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Propagation Characteristics of Single-mode Step Index Linear and Non-linear Optical Fiber involving Improved Lorentzian Approximation for the Fundamental Mode

Abstract: In this paper we approximate the fundamental mode in single mode step-index linear and nonlinear optical fiber by single Lorentzian parameter within scalar variational framework with improved version of previous field. We show that it improves in performance in comparison to earlier one in wider range of V and works well in low-V region more accurately in terms of propagation constant, group delay and wave-guide dispersion.

Keywords: propagation constant, single mode fiber, step index fiber, wave guide dispersion, group delay, Lorentzian approximation

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1 Introduction

Single mode conventional and photonic crystal fibers are well known for processing information through the fundamental mode. In single mode graded Index fiber, if one knows accurately the fundamental mode for the corresponding profile, then one can obtain the field and thereby spot size and the various propagation characteristics of fibers like splice loss, bending loss etc. [1]. Further, analytical solutions exist only in case of step index fibers in terms Bessel and modified Bessel functions. But for graded profiles, one has to take resort to exact numerical and approximate methods like variational, perturbation ones. The variational analysis demands a simple and accurate approximation of the fundamental mode in terms of optimization parameters to be optimized through the optimization of the propagation constant. Various modal functions are still being proliferated in literature in order to suitably approximate the field [2]. The Gaussian approximation of the fundamental mode [3] is popular but does not give much accurate result. Many approximations modifying the Gaussian function [4, 5] along the radial distance have been proposed but inspite of producing very accurate result they require lot of computational time. Very recently we have shown that Lorentzian approximation of the fundamental modal field for linear and nonlinear single moded graded index fibers works excellently in low v-region in comparison to the Gaussian approximation. However, applicability of this function with the extension of this region of v deserves important attention. In this paper, we propose an improved Lorentzian approximation of the fundamental mode which approximates the core region in terms of Lorentzian function and exponential based function in the cladding. It shows better performance compared to other functions [6, 7].

2 Analysis

The refractive index profile is represented as

$$\begin{aligned} n^2(R) &= n_1^2\left(1-\delta f(R)\right) \quad \text{for } R<1 \\ &= n_1^2\left(1-\delta\right) = n_0^2 \quad \text{for } R>1 \end{aligned} \tag{1}$$

where n_1 and n_0 are axial and cladding refractive indices and $\delta = (n_1^2 - n_0^2)/n_1^2$ and $f(R) = R^q$; here in this relation if $q \to \infty$ we shall get the step index fiber. We start with the Improved Lorentzian as follows:

$$\psi = \frac{A}{1+\left(\frac{R^2}{\omega_0^2}\right)} \quad \textit{for } 0 \le R \le 1 \tag{2a}$$

$$\psi = \frac{Ae^{1/R_0}}{1+\frac{1}{\omega_0^2}} \frac{1}{\sqrt{R}} e^{-R/R_0} \quad \textit{for } 1 < R \le \infty \tag{2b}$$

with R being normalized radial distance $R = r/a$; a is the core radius. R_0, ω_0 are Lorentzian parameters and A is constant. We get the relation between the parameters by differentiating both the function and equating afterwards with the value of $R = 1$. This gives the relation between R_0 and ω_0 as follows.

$$-\frac{2A}{\omega_0^2(1+\frac{1}{\omega_0^2})^2} = -\frac{A}{R_0\left(1+\frac{1}{\omega_0^2}\right)} - \frac{A}{2\left(1+\frac{1}{\omega_0^2}\right)}$$

and subsequently,

$$R_0 = \frac{2(1+\omega_0^2)}{3-\omega_0^2} \tag{3}$$

The wave equation obtained from Maxwell's equations of electromagnetic wave propagation for the fundamental mode is,

$$\frac{1}{R}\frac{d}{dR}\left(R\frac{d\psi}{dR}\right) + a^2\left(k_0^2 n^2(R) - \beta^2\right)\psi(r) = 0 \tag{4}$$

where $\psi = \psi(R)$ is the electric field and k_0 is the free space wave vector with β as propagation constant; again the above refractive index $n(R)$ under non linearity is expressed as

$$n^2(R) = n_L^2(R) + 2n_L(R)n_2 I \tag{5}$$

where $n_L(R)$ is linear refractive index, n_2 being the non linear coefficient of refractive index in m^2/w and the local power intensity I is given by

$$I = \frac{1}{2}n_0 c\varepsilon_0 |\psi(R)|^2 \tag{6}$$

Here c being the velocity of light in free space, and ε_0 is dielectric permittivity in free space. Putting the value of I of (6) in (5) we get,

$$n^2(R) = n_L^2(R) + \alpha|\psi(R)|^2 \tag{7}$$

$$\alpha = n_0^2 n_2 c\varepsilon_0 \tag{8}$$

where (4) can be written as

$$\beta^2 = \frac{k_0^2\int_0^\infty [n_L{}^2(R) + \alpha\psi^2(R)]\psi^2(R)dR - \frac{1}{a^2}\int_0^\infty R(\frac{d\psi(R)}{dR})^2 dR}{\int_0^\infty \psi^2(R)RdR} \tag{9}$$

After little computation, U^2 becomes

$$\frac{V^2\int_1^\infty \psi^2(R)RdR + \int_0^\infty (\frac{d\psi(R)}{d})^2 RdR + V^2\int_0 F(R)\psi^2(R)RdR - V \int_0^\infty \frac{\alpha}{(n_1^2 \quad n_0^2)}\psi^4(R)RdR}{\int \quad (R)RdR} \tag{10}$$

where $U^2 = a\sqrt{k_0^2 n_1{}^2(R) - \beta^2}$ and $V^2 = a^2 k_0^2(n_1^2 - n_0^2)$. Here, U and V are the normalized propagation constant and normalized frequency.

We know that $f(R)$ is zero for step index fiber the equation (10) reduces to

$$U^2 = \frac{V^2\int_1^\infty \psi^2(R)\rho dR + \int_0^\infty (\frac{d\psi(R)}{dR})^2 RdR - \gamma V^2\int_0^\infty \psi^4(R)RdR}{\int_0^\infty \psi^2(\rho)\rho d\rho} \tag{11}$$

Now we propose the Improved Lorentzian approximation of the fundamental mode as given in (2a) and (2b) in (11). After integrating, the final relation of

$$U^2 = \frac{A+B+C}{D} \tag{12}$$

where

$$A = \frac{V^2 R_0}{2} \tag{13}$$

$$B = \frac{1}{3}[\left(\frac{4R_0}{3R_0-2}\right)^3 - 1]\left(\frac{4R_0}{3R_0-2}\right)^{-2} \tag{14}$$

$$C = e^{2/R_0}\left(\frac{4R_0}{3R_0-2}\right)^{-1}\left[\frac{e^{-2/R_0}}{2R_0} + \frac{1}{R_0}\int_{2/R_0}^\infty e^{-z}\frac{dz}{z} + R_0\int_{2/R_0}^\infty e^{-z}\frac{dz}{z^2}\right] \tag{15}$$

$$D = \frac{1}{2} + \frac{3R_0-2}{8} \tag{16}$$

Above expression of U^2 is optimized to obtain the relation between Lorentzian parameter and normalized frequency V by variational approach.

3 Results and discussion

Based on the above analysis, we study how the present calculations involving single parameter Improvd Lorentzian approximation is excellently matching in the low V-region in comparison to previous version [7]. When we compare the results obtained from both approximations and those of the analytical field and exact numerical field in case of linear step profile we find improved Lorentzian

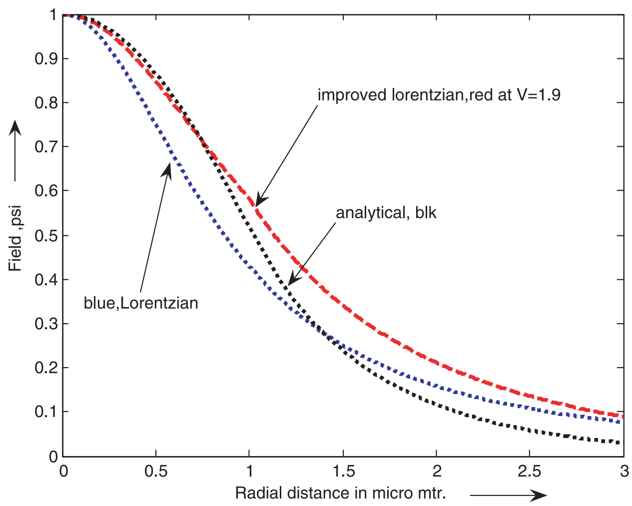

Fig. 1: Field vs. radial distance along the optical fiber at V = 1.9

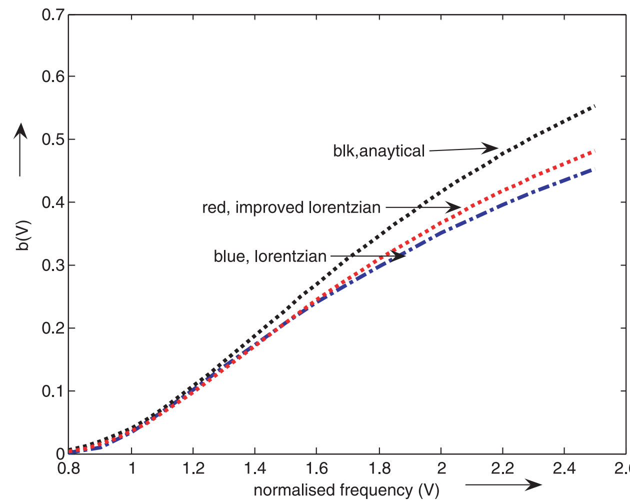

Fig. 3: b(V) (normalized propagation constant) vs. V (normalised frequency)

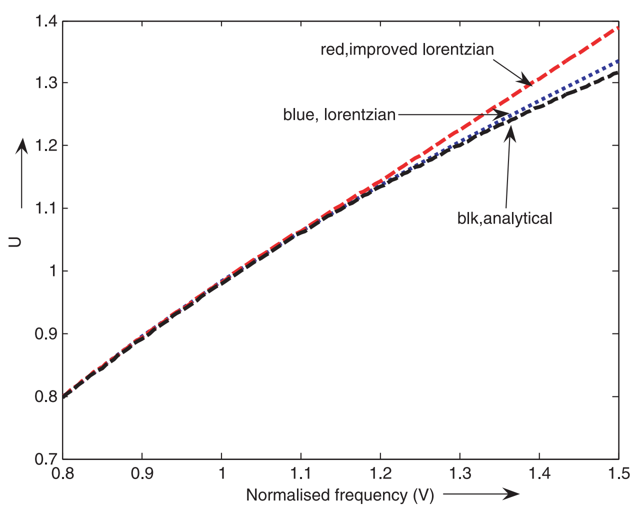

Fig. 2: U vs. V (normalised frequency) [$U^2 = k^2 n_1^2 - \beta^2$]

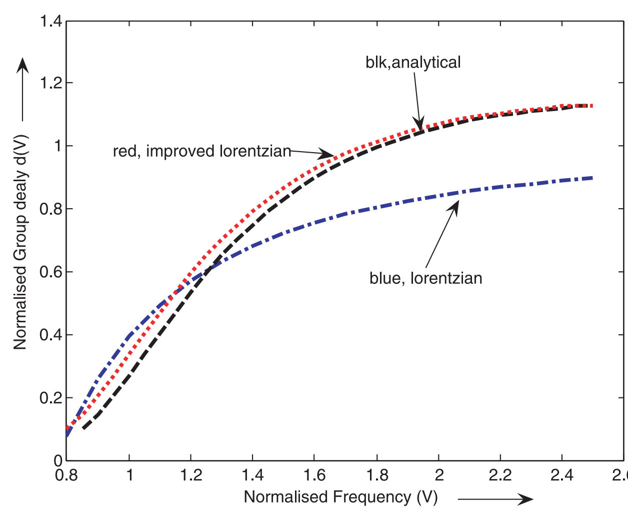

Fig. 4: d(V) (normalized group delay) vs. V (normalised frequency)

shows better matching with that of analytical. We find the fundamental field for analytical by solving Bessel and modified Bessel equation and compare the fundamental field of Lorentzian and improved Lorentzian by the method of variational approach as shown in Fig. 1. Here, analytical field is much coinciding with improved Lorentzian than ordinary Lorentzian shown by blue colour graph. The plot is shown for $V = 1.9$. Another plot is depicted in the linear optical fiber where we plot U vs V in Fig. 2 where improved Lorentzian gives same response as ordinary Lorentzian.

In Fig. 3 we present the comparison of normalized propagation constant, $b(V)$ vs. V with same three curves with blue indicating the Lorentzian, red indicating the improved Lorentzian and black as usual analytical.

Here also the improved Lorentzian better matching with analytical than ordinary Lorentzian with analytical.

In Fig. 4 we show the graph based on group delay, $d(V)$ vs. normalized frequency or V parameter for same three cases. Here also, we find the improved Lorentzian is excellently matching with analytical.

In Fig. 5 we see how group velocity dispersion behave in comparison with analytical. The improved Lorentzian shows better trend compared to ordinary Lorentzian. Here one can use improved Lorentzian for less amount of dispersion in case of bit propagation because of less effect of dispersion.

We present Table 1 to give the nonlinear effect due to three cases like improved Lorentzian, Lorentzian and exact numerical. We find in Table 1 the nonlinear values

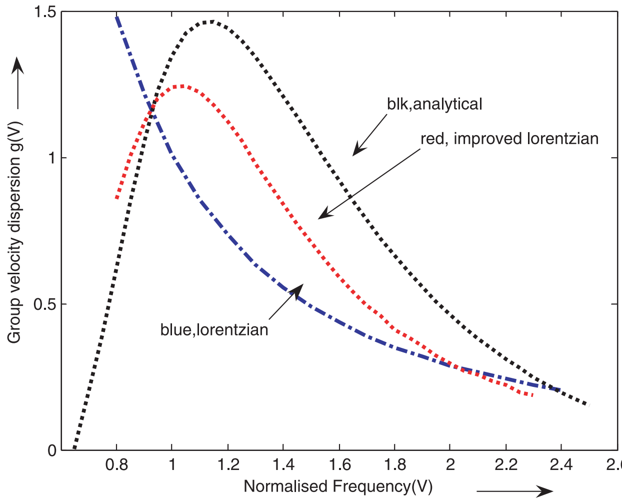

Fig. 5: g(V) (normalized waveguide dispersion) vs. V (normalised frequency)

Table 1: Nonlinear effect

V	$U_{improved\ Lorentzian}$	$U_{Lorentzian}$	$U_{exact\ nemerical}$
1.2	1.1285	1.1239	1.1204
1.3	1.1961	1.1922	1.1824
1.4	1.2885	1.2566	1.2433

of corresponding U's for different values of V's. The non-linear values taken, $\gamma = 0.062$ corresponding to 200 kW.

4 Conclusion

We propose for the first time a single parameter Improved Lorentzian approximation of the fundamental mode in single mode linear and nonlinear step index fiber. With this approximation, we develop the variational analysis, present the approximate analytical formulae for various propagation characteristics and compare them with those based on calculation involving ordinary Lorentzian, improved version and analytical or exact numerical ones. Our results show that Improved version behaves better way in comparison to Lorentzian approximation and it is more superior to Gaussian [7] and ordinary Lorentzian.

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