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Citation: Physics of Plasmas (1994-present) **21**, 092120 (2014); doi: 10.1063/1.4896714
View online: http://dx.doi.org/10.1063/1.4896714
View Table of Contents: http://scitation.aip.org/content/aip/journal/pop/21/9?ver=pdfcov
Published by the AIP Publishing

PHYSICS OF PLASMAS 21, 092120 (2014)

Rayleigh-Taylor instability in an equal mass plasma

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(Received 18 August 2014; accepted 12 September 2014; published online 29 September 2014)

The Rayleigh-Taylor (RT) instability in an inhomogeneous pair-ion plasma has been analyzed. Considering two fluid model for two species of ions (positive and negative), we obtain the possibility of the existence of RT instability. The growth rate of the RT instability as usual depends on gravity and density gradient scale length. The results are discussed in context of pair-ion plasma experiments. © 2014 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4896714]

I. INTRODUCTION

The physics of pair plasmas exhibits a considerable interest in recent studies because of its novelty as well as various applications in astrophysical and technological plasmas.[1–11] Pair plasmas consisting of electrons and positrons exist in many astrophysical environments[9,10] and inertial confinement fusion reactor using ultra intense lasers.[11] The study of RT instability in the relativistic regime of electron-positron plasmas is also an important work.[12] A pair-plasma with equal mass of two species makes the plasma space-time symmetric. The symmetry arises due to their same mobility in electromagnetic fields. This symmetry allows us to analyze new collective modes in pair plasma in contrast to the normal electron-ion plasma with wide mass difference.

In laboratory such pair plasmas have already been observed.[13–15] The formation mechanism of pair plasmas consisting of positive and negative ions (C_{60}^{+}, C_{60}^{-}) of fullerene with equal masses and slightly different temperatures, was developed by Oohara and Hatakeyama.[16] Three types of electrostatic collective mode oscillations have also been observed in such space-time symmetric pair-ion plasmas.[17–19]

Moreover, a pair-ion plasma consisting of light positive and heavy negative ions, where the temperature of positive ion is higher than that of negative ion has been produced in the laboratory experiments recently.[20,21] Finally, theoretical investigation predicts the existence of drift wave transverse to the direction of magnetic field in an inhomogeneous pair-ion plasma having different temperature of the species.[22] Also, recent experimental observation[23] and theoretical investigation[24] confirm the existence of drift wave instability in presence of ion density gradient perpendicular to the external magnetic field in such a pair-ion plasma.

Being inspired by these investigations, we were curious on RT instability in such plasmas and demonstrate that indeed there is a possibility of the existence of Rayleigh-Taylor instability in a inhomogeneous pair-ion plasma in presence of a strong magnetic field and gravity. In this work, we investigate with linear analysis of Rayleigh-Taylor (RT) mode which reveals the possibility of RT instability. It is well known that when a heavy fluid is supported by a lighter fluid against the gravity, RT instabilities are formed,[25–27] which have a general tendency to penetrate the heavier fluid down and raise the lighter fluid to the top and thereby instability is triggered. As usual in case of the pair-ion plasma, the gravity and inhomogeneity are the physical parameters to mix up two fluids and form RT instability.

The paper is organized as follows. In Sec. II, the basic equations are written down after stating the physical assumptions. The linear dispersion relation of RT instability is obtained in Sec. III. Finally, the results are concluded in Sec. IV.

II. BASIC EQUATIONS AND ASSUMPTIONS

We consider a inhomogeneous, magnetized, and collisionless pair-ion plasma composed of positive and negative ions. Then, we consider the RT perturbations in such an inhomogeneous pair-ion plasma in presence of an external uniform magnetic field $\mathbf{B} = B\hat{e}_z$ under the action of constant gravitational field $\mathbf{g} = g\hat{e}_x$, where B is the magnitude of the magnetic field, g is the magnitude of the gravitational acceleration, $\hat{e}_z$ is the unit vector along the z-direction, and $\hat{e}_x$ is the unit vector along the x-direction. We also assume that the plasma density varies only in the x-direction. Therefore, the momentum and continuity equations for the two species, namely, positive $(+)$ and negative $(-)$ ion fluids in a collisionless regime, can be expressed as

$$m_{\pm}n_{\pm}\left(\frac{\partial}{\partial t} + \mathbf{v}_{\pm}\cdot\nabla\right)\mathbf{v}_{\pm} = \pm e n_{\pm}\left(\mathbf{E} + \frac{1}{c}\mathbf{v}_{\pm}\times\mathbf{B}\right) - \nabla p_{\pm} + m_{\pm}n_{\pm}\mathbf{g}, \quad (1)$$

$$\frac{\partial n_{\pm}}{\partial t} + \nabla\cdot(n_{\pm}\mathbf{v}_{\pm}) = 0, \quad (2)$$

where $m_{\pm}$ are the masses, $\mathbf{v}_{\pm}$ are the velocities, $n_{\pm}$ are the densities, and $p_{\pm}$ are the pressure of ions. The notation +/– stands for positive/negative ions. The equation of state is isotropic for both ions, i.e., $p_{\pm} = n_{\pm}T_{\pm}$. The quasi neutrality condition can be expressed as

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$$\nabla \cdot \mathbf{E} = 4\pi e(n_+ - n_-). \tag{3}$$

For the long wavelength limit (i.e., $k\lambda_D \ll 1$, where k is the wave number and λ_D is the Debye length), the spatial variation of electric field may be ignored throughout the plasma so that we can assume quasi neutrality condition, i.e., $n_+ \approx n_- = n$.

III. LINEAR ANALYSIS OF RAYLEIGH-TAYLOR INSTABILITY

To study the RT instability, we consider the low-frequency $\omega \ll \Omega_\pm (= |e|B/m_\pm c)$ and electrostatic oscillations ($\nabla \times \mathbf{E} = 0$), i.e., $\mathbf{E} = -\nabla\varphi$, where $\Omega_\pm$ and φ are the ion cyclotron frequency and electrostatic potential, respectively. Because of this low-frequency assumption, the velocities of pair ions, directed perpendicular to the magnetic field, can be expressed from Eq. (1) as

$$\mathbf{v}_{+\perp} = \frac{c}{B}\hat{e}_z \times \nabla\varphi + \frac{cT_+}{enB}\hat{e}_z \times \nabla n - \frac{1}{\Omega_+}\hat{e}_z \times \mathbf{g} - \frac{c}{B\Omega_+}\frac{d}{dt}\nabla_\perp\varphi, \tag{4}$$

$$\mathbf{v}_{-\perp} = \frac{c}{B}\hat{e}_z \times \nabla\varphi - \frac{cT_+}{enB}\hat{e}_z \times \nabla p_- + \frac{1}{\Omega_-}\hat{e}_z \times \mathbf{g} + \frac{c}{B\Omega_-}\frac{d}{dt}\nabla_\perp\varphi. \tag{5}$$

In the right hand side of the above equations, the first term is the $\mathbf{E}\times\mathbf{B}$ drift, the second term is the diamagnetic drift, the third term is the gravitational drift, and the last term is the polarization drift velocities. It can be shown that the $\mathbf{E}\times\mathbf{B}$ velocity is much greater than the diamagnetic drift velocity and polarization drift is in order of magnitude less than the $\mathbf{E}\times\mathbf{B}$ drift velocity. Moreover, the $\mathbf{E}\times\mathbf{B}$ drift is independent of charge and mass and therefore same for both positive and negative ions. For simplicity, we ignore the parallel dynamics because the plasma particles feel the magnetic effect only in the direction perpendicular to the magnetic field. Therefore, ignoring the parallel dynamics from continuity equations, we have

$$\frac{\partial n}{\partial t} + \nabla_\perp \cdot (n\mathbf{v}_{\pm\perp}) = 0. \tag{6}$$

Adding and subtracting separately for positive and negative ions from the above Eq. (6), we have

$$\frac{\partial n}{\partial t} + \frac{1}{2}\nabla_\perp \cdot \left[n(\mathbf{v}_+ + \mathbf{v}_-)_\perp\right] = 0, \tag{7}$$

$$\nabla_\perp \cdot \left[n(\mathbf{v}_+ - \mathbf{v}_-)_\perp\right] = 0. \tag{8}$$

As reflected in the above equations, the sum and difference of the directed velocities of positive and negative ions may be obtained from Eqs. (4) and (5) as

$$(\mathbf{v}_+ + \mathbf{v}_-)_\perp = 2\frac{c}{B}\hat{e}_z \times \nabla\varphi + \frac{c(T_+ - T_-)}{enB}\hat{e}_z \times \nabla n - \left(\frac{1}{\Omega_+} - \frac{1}{\Omega_-}\right)\hat{e}_z \times \mathbf{g} - \frac{c}{B}\left(\frac{1}{\Omega_+} - \frac{1}{\Omega_-}\right)\frac{d}{dt}\nabla_\perp\varphi, \tag{9}$$

$$(\mathbf{v}_+ - \mathbf{v}_-)_\perp = \frac{c(T_+ + T_-)}{enB}\hat{e}_z \times \nabla n - \left(\frac{1}{\Omega_+} + \frac{1}{\Omega_-}\right)\hat{e}_z \times \mathbf{g} - \frac{c}{B}\left(\frac{1}{\Omega_+} + \frac{1}{\Omega_-}\right)\frac{d}{dt}\nabla_\perp\varphi. \tag{10}$$

Then, using Eqs. (8) and (10), we have

$$\frac{c}{B}\left(\frac{1}{\Omega_+} + \frac{1}{\Omega_-}\right)\nabla_\perp \cdot \left[n\frac{d}{dt}\nabla_\perp\varphi\right] + \left(\frac{1}{\Omega_+} + \frac{1}{\Omega_-}\right)\nabla_\perp \cdot [n(\hat{e}_z \times \mathbf{g})] = 0. \tag{11}$$

Next, we take the input from the experimentally observed pair-ion plasma.[18,19] According to the observations from these experiments, we consider that the masses of positive and negative ions are equal as they are generated by the same source (fullerene ion source), i.e., $m_+ = m_- = m$. However, their temperatures are slightly different [range of (0.3–0.5) eV], because of the different charging processes of the ions. In this analysis, we have defined a new temperature variable $T_s = (T_+ + T_-)/2$. For equal mass of both positive and negative ions, the magnitude of cyclotron frequencies are equal, i.e., $\Omega_+ = \Omega_- = \Omega$.

As mentioned before, polarization drift velocity is in one order magnitude less than the $\mathbf{E}\times\mathbf{B}$ drift, therefore, neglecting polarization drift convection of density in such equal mass plasma, we have

$$\rho_s^2\frac{d}{dt}\nabla_\perp^2\phi + \frac{g}{\Omega}\frac{\partial\tilde{n}}{\partial y} = 0. \tag{12}$$

With the help of temperature variable T_s, we normalize the electrostatic potential as $\phi = e\varphi/T_s$. The normalized variable $\tilde{n}(= n_1/n_0)$ denotes the density perturbation. The physical variable c_s is the acoustic speed and ρ_s is the Larmor radius, defined as

$$c_s = \sqrt{\frac{T_s}{m}}, \rho_s = \frac{c_s}{\Omega}.$$

Assuming equal mass plasma, using Eq. (9), Eq. (7) can be written as

$$\left(\frac{\partial}{\partial t} + \rho_s c_s \hat{e}_z \times \nabla_\perp\phi \cdot \nabla_\perp\right)\ln n = 0. \tag{13}$$

Therefore, the basic normalized equations to study this mode are

$$\left(\frac{\partial}{\partial t} + \hat{e}_z \times \nabla\phi \cdot \nabla\right)\nabla_\perp^2\phi + \frac{g}{\rho_s\Omega^2}\frac{\partial\tilde{n}}{\partial y} = 0, \tag{14}$$

$$\left(\frac{\partial}{\partial t} + \hat{e}_z \times \nabla_\perp\phi \cdot \nabla_\perp\right)\tilde{n} + \frac{\rho_s}{L_n}\frac{\partial\phi}{\partial y} = 0, \tag{15}$$

where time and space scales are normalized in units of cyclotron time Ω^{-1} and Larmor radius ρ_s. To define the inhomogeneity of the plasma, we consider the initial density $n_0 \equiv n_0(x)$ and assume the density distribution to be

$$n_0(x) = n_{00}\exp(-x/L_n),$$

where L_n is the density gradient scale length.

Finally, for the linear analysis, we linearize Eqs. (14) and (15) and assume that the density and potential perturbations are proportional to $\exp[i(k_x x + k_y y - \omega t)]$. All these yield the following linear dispersion relation in a dimensional from

$$\omega^2 = -\left(\frac{k_y}{k_\perp}\right)^2 \frac{g}{L_n},$$

where $k_\perp(k_y)$ and ω are the wave number and frequency, respectively. The above dispersion relation gives the expression for RT instability growth rate as

$$\gamma = \left(\frac{k_y}{k_\perp}\right)\sqrt{\frac{g}{L_n}}.$$

One can also observe from the dispersion relation that inhomogeneity in the plasma and the gravitational acceleration are responsible for RT instability and the square root of the density gradient scale is inversely proportional to the growth rate of RT instability. Thus, if we have a inhomogeneous pair ion plasma situated under the gravitational force, as usual, there is a possibility of existence of RT instability.

IV. CONCLUSION

In this work, we have investigated in detail about the possibility of the existence of the RT instability in a inhomogeneous, magnetized pair-ion plasma under the action of gravity. The mathematical analysis presented here for equal mass plasma is a bit different from normal electron ion plasma. This is because, a wide mass difference in an electron ion plasma, one can neglect electron inertial effect (e.g., polarization drift) as compared to ions. In the present situation, since the masses of both species are equal hence their polarization drifts are equally important and have been taken into account fruitfully. For simplicity, we ignore the parallel dynamics of the species in momentum and continuity equations. To realize this RT instability in experiment, we consider a pair-ion plasma with equal masses and different temperatures of positive and negative ions as per the experimental observation.[16–19] However, the linear analysis indicates that the formation of RT instability does not depend on the masses of the two species as the growth rate $\gamma = (k_y/k_\perp)(g/L_n)^{1/2}$ is independent of the mass of any species. So, our analysis of RT instability is also applicable to another pair-ion plasma with different mass ($m_+ \ll m_-$) and temperature ($T_- \ll T_+$) observed experimentally.[20,21] Finally, our analysis clearly demonstrates that under a favorable condition of gravity with respect to density inhomogeneity, RT instability in a pair-ion plasma is profoundly present and perhaps experimentally realizable.

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