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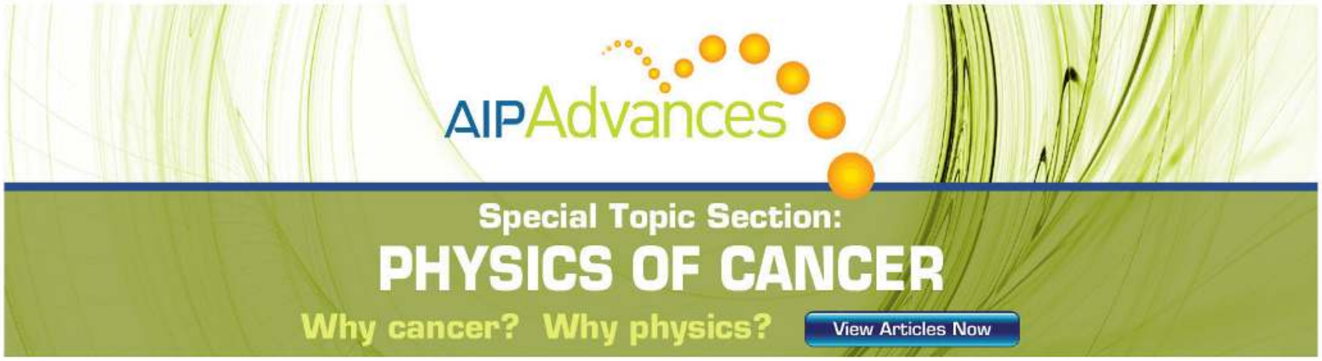

Role of positively charged dust grains on dust acoustic wave propagation in presence of nonthermal ions

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An expression for ion current flowing to the dust grains is proposed, when dust charge is positive and the ions are nonthermal. Secondary electron emission has been considered as the source of positive charging of the dust grains. Investigation shows that presence of positively charged dust grains along with thermal electrons and nonthermal ions generate purely growing dust acoustic waves for both the cases of ion nonthermal parameter greater than one and less than one. In the later case, the growth is conditional. © *2013 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4817739]

Although typical micron sized dust grains in space or laboratory plasmas are often negatively charged because of collisions with mobile electrons, there are environment where grains may also take positive charges. The emission of electrons from the surface of a dust particle may provide condition of positive charging. If a plasma contains positively charged dust grains, electron density will be larger than ion density and hence waves and instabilities will be produced there.[1–5] Since positively charged dust occurs in the presence of strong ultraviolet (UV) radiation or fast electrons, plasma with positively charged dust particles occurs widely in space and also in the earth's mesosphere.[6–9]

In this Brief Communication, we have investigated dust acoustic wave propagation in a complex plasma in presence of Boltzmann distributed electrons, nonthermal ions,[20–25] and positively charged dust grains. Secondary electron emission from dust grains has been considered the source of positive dust charging, which is an important emission process that plays important role on dust charging mechanisms.[10–20,24–26] Expression for nonthermal ion current flowing to the negatively charged dust grains with its effect on dust acoustic wave propagation was earlier investigated.[28–30] In this brief communication, an expression for nonthermal ion current flowing to the positively charged dust grains is being proposed for the first time with its effect on the dispersion relation of the propagating dust acoustic waves.

To calculate this expression, we have used the following three dimensional equilibrium state ion velocity distribution function satisfying collisionless Vlasov equation with a population of fast energetic ions,[29]

$$F_i(v_i) = F_i(v_x, v_y, v_z) = \frac{n_{i0}}{(1+3a)}\left(\frac{1}{2\pi v_{ti}^2}\right)^{3/2} \times \left[1 + 4\mathrm{a}\left(\frac{1}{2}\frac{v_x^2}{v_{ti}^2} + \frac{\Phi}{\sigma_i}\right)^2\right]\exp\left(-\frac{v_x^2 + v_y^2 + v_z^2}{2v_{ti}^2} - \frac{\Phi}{\sigma_i}\right), \tag{1}$$

where a is the ion nonthermal parameter that determines the extent of the deviation away from the usual Maxwellian distribution. Nonthermality increases with increasing value of this parameter "a"; T_i, $m_i, v_{ti}\left(=\sqrt{\frac{K_B T_i}{m_i}}\right)$ are the temperature, mass and thermal velocity and of ions; $\Phi = \frac{e\varphi}{K_B T_e}$ and $\sigma_i = \frac{T_i}{T_e}$, where T_e is the electron temperature, φ is the electrostatic potential and K_B is the Boltzmann constant.

Using orbit motion limited (OML) theory,[27] we have calculated the expression of ion current flowing to the positively charged dust grains in the form,

$$I_i = \pi r_0^2 e\sqrt{\frac{8T_i}{\pi m_i}}\frac{n_{i0}}{(1+3a)}\exp\left(-\frac{\Phi}{\sigma_i}\right) \times \left[\left(1 + \frac{24a}{5}\right) - \frac{8a}{3}\frac{\Phi}{\sigma_i}\left(2 + \frac{\Phi_d}{\sigma_i}\right) + \frac{4a}{5}\left(5 + \frac{\Phi^2}{\sigma_i^2} + \frac{\Phi_d^2}{\sigma_i^2}\right) + \frac{16a}{5}\frac{\Phi_d}{\sigma_i}\right]\exp\left(-\frac{\Phi_d}{\sigma_i}\right), \tag{2}$$

where $\Phi_d = \frac{eq_d}{r_0 T_e}$, r_0 is the grain radius. We have also calculated the average kinetic energy of nonthermal ions in the form,

$$E_{av} = \frac{3}{2}\left(\frac{1+7a}{1+3a}\right)KT_i. \tag{3}$$

It shows that average kinetic energy of nonthermal ions is greater than average kinetic energy of thermal ions $\frac{3}{2}$ KT_i. Thus, nonthermal ions present in dusty plasma reaches faster to the dust grains than thermal ions and are more efficient to overcome the repeling positive grain surface potential. This enhances positive flux to the dust grains.

The orbital motion limited (OML) theory based expression for primary electron current flowing to the and secondary electron current flowing out of the positively charged dust grains are,[27]

$$I_e = -\pi r_0^2 e\sqrt{\frac{8T_e}{\pi m_e}}n_e\left(1 + \frac{eq_d}{r_0 T_e}\right), \tag{4}$$

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$$I_e^s = 3.7\delta_M \pi r_0^2 e\sqrt{\frac{8T_e}{\pi m_e}} n_e \left(1+\frac{eq_d}{r_0 T_e}\right) \times \exp\left(\frac{eq_d}{r_0 T_e} - \frac{eq_d}{r_0 T_s}\right) F_{5,B}(x), \tag{5}$$

where m_e is the electron mass, T_e and T_s are the primary and secondary electron temperatures, δ_M is the maximum yield of secondary electrons which occurs when the impinging electron has the kinetic energy E_M. The function $F_{5,B}(\mathrm{x})$ is given by

$$F_{5,B}(x) = x^2 \int_B^{\infty} u^5 \exp[-(\mathrm{x}u^2 + \mathrm{u})]\mathrm{du}$$

$$\text{with } x = \frac{E_M}{4T_e} \quad \text{and} \quad \mathrm{B} = \sqrt{\frac{\frac{eq_d}{r_0 T_e}}{x}}.$$

In equilibrium state with positively charged dust grains, the equilibrium dust charge is, $(q_d)_{eq} = q_{d0} = Z_{d0}e$. Setting $n_i(\Phi = 0) = n_{i0}$, $n_e(\Phi = 0) = n_{e0}$ in the zero current equation $I_{e0} + I_{i0} + I_{e0}^s = 0$, and using the expressions for I_{e0}, I_{i0}, I_{e0}^s as given by Eqs. (2), (4), and (5), with $\Phi = 0$, we obtain the equilibrium ion-electron density ratio in the form,

$$\frac{n_{i0}}{n_{e0}} = \frac{1}{\sqrt{\sigma_i}}\sqrt{\frac{m_i}{m_e}} \frac{(1+z)(1+3a)\exp\left(\frac{z}{\sigma_i}\right)}{(1+A(a))} \alpha_{1s}, \tag{6}$$

where

$$\mathrm{A(a)} = \frac{4a}{5}\left[6 + \frac{z^2}{\sigma_i^2} + 5\frac{z}{\sigma_i}\right]; \quad \sigma_i = \frac{T_i}{T_e},$$

$$\alpha_{1s} = 1 - 3.7\delta_M \exp\left(z - \frac{z}{\sigma_s}\right) F_{5,B}(x). \tag{7}$$

The grain charging frequency for positively charged dust grains with secondary electron emission is,

$$\nu_{ch} = -\left(\frac{\partial I_{tot}}{\partial q_d}\right)_{eq} = \frac{\frac{r_0}{\lambda_{Di}}}{\sqrt{2\pi}} \omega_{pi} \exp\left(-\frac{z}{\sigma_i}\right) \frac{G(a)}{(1+3a)} \times \left[1 + \sigma_i \alpha_{2s} \frac{(1+A(a))}{(1+z)G(a)}\right], \tag{8}$$

where $\alpha_{2s} = 1 - 3.7\delta_M F_{5,B_0}(\mathrm{x})\left[1 - (1+z)(1-\frac{1}{\sigma_s}) + \frac{z^2}{2x}\frac{\exp[-z-\sqrt{z/x}]}{F_{5,B_0}(\mathrm{x})}\right]\exp[z - z/\sigma_s]$, $\mathrm{G(a)} = \frac{4a}{5}[2 + \frac{z^2}{\sigma_i^2} + 2\frac{z}{\sigma_i}] + 1$, $\sigma_s = \frac{T_s}{T_e}$, $B_0 = \sqrt{\frac{z}{x}}$, $\lambda_{Di} = \frac{v_{ti}}{\omega_{pi}}$, ω_{pi} is the ion plasma frequency.

Substitution of the expressions for I_i, I_e and I_e^s from Eqs. (2), (4), and (5) in the dust charging equation,

$$\frac{\partial q_d}{\partial t} + v_d \frac{\partial q_d}{\partial x} = I_e + I_i + I_e^s = I_{tot}, \tag{9}$$

and its linearization about the equilibrium gives the first order dust charge variation,

$$q_1 = -\frac{\frac{r_0}{\sqrt{2\pi}}\frac{\omega_{pe}^2}{v_{te}}}{\nu_{ch} - i\omega} \frac{1+z}{z}\left[\alpha_{1s} + \frac{F(a)}{1+A(a)}\right]\varphi, \tag{10}$$

where $\mathrm{F(a)} = \frac{1}{\sigma_i}\left[\left(1 + \frac{152}{15}a\right) + \frac{88}{15}a\frac{z}{\sigma_i} + \frac{4a}{5}\frac{z^2}{\sigma_i^2}\right]$ and q_d is the dust charge at time t.

As the plasma under consideration consists of Boltzmann distributed primary electrons with temperature T_e, secondary electrons with temperature T_s, nonthermal ions and positively charged inertial dust grains, their number densities satisfy the equilibrium charge neutrality condition,

$$n_{io} - n_{e0} - n_{s0} + z_{d0} n_{d0} = 0, \tag{11}$$

and the basic equations,

$$n_e = n_{e0} \exp\left(\frac{e\varphi}{T_e}\right), \tag{12}$$

$$n_s = n_{s0} \exp\left(\frac{e\varphi}{T_s}\right), \tag{13}$$

$$n_i = n_{i0}\left[1 + \frac{4a}{1+3a}\left(\frac{\Phi}{\sigma_i} + \frac{\Phi^2}{\sigma_i^2}\right)\right]\exp\left(-\frac{\Phi}{\sigma_i}\right), \tag{14}$$

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d v_d) = 0, \tag{15}$$

$$\frac{\partial v_d}{\partial t} + v_d \frac{\partial v_d}{\partial x} = -\frac{q_d}{m_d}\frac{\partial \varphi}{\partial x} - \frac{T_d}{n_d m_d}\frac{\partial n_d}{\partial x}. \tag{16}$$

Equations (9) and (12)–(16) are closed with the Poisson equation,

$$\frac{\partial^2 \varphi}{\partial x^2} = -4\pi\,(en_i - en_e - en_s + q_d n_d). \tag{17}$$

Here m_d is the mass of the charged dust grains moving with velocity v_d and n_d is the dust number density, other symbols have already defined.

Linearizing these basic equations about their equilibrium values, we obtain the first order perturbed number densities,

$$\delta n_e = n_{e0}\,\Phi, \quad \delta n_i = n_{i0}\frac{a-1}{1+3a}\frac{\Phi}{\sigma_i}, \quad \delta n_s = -n_{s0}\frac{\Phi}{\sigma_s},$$
$$\frac{\delta n_d}{n_{d0}} = \frac{k^2 c_{da}^2}{\omega^2 - k^2 v_{td}^2}\Phi, \tag{18}$$

for primary electrons, ions and secondary electrons, and dust grains.

Substituting δn_e, δn_i, δn_s, δn_d from Eq. (18) and q_1 from Eq. (10) in the linearized Poisson equation, we obtain the following dispersion relation for dust acoustic waves in the long wavelength approximation:

$$1+\frac{1}{\sigma_s}\frac{n_{s0}}{n_{e0}}-\frac{n_{i0}}{n_{e0}}\frac{(a-1)}{(1+3a)}\frac{1}{\sigma_i}+\frac{z_{d0}n_{d0}}{n_{e0}} \times\left[\frac{\nu_{ch}}{I(a)(\nu_{ch}-i\omega)}+\frac{k^2c_{da}^2}{\omega^2-k^2v_{td}^2}\right]=0. \qquad (19)$$

From this dispersion relation, we have calculated the real part ω_{r} and imaginary part ω_i of the wave frequency $\omega=\omega_{\mathrm{r}}+i\omega_i$ in the form,

$$\frac{\mathrm{w_r^2}}{\mathrm{k^2c_{da}^2}}=\frac{\mathrm{v_{td}^2}}{\mathrm{c_{da}^2}}-\frac{\mathrm{I(a)}}{1+\mathrm{p(a)}},\qquad \omega_{\mathrm{i}}=\frac{\mathrm{k^2c_{da}^2}}{\nu_{\mathrm{ch}}}\frac{\mathrm{I(a)}}{2(1+\mathrm{p(a)})^2}, \qquad (20)$$

where $I(a)=\frac{G(a)}{(1+3a)}e^{\frac{-z}{\sigma_i}}[1+\frac{\sigma_i\alpha_{2s}}{(1+z)\alpha_{1s}}\frac{(1+A(a))}{G(a)}]$, $p(a)=(1+\frac{1}{\sigma_s}\frac{n_{s0}}{n_{e0}}-\frac{n_{i0}}{n_{e0}}\frac{a-1}{1+3a}\frac{1}{\sigma_i})\frac{1}{\frac{z_{d0}n_{d0}}{n_{e0}}}$, other symbols have their usual meaning.

Since ∂_M is the ratio of the emitted electrons to the incident electrons, higher values of ∂_M correspond to the higher number of electrons emitted from dust grains. This makes the equilibrium grain charge positive. For MgO material ∂_M varies from $3\sim25$ and $E_M(\mathrm{eV})\approx400$–1500.[26] We have considered *here* $\partial_M=24$ *as* our purpose is to study the effect of ion nonthermality on dust acoustic wave propagation with positively charged dust grains. For secondary electron emission, $E_M/4T_e\gg1$ and $3.7\partial_M\sim E_M/4T_e$.[26]

In presence of positively charged dust grains $\frac{n_{i0}}{n_{e0}}<1$ because in equilibrium, secondary electron population is very low compared to primary electron population. The value of $\frac{n_{i0}}{n_{e0}}$ depends on different values of $z=\frac{z_{d0}e^2}{r_0T_e}$, $\sigma_i=\frac{T_i}{T_e}$, ∂_M, $\sigma_s=\frac{T_s}{T_e}$, etc.

Figures 1 and 2 are plotted for ω_r^2 against z and Figures 3, 4 are plotted for ω_i against z for $a<1$ and $a>1$, respectively. In all cases, we have considered the parameters $\sigma_i=1.0$, $\partial_M=24$, $\sigma_s=1.01$. Figures 1 and 2 show that $\omega_r^2<0$ for both ranges $a<1$ and $a>1$, i.e., in both cases, ω_r is imaginary and hence wave frequency has no real part. Figures 3 and 4 show that $\omega_i<0$ for $a<1$ and $\omega_i>0$ for $a>1$. This implies positively charged dust grains generate purely growing dust acoustic waves when nonthermal parameter $a>1$. In case of $a<1$, it may also generate purely growing dust acoustic waves provided $|\omega_r|>|\omega_i|$.

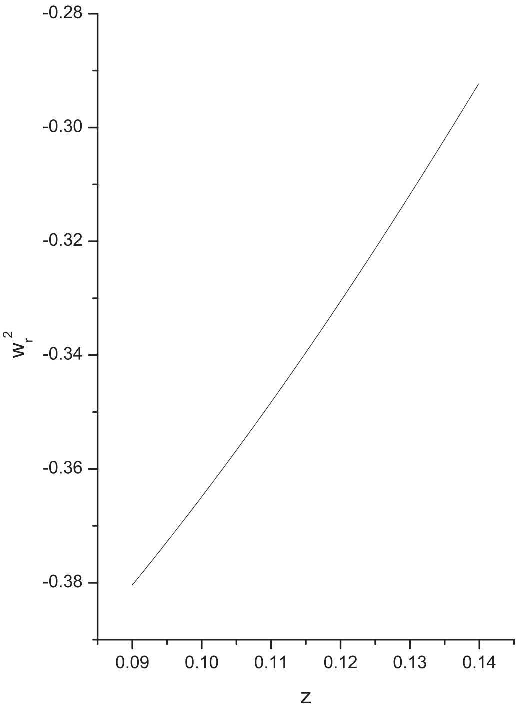

FIG. 2. Plot of the normalised squared real frequency $\mathrm{w_r}^2$ vs z for positively charged dust grain for different plasma parameters and a greater than 1.

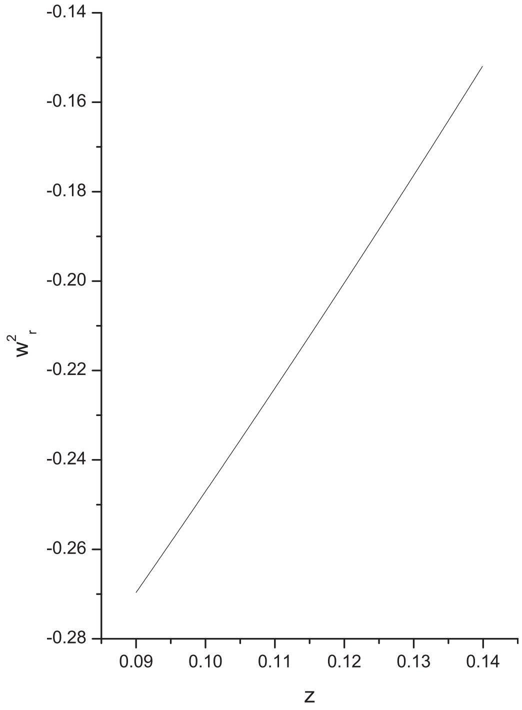

FIG. 1. Plot of the normalised squared real frequency $\mathrm{w_r}^2$ vs z for positively charged dust grain for different plasma parameters and a less than 1.

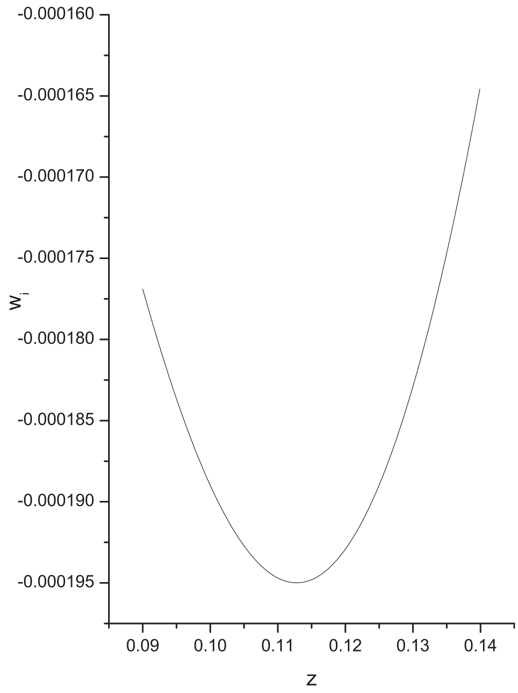

FIG. 3. Plot of the normalised imaginary frequency $\mathrm{w_i}$ vs z for positively charged dust grains for different plasma parameters for a less than 1.

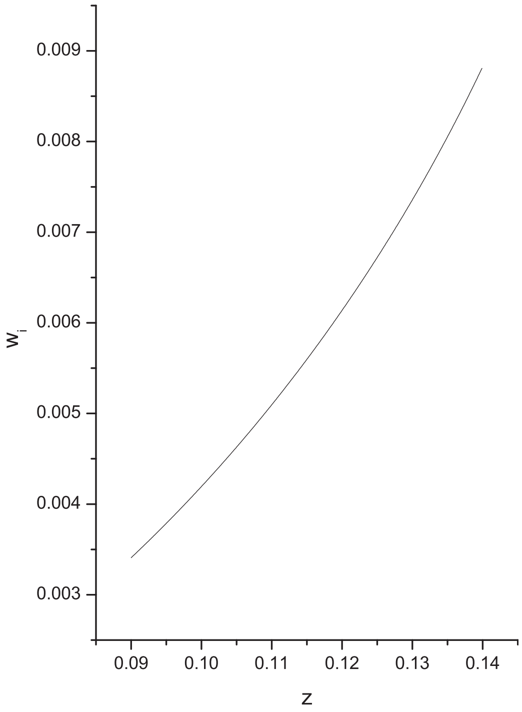

FIG. 4. Plot of the normalised imaginary frequency w_i vs z for positively charged dust grains for different plasma parameters for a greater than 1.

Thus positively charged dust grains are enable to transfer more energy to the dust acoustic waves than negatively charged dust grains.

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