# Reply to comments of A. K. Kapila

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# COMMENTS

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## Comments on "Inertial oscillations and multiple boundary layers in an unsteady rotating flow"

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In a recent paper, Debnath and Mukherjee[1] have considered a rotating flow bounded by a porous disk. They have examined the unsteady flow generated by impulsively setting the plate into oscillation and translation in its own plane, while suction (or blowing) is also switched on. A main result of the paper is that immediately following the impulsive start, the flow near the plate consists of a Rayleigh layer and two additional boundary layers of thicknesses depending upon suction. This conclusion is based upon an asymptotic analysis for $t \to 0$ which appears to be incorrect.

The precise mistake lies in the expansion of the Laplace transform $\bar{q}$ [Eq. (1), Ref. 1] where $(S^2 + 4iE + 2pE)^{1/2}$ has been approximated by $(S^2 + 2pE)^{1/2}$ instead of $(2pE)^{1/2}$, for large $p$. The correct expansion is

$$\bar{q}(z,p) \approx \exp(-Sz/2)\, p^{-1}[a + b - (U/U^*)] \exp[-\tfrac{1}{2}z(2pE)^{1/2}],$$

leading to the inverse

$$q(z,t) \approx \exp(-Sz/2)[a + b - (U/U^*)]\operatorname{erfc}\left(\frac{z}{4}\sqrt{\frac{2E}{t}}\right).$$

[The same result can be obtained by expanding the exact solution [Eq. (7), Ref. 1] for $t \to 0$.]

Thus, the solution for $t \ll 1$ consists of the Rayleigh layer alone, which is as it should be, since the fluid has not yet had time to feel the oscillations of the plate.

[1] L. Debnath and S. Mukherjee, Phys. Fluids 17, 1372 (1974).

## Reply to comments of A. K. Kapila

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The authors appreciate the comments made by Dr. A. K. Kapila on our[1] solution (3) for small time. We would like to point out that the study of the effects of suction on boundary layer flows is of interest and one of our objectives[1] was to examine such effects on the solution and the associated boundary layer. In doing so, the asymptotic expansion of the Laplace transform solution $\bar{q}$ given by (1) has been performed for large $p$ such that the effects of suction remain present in the solution of the problem. This leads to an apparent discrepancy between our exact solution and the asymptotic solution for small time which possibly needs a careful analysis. However, the expansion of $\bar{q}$ given by Dr. Kapila eliminates the effects of suction on the boundary layer thickness. The solution and the associated boundary layer thickness incorporating the effects of suction are well known in the literature,[2] and hence such effects seem to be inherent in the boundary layer flows. Apparently, the small time solution of Kapila needs a physical justification.

[1] L. Debnath and S. Mukherjee, Phys. Fluids 17, 1372 (1974).
[2] L. Rosenhead, *Laminar Boundary Layers* (Clarendon, Oxford, England, 1963), pp. 141 and 241.