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Propagation of pulses at optical wavelengths through fog-filled medium

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Abstract. The pulse propagation at optical wavelengths (0.4-10.0 μm) through the fog-filled medium has been studied considering the effects of differential attenuation and phase dispersion on the spectrum of the propagated signal. The pulse distortion, in terms of percentage change in the width of a Gaussian pulse, has been obtained using a closed solution of Fourier integral for the time domain representation of the pulse. This technique, initially developed by *Forrer* [1958], approximates the propagation constants by a truncated Taylor series. It has been found that the pulse distortion can be quite severe for pulses of width 0.001 ns or smaller, depending on wavelength, fog type, pulse width, and path length. Both broadening and compression of pulses can occur according to the values of second derivatives of the propagation constants.

1. Introduction

Propagation of pulsed optical signals through the atmosphere has received considerable attention in view of its application in areas such as atmospheric optical communication, precise ranging, and remote sensing. In the area of communiction, during recent years, fiberless optical communication links through the atmosphere have been studied as a short-term alternative to optical fiber links, and as communication links for remote areas [*Chaimowitz and Cole*, 1989, 1990]. Also planned are laser communication experiments over the satellite-to-earth path [*Araki et al.*, 1993]. Optical communication systems are operating at tens of gigabits per second, and higher bit rate systems are contemplated for future applications. However, for transmission of very short pulses, the signal bandwidth becomes large so that the atmospheric response to the signal propagation may show marked variation within the bandwidth, resulting in the distortion of received pulses in the form of compression or broadening. Consequences of these effects may be an increase in the intersymbol interferences in high-rate data communication systems.

The pulse propagation through the atmosphere has been investigated by a number of workers considering a two-frequency mutual coherence function for a random medium [*Hong and Ishimaru*, 1976; *Liu*, 1977; *Ito and Furutsu*, 1980; *Ito*, 1980]. The pulse distortion computed with this technique is caused by the incoherent component of the signal under forward scattering conditions. In this case the pulse broadening occurs only when the incoherent component is significant and when the coherence bandwidth is small, which occurs when the receiving angle is narrow.

Another approach in this investigation is based on a technique that was first used by *Forrer* [1958] for studying the pulsed transmission through a waveguide. In this case the channel transfer function is approximated by a truncated polynomial function of frequency, which is valid when changes of transfer function with frequency are not too sharp. The pulse distortion in the coherent component of the signal under forward scattering conditions is due to differential attenuation and phase dispersion which the signal suffers during propagation through the atmosphere. The distortion becomes significant when the bandwidth of the signal, as in case of picosecond pulses, is large. This technique has been applied to obtain pulse distortion of HF in the ionosphere by *Terina* [1967] and at millimeter wavelengths in the atmosphere by several workers [*de Medeiros Filho*, 1981; *Gibbins*, 1990; *Maitra et al.*, 1993].

The atmospheric optical communication links would be set up in the window regions of the electromagnetic spectrum where the variation of the clear air attenuation with the wavelength is small without causing

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significant distortion of the signal. However, hydrometeors and turbulence present in the atmosphere would effect the propagation. Although rain reduces the optical visibility significantly, the pulse distortion would be insignificant as visibility through rain does not change appreciably with wavelengths in the optical region [*Olsen et al.*, 1978]. Also, the pulse distortion due to atmospheric turbulence is negligible at optical wavelengths [*Hong and Ishimaru*, 1976]. Fog is the worst offender in reducing visibility and causing the distortion of optical pulses. Observations of optical pulse dispersion through fog have been reported by *Mooradian et al.* [1974].

In the present paper, pulse distortions due to propagation through fog have been theoretically investigated in the wavelength region 0.4-10.0 μm, following the technique of *Forrer* [1958]. The study has been made for different fog models, path lengths, and pulse widths to assess the extent of distortions under varying propagation conditions.

2. Theoretical Background

For very short pulses a Gaussian shape can be taken as realistic; it has the form in the time domain as follows:

$$f(t) = \exp(-bt^2) \tag{1}$$

The width of the pulse at the 10% level is

$$T_0 = \left[\frac{4 \ln (10)}{b}\right]^{1/2} \tag{2}$$

The spectrum of the pulse has also the Gaussian form:

$$F(\omega) = \exp(-\omega^2/4b) \sqrt{(\pi/b)} \tag{3}$$

Now, if a pulsed carrier wave at frequency ω_0 travels through the fog-filled medium over a distance L, the received pulse in time domain can be expressed as

$$r(t, L) = (1/2\pi) \int_{-\infty}^{\infty} F(\omega) \exp [j(\omega + \omega_0)t - \Gamma L] \, d\omega \tag{4}$$

Here the complex propagation constant for the fog medium can be expressed by *Foldy*'s [1945] approximation

$$\Gamma(\omega) = \alpha(\omega) + j\beta(\omega) = (\lambda^2/2\pi) \int_0^{\infty} S(0, D) \, N(D) \, dD \tag{5}$$

where $\alpha(\omega)$ is the extinction coefficient and $\beta(\omega)$ is the phase delay coefficient. $S(0, D)$ is the complex forward scattering amplitude and $N(D)$ is the fog drop-size distribution.

To solve (4), α and β are expanded in terms of Taylor series about the center frequency ω_0, retaining the first three terms as follows:

$$\alpha(\omega) = \alpha(\omega_0) + \alpha'(\omega_0)(\omega - \omega_0) + \{\alpha''(\omega_0)/2\}(\omega - \omega_0)^2 \tag{6a}$$

$$\beta(\omega) = \beta(\omega_0) + \beta'(\omega_0)(\omega - \omega_0) + \{\beta''(\omega_0)/2\}(\omega - \omega_0)^2 \tag{6b}$$

where primes indicate the first and second derivatives.

The substitution of (6) allows the integral in (4) to be obtained in closed form if the following condition is satisfied [*Forrer*, 1958]:

$$1 + 2b\alpha''L > 0 \tag{7}$$

Terina [1967] obtained the integral solution of (4) to give the received pulse waveform as

$$r(t^*, L) = A(L) \exp [-kb(t^* + \zeta)^2] . \exp [-j(\omega^* t^* + \delta)] \tag{8}$$

where t^* is the retarded timescale given by

$$t^* = t - \beta' L$$

The new phase of the carrier is given by (*Gibbins*, 1990)

$$\delta = (\omega\beta' - \beta)L - \frac{2b^2\beta''L(\alpha'L)^2}{(1+2b\alpha''L)^2 + (2b\beta''L)^2} - \frac{1}{2} \text{ arc tg} \left[\frac{2b\beta''L}{1+2b\alpha''L}\right]$$

The peak amplitude of the pulse changes by a factor

$$A(L) = \alpha L\ [(1+2b\alpha''L)^2 + (2b\beta''L)^2]^{-1/4} \cdot \exp\left[\frac{b(\alpha' L)^2}{1+2b\alpha''L}\right]$$

The additional group lag time is

$$\zeta = \frac{2b\alpha'\beta''L^2}{1+2b\alpha''L}$$

The new carrier mode frequency, ω^*, at which maximum energy is received is given by

$$\omega^* = \omega_0 + \Delta\omega + \chi t^* \qquad (9)$$

where shift in carrier frequency is

$$\Delta\omega = -2kb\alpha' L$$

The term χt^* corresponds to the imposed frequency modulation on the carrier, known as a "chirp," where

$$\chi = \frac{2b^2\beta''L}{(1+2b\alpha''L)^2 + (2b\beta''L)^2} \qquad (10)$$

It is evident that the received pulse envelope $\exp[-kb(t^*+\zeta)^2]$ remains Gaussian, but the pulse width changes by a factor $\sqrt{k}$, where k is given by

$$k = \frac{1+2b\alpha''L}{(1+2b\alpha''L)^2 + (2b\beta''L)^2} \qquad (11)$$

The modified width of the received pulse under the convergence condition (7) is therefore given by

$$T = T_0 / \sqrt{k} \qquad (12)$$

In case the condition (7) is not satisfied, the pulse shape will no longer remain Gaussian, and the waveform given by (4) has to be obtained by numerical techniques.

In order to obtain the propagation constants of the fog medium, the forward scattering amplitude $S(0, D)$ has been calculated for spherical drops using the Mie-Stratton algorithm [*Stratton*, 1941]. The complex refractive indices are obtained with *Ray*'s [1972] model for wavelengths above 2 μm and for the 0.4-2 μm region the values were obtained from *Hale and Querry* [1973]. The use of the Mie scattering theory for the fog medium at optical wavelengths has been shown to be valid by *Rensch and Long* [1970].

The drop-size distribution of the fog medium can be specified by a modified gamma distribution as follows [*Deirmendjian*, 1964]:

$$N(D) = a(D/2)^{\eta} \exp[-c(D/2)^{\gamma}] \qquad (13)$$

Here a, c, η, and γ are the distribution parameters to be specified for a particular type of fog.

3. Results of Numerical Calculations

The pulse distortion, in terms of the percentage change in pulse width, has been studied in the wavelength region 0.4-10.0 μm with respect to the type of fog (drop-size distribution), pulse width, and path length. A positive value of percentage distortion corresponds to pulse broadening, and a negative value to pulse compression.

As already mentioned, the propagation constants $\alpha(\omega)$ and $\beta(\omega)$ have been obtained using the Mie scattering theory. The second derivatives $\alpha''(\omega)$ and $\beta''(\omega)$ have been calculated with a numerical differentiation technique based on an extension of the Neville algorithm for numerical quadrature [*Lyness and Moler*, 1969]. The derivatives are obtained from 21 values of $\alpha(\omega)$ and $\beta(\omega)$ evaluated around the carrier frequency over the bandwidth of the pulsed signal. Gaussian pulses of width (at 10% level) of 0.005, 0.001, and 0.0005 ns have, respectively, 3-dB bandwidths of 320, 1600, and 3200 GHz.

In this study, three types of fog, namely, cumulus, stratocumulus and nimbostratus, have been considered. The parameters of drop-size distribution, used in relation (13), of these fog types are given in Table 1.

Figure 1 shows the percentage distortion of a pulse of width 0.001 ns over a path length of 1 km for cumulus, stratocumulus, and nimbostratus fog models. The distortion is quite large in several wavelength regions, showing several prominent peaks in its value, peaks which are both positive and negative. There are several

Table 1. Types of Fog Considered

Fog Type	Parameter a	c	η	γ
Cumulus	2.604	0.5	3.0	1.0
Stratocumulus	0.4369	0.8	5.0	1.0
Nimbostratus	7.676	0.425	2.0	1.0

gaps shown in the curves. These are the regions where the convergence condition (7) is not satisfied, indicating that the pulse shape is no longer Gaussian, and hence the distortion value cannot be obtained using relation (12). The patterns of variations of the distortions due to three types of fog are more or less similar though the peaks have different amplitudes. The overall distortion is most severe for cumulus fog.

Figure 2 shows the effect of path length on pulse distortion. The distortion of a pulse with a width of 0.001 ns through stratocumulus fog for 0.5-, 1-, and 1.5-km path lengths are shown in this figure. It is observed that the distortion increases with the path length, the peaks being more prominent for longer paths.

Figure 3 depicts the distortion of pulses of width 0.005, 0.001, and 0.0005 ns for stratocumulus fog over a path length of 1 km. It is evident that the distortion increases as the pulse becomes narrower. The distortion is small for a 0.005-ns pulse. For the 0.0005-ns pulse

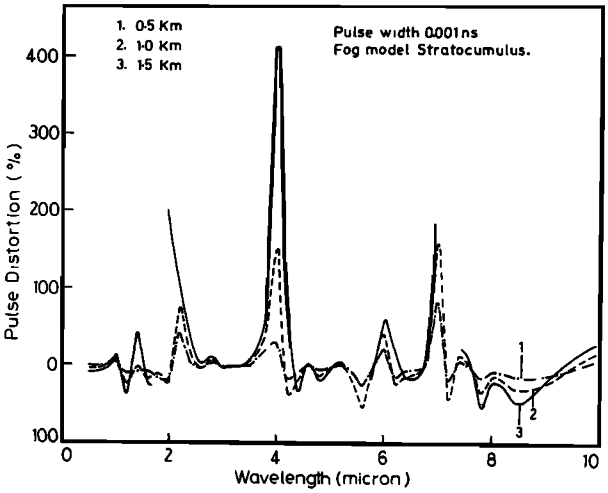

Figure 2. Distortion of a pulse with width 0.001 ns for the stratocumulus fog model over different path lengths.

the distortion is rather severe, the pulse shape often not remaining Gaussian, as indicated by the breaks in the curve.

4. Discussion

Pulse distortions can be very large depending on wavelength, type of fog, and path length. The above parameters should go as inputs to designing a high-rate data communication link through the atmosphere at optical wavelengths. As already indicated, the pulse

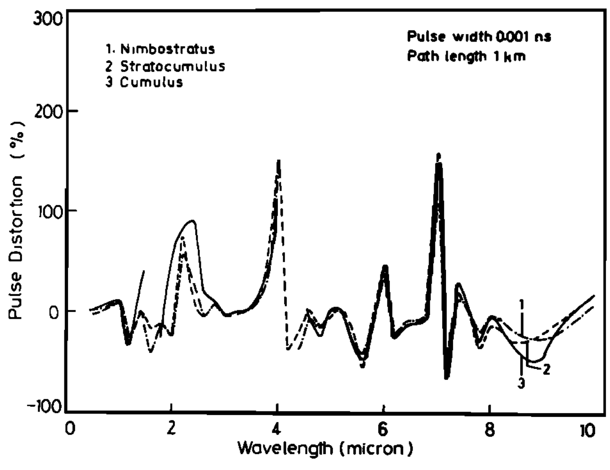

Figure 1. Pulse distortion over a 1-km path length for pulse width of 0.001 ns for different fog models.

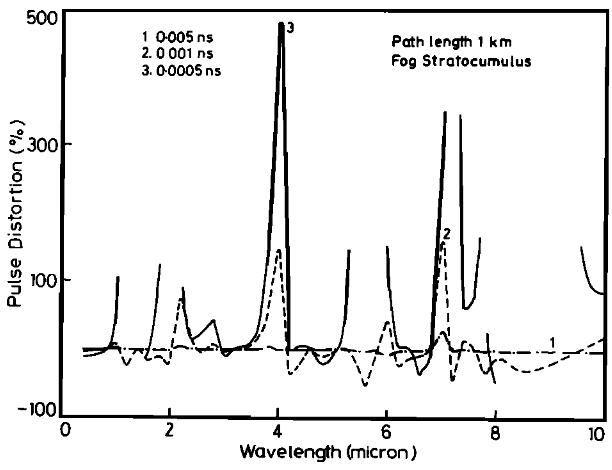

Figure 3. Pulse distortion over a 1-km path length for the stratocumulus fog model for different pulse widths.

distortion occurs due to variation of the propagation constants within the bandwidth of the propagated signal. The variations of extinction coefficient and phase delay coefficient with the wavelength for the three types of fog are shown in Figures 4 and 5 respectively. It may be observed that the peaks in distortion values are associated with noticeable changes in the slope of the curves for propagation constants given in Figures 4 and 5. In fact, the distortion depends on the second derivative values of propagation constants.

In the present study, both broadening and compression of pulses are obtained. The pulse broadening occurs when the higher-frequency components of the signal are more attenuated than the lower-frequency components. The mathematical condition for pulse broadening, in the present case as obtained from (11) and (12) , is

$$\frac{T}{T_0} = \left[\frac{(1+2b\alpha''L)^2 + (2b\beta''L)^2]}{1+2b\alpha''L} \right]^{1/2} > 1 \quad (14)$$

As $(2b\beta''L)^2$ is always positive and $(1+2b\alpha''L)$ is also always positive according to the convergence condition (7), the inequality (14) will always be satisfied when

$$2b\alpha''L > 0 \quad (15)$$

Hence the broadening will always occur whenever α'' is positive. However, the broadening will also occur for negative values of α'' up to a certain limit as indicated by the inequality (14).

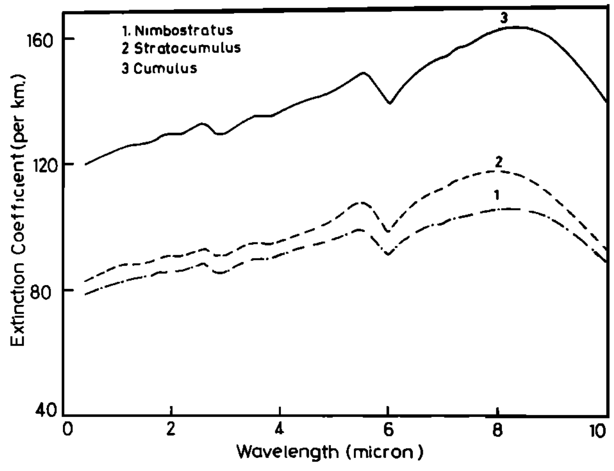

Figure 4. Extinction coefficients for different fog models.

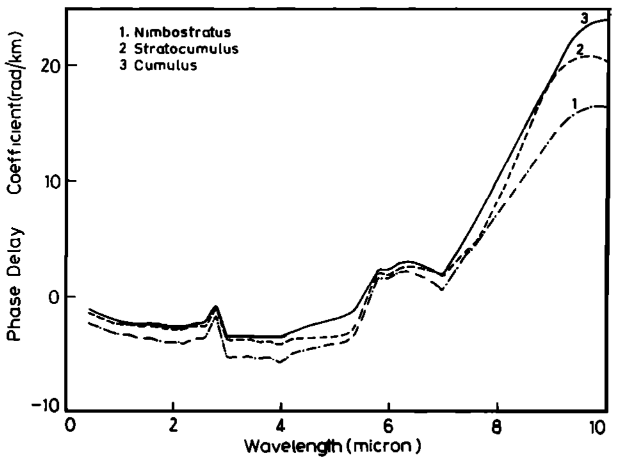

Figure 5. Phase delay coefficients for different fog models.

The pulse compression occurs because the atmosphere behaves as a dispersive delay line to optical waves. As already mentioned, the propagated signal has an imposed frequency modulation on the carrier within the pulse. Under certain condtions, the frequencies at the leading edge of the pulse may be delayed more than the frequencies at the trailing edge, thus causing pulse compression. The mathematical condition for pulse compression in the present case is

$$\frac{T}{T_0} < 1$$

which simplifies to

$$2bL\,(\alpha''^2 + \beta''^2) < -\,\alpha'' \quad (16)$$

Since the left-hand side of the above inequality is always positive, α'' must be negative whenever pulse compression occurs. However, a negative value of α'' does not ensure pulse compression. The additional condition to be fulfilled for pulse compression to occur is that the magnitude of α'' should be greater than the left-hand side of the inequality (16), which in turn depends on the pulse width and the path length.

It may be mentioned that the attenuation of optical pulses can be mitigated, to a certain extent, by increasing the transmitted power. However, for a high-rate data communication system, the pulse broadening is a more concerning factor since the pulse attenuation may not be as impairing as the overlapping of pulses that increases the intersymbol interference.

It is seen that the pulse distortion may significantly depend on the type of fog. The cumulus fog is responsible for highest extinction and phase delay among the three types of fog and, consequently, causes most severe pulse distortion on the whole. The fog models used in the present study are well quoted in the literature [*Silverman and Sprague*, 1970] . The information on different types of fog available on a statistical basis can be used to obtain a reasonable prediction capability for pulse distortion occurrences.

The decrease of pulse width causes an increase of the bandwidth of the signal. The effect of differential attenuation and phase dispersion should usually be more pronounced on the signal with larger bandwidth, causing greater distortion to narrower pulses.

The accuracy of the present technique greatly depends on the validity of the assumption that the third- and higher-order derivatives of propagation constants are negligible in the Taylor series expressions. To examine this point, third- and fourth-order derivatives have been calculated. It is found that when the distortion is significant (greater than 50%), the third-order derivative is usually two orders magnitude smaller than the second-order derivative, and the fourth-order is further two orders of magnitude lower. For lower distortions the higher-order derivatives are still less significant. Hence the three term Taylor series expressions for the propagation constants are practically valid for most of the propagation conditions considered in the present study.

No experimental observations are available in the open literature on subnanosecond-pulse propagation at optical wavelengths through the atmosphere with which the present numerical calculations can be compared. However, *Forrer*'s [1958] technique has been applied to study the propagation of short pulses at millimeter wavelengths, the results of which compare well with experimental measurements [*Glutsyuk et al.*, 1977], possibly indicating that the technique can be applied to other wavelengths at which the atmosphere behaves in a similar manner. In the absence of experimental observations the present results may indicate the extent of distortions of subnanosecond optical pulses propagating through the fog, which is the worst offender as far as the optical propagation through the atmosphere is concerned.

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