shock wave in a two-dimensional dusty plasma crystal

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Two-dimensional (2D) shock structures of longitudinal dust lattice wave (LDLW) in a hexagonal Yukawa crystal are studied. The nonlinear evolution equation derived for dusty plasma crystal is found to be a 2D Burgers' equation, where the Burgers' term, i.e., the dissipation is provided by "hydrodynamic damping" due to irreversible processes that take place within the system. Analytical and numerical solutions of this equation on the basis of crystal experimental parameters show the development of compressional shock structures of LDLW in 2D dusty plasma crystal. The shock strength decreases (increases) with the increase in lattice parameter κ (angle of propagation of the nonlinear wave). The results are discussed in the context of 2D monolayer hexagonal dusty plasma crystal experiments. © *2009 American Institute of Physics*. [doi:10.1063/1.3240339]

I. INTRODUCTION

Dusty plasma crystals can be created by adding spherical macroscopic (micron-sized) dust particles to ion-electron plasma. The dust particles then charge up (by collecting ions and electrons) to a large negative (usually) charge Q (as electron thermal velocity is much greater than that of ion) and form "complex plasma." Complex plasmas can exist in solid, liquid, and gaseous states and exhibit phase transition.[1] Dust particles interact with each other collectively and their repulsive interparticle potential is shielded by the ion-electron plasma, characterized by a Debye length λ_D. The dust particle coupling is measured by the parameter $\Gamma(=Q^2e^{-\kappa}/4\pi\epsilon_0\Delta T)$, where Δ is the lattice spacing (interdust spacing), T is the dust particle temperature, and $\kappa(=\Delta/\lambda_D)$ is the lattice parameter. When the dust particle potential energy exceeds its kinetic energy, i.e., $\Gamma \gg 1$, the plasma becomes strongly coupled and forms ordered structures.[1] The formation and dynamics of dusty plasma crystals ($\Gamma > \Gamma_{cr}$) have been investigated in various laboratory experiments.[2,3]

Different wave modes can exist in dusty plasma crystals. In one-dimensional (1D) dusty plasma crystals, theory predicts compressional dust lattice waves[4,5] (DLWs) and transverse vertical waves.[6] In a two-dimensional (2D) lattice, longitudinal (compressional),[7] transverse shear,[8,9] and sloshing[10] modes were observed in laboratory experiments. All these wave modes are the elastic deformations of lattice. The most surprising discovery was the realization that the dusty plasma crystals sustain the nonlinear coherent structures. In 2D lattice, nonlinear phenomena such as formation of Mach cones or wakes, solitons, wave-wave interactions, and shocks were observed in experiments.[11–15] Previously, both experimental observations[16] and theoretical investigations[17,18] revealed that shock wave also exist in weakly coupled ($\Gamma \ll 1$) dusty plasma and even in strongly coupled quasicrystal state ($1 \ll \Gamma < \Gamma_{cr}$).[19]

It is seen that in the absence of any dissipation, competition between the wave breaking nonlinearity and dispersion of waves can lead to the appearance of solitons in monolayer hexagonal 2D dusty plasma crystal.[20,21] However, it is well known that in the presence of dissipation, shock wave is generated due to the balancing between the nonlinearity and the combined action of dispersion and dissipation. In case of small but finite amplitude wave, when the dissipation dominates over dispersion, the shock is described by the Burgers' equation (no dispersion),[22,23] which exhibits monotonic shock structure, whereas the shock is described by Korteweg–de Vries–Burgers' equation (both dissipation and dispersion),[23] which exhibits oscillatory shock structure when dissipation is weak. Recently, both the monotonic shock structure (no dispersion)[24] and the oscillatory shock structure (both dissipation and dispersion)[25] of longitudinal DLW in 1D lattice have been investigated. However, the shock wave propagation characteristics in 2D lattice are still to be investigated.

In this paper, the shock propagation characteristics of small but finite amplitude longitudinal DLW in a monolayer hexagonal 2D dusty plasma crystal are studied when dissipation is strong. The "hydrodynamic damping" that arises due to finite velocity of the internal motion of the system is proposed to be a possible mechanism for the shock wave in monolayer hexagonal 2D dusty plasma crystal. It is shown that the nonlinear LDLW is governed by a 2D Burgers' equation. The implications of the results with the shock experiment[15] in 2D dusty plasma crystal are also discussed.

The paper is organized in the following manner. Section II contains the theoretical model with physical assumptions. The 2D Burgers' equation that describes the propagation of nonlinear DLW is derived in Sec. III. Section IV deals with the steady state (traveling wave) solution of the nonlinear equation. The numerical solutions on the basis of experimental plasma parameters are explained in Sec. V. Finally, a summary of the results in connection with the shock wave experiment is presented in Sec. VI.

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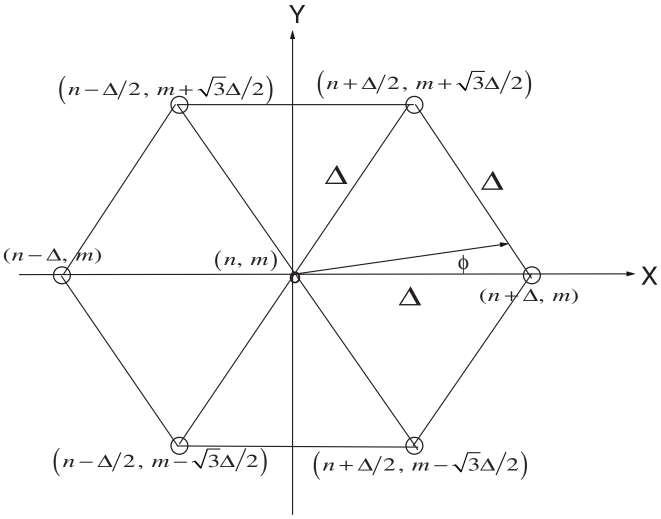

FIG. 1. Elementary monolayer hexagonal 2D crystal centered at (n,m) with lattice spacing Δ. The quasilongitudinal wave propagation vector makes an angle ϕ with the x-axis.

II. PHYSICAL ASSUMPTIONS: THEORETICAL MODEL

The physical assumptions to formulate the problem theoretically are the following:

(i) The dusty plasma consists of electrons, ions, and small particles of solid matter, i.e., the dust grains. The dust grains gain negative charge Q and the mass of it is m_d. A monolayer 2D hexagonal lattice structure is considered (Fig. 1). In the unperturbed situation, the particle coordinate is $(x_s=\Delta(n+m/2),\ y_s=m\sqrt{3}\Delta/2)$, where Δ is the interdust distance, i.e., lattice spacing and $s=(n,m)$ denotes a pair of integers such that $s=\{(0,\pm 1),(\pm 1,0),(1,-1),(-1,1)\}$. The interaction potential between each particle is the Yukawa potential[26] and the corresponding interdust force between sth and $\acute{s}$th particle is

$$\vec{F}_{\acute{s},s}=-\vec{\nabla}_{r_s}\left[\frac{Q^2\exp\left(-\frac{r_{\acute{s},s}}{\lambda_D}\right)}{4\pi\epsilon_0 r_{\acute{s},s}}\right] = \frac{Q^2\exp\left(-\frac{r_{\acute{s},s}}{\lambda_D}\right)}{4\pi\epsilon_0 r_{\acute{s},s}}\left(\frac{1}{\lambda_D}+\frac{1}{r_{\acute{s},s}}\right)\left(\frac{\vec{r}_{\acute{s}}-\vec{r}_s}{r_{\acute{s},s}}\right), \quad (1)$$

where $r_{\acute{s},s}=|\vec{r}_{\acute{s},s}|=|\vec{r}_{\acute{s}}-\vec{r}_s|$ and λ_D is the plasma Debye Length. All Q, m_d, and λ_D are assumed to be constant. In the presence of weak external force, it is assumed that $\vec{r}_{\acute{s},s}=\vec{r}_{\acute{s},s}(0)+\vec{D}(\acute{s},s)$, where $\vec{r}_{\acute{s},s}(0)$ is the relative equilibrium position and $\vec{D}(\acute{s},s)$ is the relative displacement of the sth lattice. Note that $\vec{D}(\acute{s},s)=[d_l(\acute{s})-d_l(s)]\hat{x}+[d_t(\acute{s})-d_t(s)]\hat{y}$, where $d_l(\cdots)$ and $d_t(\cdots)$ are, respectively, the longitudinal and transverse displacement components.

(ii) Experimental observations, 2D molecular dynamics simulation[11] and theoretical investigation[13] reveal that, for glow-discharge plasma, the nearest neighbor approximation is well justified for 2D monolayer hexagonal dusty plasma crystals for lattice parameter $\kappa(=\Delta/\lambda_D)>1$. Thus here it is assumed that $\Delta>\lambda_D$, i.e., $\kappa>1$, so that the dust particles in this monolayer 2D hexagonal crystal interact with their six nearest neighbor.

(iii) The waves in a crystals are the elastic deformation of the lattice. In the actual motion, the elastic body is not in thermodynamic equilibrium at every instant because of finite velocity of internal motion of the system. Therefore, processes will take place within the system, which tends to return the elastic body to thermodynamic equilibrium and as a result the motion becomes irreversible. This indicates energy dissipation takes place within the system, which may be referred to, as in fluids, as hydrodynamic damping or "viscosity."[27] The corresponding dissipative force can be expressed through the following discrete relation:[24,25]

$$\vec{F}_{\acute{s},s}(\mathrm{diss})=m_d\nu_{\mathrm{diss}}\frac{\partial\vec{D}(\acute{s},s)}{\partial t} = m_d\nu_{\mathrm{diss}}\frac{\partial\{[d_l(\acute{s})-d_l(s)]\hat{x}+[d_t(\acute{s})-d_t(s)]\hat{y}\}}{\partial t}, \quad (2)$$

where ν_{diss} is the damping (hydrodynamic) frequency. This hydrodynamic damping mechanism is suggested for the generation of shock wave in 2D dusty plasma crystals.

(iv) For strong dissipation, to incorporate the above dissipative effects, it is assumed that the hydrodynamic damping frequency ν_{diss} is large but finite compared to the dust lattice frequency $\omega_L(=C_{DL}/\Delta)$, where

$$C_{DL}=\sqrt{\frac{Q^2e^{-\kappa}(\kappa^2+2\kappa+2)}{4\pi\epsilon_0 m_d\kappa\lambda_D}} \quad (3)$$

is the dust lattice speed in the nearest neighbor approximation.[4,12]

(v) An external force $[=\vec{F}_{\acute{s},s}(\mathrm{ext})]$ is often introduced in complex plasma experiments[12,14,15] for the initial laser excitation and/or the parabolic confinement. However, for the first step this force term is to be omitted in the present model. Also note that the damping due to neutral drag is to be omitted as the neutral drag does not play any direct role in the formation of nonlinear structures such as soliton[20] and shock[24,25] in dusty plasma crystal, but plays a predominant role in the life and death of soliton and shock.

III. EVOLUTION EQUATION: 2D NONLINEAR WAVE EQUATION

The foregoing assumptions lead to the following equation of motion of the sth particle in the X-direction and Y-direction:

$$\frac{\partial^2 d_l}{\partial t^2} = \frac{\left(\Sigma_{s\neq \acute{s}} F_{s,\acute{s}}\right)_x}{m_d} + \nu_{\text{diss}} \frac{\partial\left[\Sigma_{s\neq \acute{s}} \vec{D}(\acute{s},s)\right]_x}{\partial t},$$
$$\frac{\partial^2 d_t}{\partial t^2} = \frac{\left(\Sigma_{s\neq \acute{s}} F_{s,\acute{s}}\right)_y}{m_d} + \nu_{\text{diss}} \frac{\partial\left[\Sigma_{s\neq \acute{s}} \vec{D}(\acute{s},s)\right]_y}{\partial t}, \quad (4)$$

where $s,\acute{s} \in \{(0,\pm 1),(\pm 1,0),(1,-1),(-1,1)\}$. In order to study the propagation characteristics of small but finite amplitude quasilongitudinal nonlinear DLW, the following stretched coordinates are introduced:

$$\xi = \epsilon \frac{(x - Vt)}{\Delta}, \quad \eta = \epsilon^{3/2} \frac{y}{\Delta}, \quad \tau = \epsilon^2 \omega_L t, \quad (5)$$

where ϵ is a small parameter that indicates the magnitude of the rate change and V is the wave velocity. It is to be noted that, in such a quasilongitudinal wave, the transverse displacements has a higher-order smallness than the amplitude of longitudinal displacements ($d_t = \sqrt{\epsilon} d_l$) and thus the dependent variables (d_l, d_t) are expanded in powers of ϵ in the following way:

$$d_l = d_l^{(0)}(\xi,\eta,\tau) + \epsilon d_l^{(1)} + \cdots;$$
$$d_t = \epsilon^{1/2} d_t^{(0)}(\xi,\eta,\tau) + \epsilon^{3/2} d_t^{(1)} + \cdots; \quad \frac{\partial d_l^{(0)}}{\partial \xi} = \Delta u. \quad (6)$$

Also due to the assumption (iv) for the consistent perturbation, it is assumed that

$$\bar{\nu}_{\text{diss}} = \frac{\nu_{\text{diss}}}{\omega_L} \sim O(1). \quad (7)$$

To consider the continuum approximation, it is assumed that the characteristic scale length L (the typical scale length of the wave form that can be the width of a pulse or the wavelength of a sinusoidal wave) is much larger than the lattice spacing Δ so that $s=(n,m)$ can be considered as quasicontinuous variable (coordinate). Expansion of $D(\acute{s},s)$ in Taylor's series retaining the terms $O(\Delta/L)^4$ (Refs. 20 and 21) in the nearest neighbor approximation and substitution of Eqs. (5)–(7) in Eqs. (1) and (4) yields the following relations in the lowest order of ϵ:

$$V^2 = \frac{9}{8} C_{DL}^2 \Rightarrow V = \pm \frac{3}{2\sqrt{2}} C_{DL} \quad (8)$$

and

$$\frac{\partial^2 d_t^{(0)}}{\partial \xi^2} = \frac{\partial^2 d_l^{(0)}}{\partial \xi \, \partial \eta}. \quad (9)$$

Finally, the usual perturbation analysis yields [keeping the terms $O(\epsilon^3)$] the following (nondimensional) 2D Burgers' equation:

$$\frac{\partial}{\partial \xi}\left[\frac{\partial u}{\partial \tau} - \Lambda u \frac{\partial u}{\partial \xi} - \mu \frac{\partial^2 u}{\partial \xi^2}\right] + \gamma \frac{\partial^2 u}{\partial \eta^2} = 0, \quad (10)$$

where

$$\Lambda = \frac{3}{16\sqrt{2}}\left[\frac{2\kappa^3 + 5\kappa^2 + 10\kappa + 10}{\kappa^2 + 2\kappa + 2}\right], \quad \gamma = \frac{3}{4\sqrt{2}}, \quad (11)$$

and

$$\mu = \frac{3\bar{\nu}_{\text{diss}}}{4} = \frac{3}{4}\left(\frac{\nu_{\text{diss}}}{\omega_L}\right). \quad (12)$$

The expression for coefficient of nonlinearity Λ [Eq. (11)] shows that it increases with the increase in the lattice parameter κ. The above Eq. (12) shows that the Burgers' term $\mu \propto \bar{\nu}_{\text{diss}}$, i.e., proportional to normalized hydrodynamic damping frequency. Thus the Burgers' term originates from the hydrodynamical damping. The presence of Burgers' term in Eq. (10) implies the possibility of existence of shock structure and hence the dissipative effect due to hydrodynamic damping is responsible for the generation of shock wave in 2D complex (dusty) plasma.

IV. STEADY STATE SOLUTION: TRAVELING WAVE SOLUTION

The 2D Burgers' Eq. (10) is analytically exactly solvable. To find the steady state (traveling wave) solution of Eq. (10), it is assumed that $u(\xi,\eta,\tau)$ depends on (ξ,η,τ), through χ, where χ is defined by

$$\chi = V_f \tau - \xi - p\eta = \frac{\epsilon}{\Delta}\left[C_{DL}\left(\frac{3}{2\sqrt{2}} + \epsilon V_f\right)t - x - \sqrt{\epsilon} p y\right], \quad (13)$$

where V_f is the velocity of the moving wave front and p is related to ϕ by $\tan \phi = \sqrt{\epsilon} p$, so that ϕ is the angle between the direction of propagation and x-axis. The Mach number M, which is the ratio of nonlinear wave velocity to linear wave velocity, is given by

$$M = \frac{C_{DL}(3/2\sqrt{2} + \epsilon V_f)}{C_{DL}} = 1.1 + \epsilon V_f \gg 1. \quad (14)$$

Now the transformation of Eq. (10) to the wave frame χ [Eq. (13)] and integration of the transformed equation with respect to χ subject to the boundary conditions u, $du/d\chi$, and $d^2u/d\chi^2$, all$\rightarrow 0$ at $\chi \rightarrow -\infty$ yields the following traveling wave solution:

$$u = -\frac{(V_f + \gamma p^2)}{\Lambda}\left[1 + \tanh\left(\frac{\chi}{\delta}\right)\right], \quad (15)$$

where δ is the shock width (shock transition width region[23]) and is given by

$$\delta = \frac{2\mu}{[\gamma \tan^2 \phi + (M - 1.1)]}. \quad (16)$$

The shock strength (shock height) is given by the following relation:

$$\epsilon(u_{\chi=\infty}-u_{\chi=-\infty})=\epsilon u_{\chi=\infty}=-\epsilon\frac{2(V_f+\gamma p^2)}{\Lambda}$$

$$=-\frac{2}{\Lambda}[\gamma\tan^2\phi+(M-1.1)]. \qquad (17)$$

Let us now briefly consider the applicability condition for the continuum approximation considered. The continuum approximation to study the shock propagation characteristics in dusty plasma crystals is valid only if the characteristic scale length is much larger than the lattice spacing Δ. The natural scale length L (normalized by Δ) of the system considered is defined as

$$L=\left|u/\left(\frac{du}{d\chi}\right)_0\right|=\delta\left|1+\tanh\left(\frac{\chi}{\delta}\right)\right|. \qquad (18)$$

This shows that the characteristic scale length is proportional to the shock width δ and in the asymptotic limit $L=2\delta$. Thus it is justified to assume

$$L\approx\delta. \qquad (19)$$

The continuum approximation is valid only if

$$L\approx\delta\gg 1\Rightarrow\bar{\nu}_{\rm diss}=\frac{\nu_{\rm diss}}{\omega_L}\gg\frac{2}{3}[\gamma\tan^2\phi+(M-1.1)]. \qquad (20)$$

V. NUMERICAL SOLUTION: SHOCK STRUCTURE

To analyze the analytical results numerically, the following representative dusty plasma crystal experimental parameters are used:[11] $\Delta=256$ μm, dust grain radius $a=5.45$ μm, $m_d=5.57\times10^{-13}$ kg, $Q=1.6\times10^4 e$, and $\lambda_D=124$ and 100 μm. The other plasma parameters are $C_{DL}=23$ mm s^{-1}, $\omega_L=90.49$ s^{-1}, $\kappa=2.1$ for $\lambda_D=124$ μm and $C_{DL}=21$ mm s^{-1}, $\omega_L=80.33$ s^{-1}, and $\kappa=2.6$ for $\lambda_D=100$ μm. The angles of propagation are $\phi=5°$ and $20°$.

To justify scaling (7) [assumption (iv)] and the continuum approximation criteria (20), it is chosen that $\bar{\nu}_{\rm diss}=5(\nu_{\rm diss}=5\omega_L)$. On the basis of these plasma parameters, shock structures are shown in Fig. 2 [(a) $M=2.1$ and (b) $M=2.7$]. The transition from the far upstream value to the far downstream value is of monotonic nature.

The velocity of the particles moving in the wave is given by

$$V_x=\frac{\partial d_l}{\partial t}=-\epsilon C_{DL}Mu>0, \qquad (21)$$

as Mach number $M>0$ and $u<0$ [Eq. (11); $\Lambda>0$]. The particles move forward in the direction of shock propagation. Thus for any value of (x,y) the velocity $(\propto -u)$ builds up from near zero value to a steady downstream value [Figs. 2(a) and 2(b)]. The dust number density perturbation N_d (normalized in units of equilibrium dust density n_{d0}) is related to u by[28]

$$u\approx -N_d\Rightarrow V_x\approx\epsilon C_{DL}MN_d>0. \qquad (22)$$

This corresponds to an increase in the particle number density as shock moves forward from upstream to downstream side and as a consequence the shock is compressive in nature in agreement with the experimental observations.[11,15]

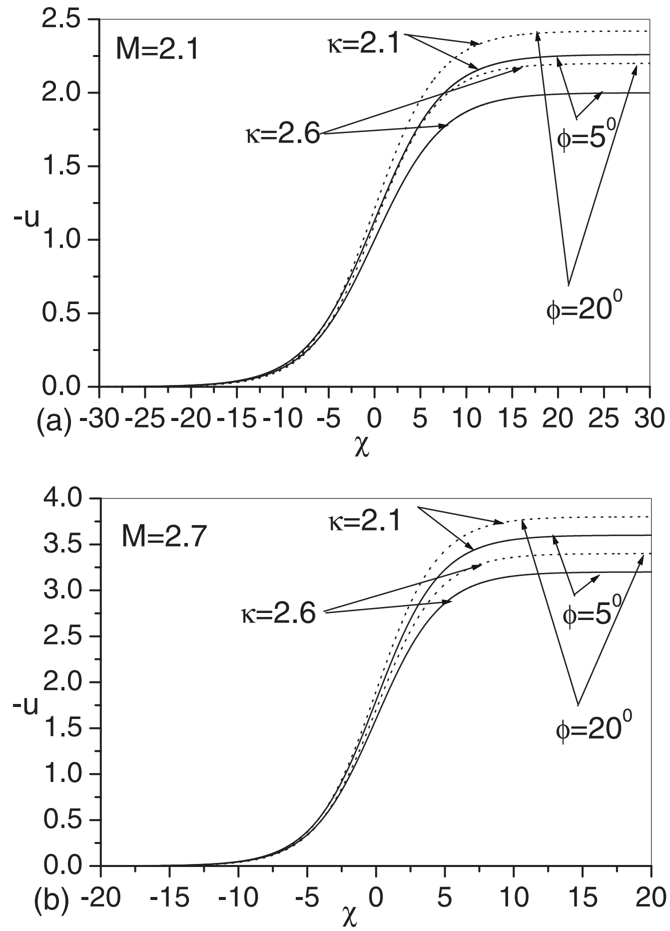

FIG. 2. Shock profile of particle velocity $(\propto -u)$ in dusty plasma crystal. Mach numbers are (a) $M=2.1$ and (b) $M=2.7$.

Figures 2(a) and 2(b) shows the variations of particle velocity $(\propto -u)$ with χ for different Mach numbers and lattice parameters. Figures 2(a) and 2(b) are drawn for lattice parameter $M=2.1$ and $M=2.7$. The dotted curves in the figures are drawn for $\phi=20°$, whereas solid curves are drawn for $\phi=5°$. All the curves in both the figures show that with the increase of lattice parameter κ, the shock strength (shock height) decreases, whereas shock strength increases with the increase in the angle of propagation ϕ. It is also seen that the shock strength in Fig. 2(b) [$M=2.7$] is higher than that of Fig. 2(a) [$M=2.1$] and thus the shock strength increases with increase in Mach number. All these phenomena are clearly shown in Fig. 3, which shows the variation of magnitude of shock strength $|\epsilon u_{\chi=\infty}|$ [Eq. (17)] with lattice parameter κ for different angles of propagation and Mach numbers.

Also note that as shock moves from upstream to downstream side in the crystal, the dust particles are energized by the passing of shock wave to

$$E=\tfrac{1}{2}m_d u^2=\tfrac{1}{2}m_d C_{DL}^2M^2|\epsilon u_{\chi=\infty}|^2. \qquad (23)$$

The energy decreases with the increase in lattice parameter κ, whereas it increases with the increase in Mach number and angle of propagation of the wave as shown in Fig. 4.

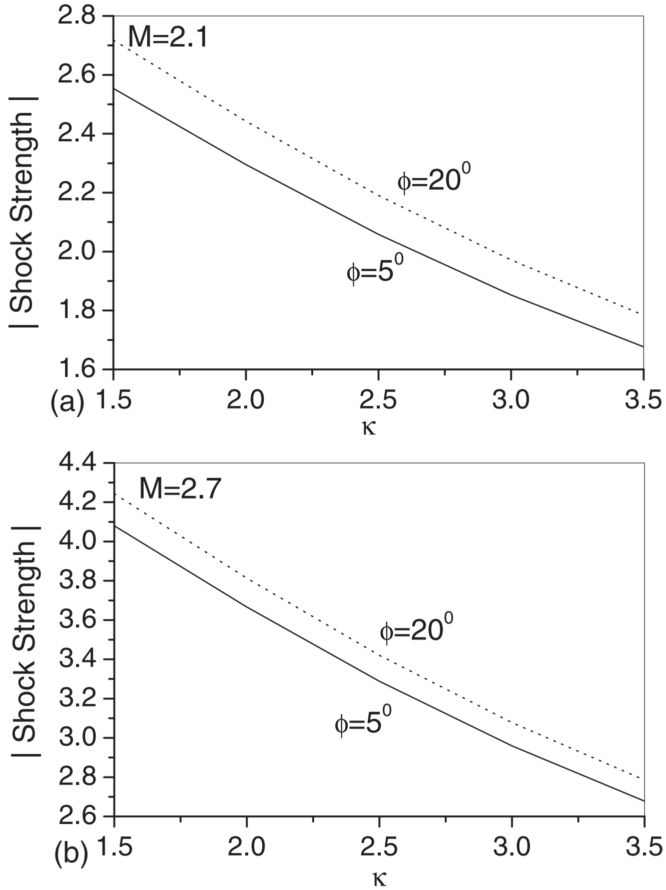

FIG. 3. Variations of magnitude of shock strength $|\epsilon u_{\chi=\infty}|$ [Eq. (17)] with lattice parameter κ for different angles of propagation and Mach numbers.

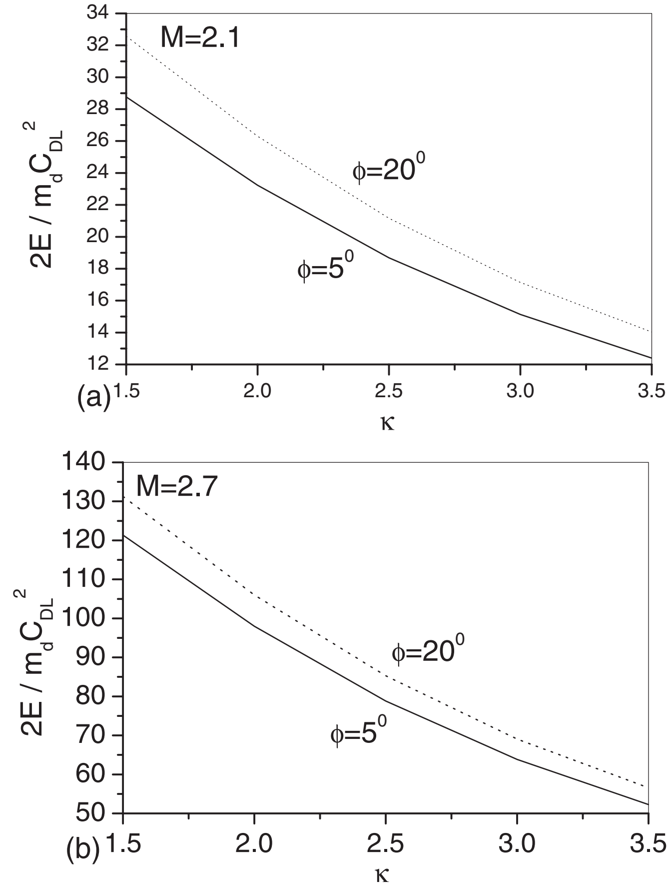

FIG. 4. Variations of energy E [Eq. (23)] to which dust grains are raised by the passing shock wave with lattice parameter κ for different angles of propagation and Mach numbers.

VI. DISCUSSIONS

Discussions in connection with shock experiment and a summary of the results are presented in this section.

(a) It is shown that the hydrodynamic damping causes dissipation represented by the term $\mu(\partial^2 u/\partial\xi^2)$ in the 2D Burgers' equation [Eq. (10)] describing the shock wave in a 2D complex (dusty) plasma.

(b) The numerical investigations on the basis of dusty plasma crystal experimental parameters show that the both shock strength and energy E (with suitable normalization) increases with the increase in Mach number (M) and angle of propagation (ϕ), whereas both decreases with the increase in lattice parameter (Figs. 3 and 4).

(c) The observed shock structure is of compressional type showing a considerable increase in dust density in qualitative agreement with the experimental observations.[11,15]

(d) From the present study, it is clear that as the shock moves forward from upstream to downstream in the dusty plasma crystal, the following physical phenomena occurs: (i) compression of dust density (shock is compressional) that implies the decrease in lattice spacing $\Delta \approx (3/4\pi n_{d0} N_d)^{1/3}$, (ii) decrease in magnitude of dust charge on individual dust particles due to packing effect,[29] (iii) increase in dust pressure, and (iv) energization of dust particles [Eq. (23), by passing shock wave], which implies the increase in kinetic temperature of dust particles. As a consequence of these physical phenomena the value of the dusty plasma coupling parameter Γ decreases as the shock moves forward in the crystal. Thus the shock can cause phase transition and hence the melting of dusty plasma crystal occurs which was observed in experiment on monolayer hexagonal 2D dusty plasma crystal.[15]

(e) The characteristic scale length $L(\approx\delta)\gg 1$ for the specified plasma parameters. Thus L is much larger than the lattice spacing Δ [Eq. (20)] for the gas-discharge plasma crystals experimental parameters and hence the continuum approximation is justified.

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