J. Opt. Commun. 19 (1998) 4, 145–148

Journal of Optical Communications

Prediction of Modal Dispersion in Single-mode Graded Index Fibers by Chebyshev Technique

S. Gangopadhyay*, S. N. Sarkar

Summary

A simple method based on Chebyshev technique is reported to predict the normalised group delay and modal dispersian in single-mode optical fibers having arbitrary index profiles with examples of step and parabolic index cores. This method contains formulation of a linear relation of $K_1(W)/K_0(W)$ with 1/W over a wide and practical range of W values, relevant to single-moded guidance in such fibers. Comparison with the exact numerical results shows that the proposed technique estimates the dispersion parameters excellently.

1 Introduction

One of the important parameters characterising a single-mode fiber is its modal spot size from which one can predict accurately the modal dispersion, bending loss etc [1–4]. Further, of various recently proposed definitions of spot size, Petermann II spot size [5] is specially convenient to predict the normalised group delay and modal dispersion. An accurate knowledge of the transverse field of the fundamental mode propagating in such fibers is essential for evaluating the modal spot size. In order to solve the scalar wave equation for an arbitrary index profile and obtain the fundamental modal field thereof, one either resorts to numerical techniques or uses approximate methods [6, 7] except in the case of step index profile where analytic solution is known. The variational techniques involving single parameter [8] as well as two parameter [9] trial functions approximating the fundamental mode of graded index fibers have been reported and it has been shown that the two parameter approximation interprets the propagation characteristics more accurately than the single parameter approximation. All these analyses, however, involve complicated computations and therefore, prescription of a suitable but simple functional form for the fundamental mode of a single-mode graded index fiber is desirable in this connection. The approximation of the fundamental modal field for graded index fiber by a Chebyshev power series has been recently reported [10]. This method is based on a prescription of a linear relation of $K_1(W)/K_0(W)$, the ratio of modified Bessel functions, with 1/W over a long and practical single-mode range of normalised propagation constant W. The coefficients of the power series are found from the solutions of simple algebraic equations obtained for different suitable Chebyshev points [10].

In this communication, we report the applicablity of the Chebyshev technique to predict the normalised group delay and modal dispersion for weakly guiding single-mode graded index fibers in a simplified manner.

2 Theory

For weakly guiding fiber, the refractive index profile is expressed as

$$n^2(R) = n_1^2[1-2\delta f(R)], \qquad R \le 1$$
$$= n_2^2, \qquad R > 1 \qquad (1)$$

Here n_1 is the refractive index on the axis of the core and n_2 is the refractive index of the cladding. We have also $\delta = (n_1^2 - n_2^2)/2n_1^2$, R=r/a where a = core radius, f(R) defining the shape of the profile; for graded index fiber, it is given by

$$f(R) = R^q, \qquad R \le 1 \qquad (2)$$

q being the profile exponent and its value being ∞ for step index fiber and 2 for parabolic index fiber.

The field ψ(R) for the fundamental mode in the core is given by

Address of authors:

Department of Electronic Science
University of Calcutta
92, A.P.C. Road
Calcutta-700009, India

* Department of Physics
Surendranath College 24-2
M.G.Road
Calcutta-700009, India

Received 16 November 1996

Journal of Optical Communications
19 (1998) 4

$$\frac{d^2\psi}{dR^2}+\frac{1}{R}\frac{d\psi}{dR}+\left(V^2(1-f(R))-W^2\right)\psi=0, \qquad R\le 1 \qquad (3)$$

along with the boundary condition

$$\left(\frac{1}{\Psi}\frac{d\Psi}{dR}\right)_{R=1}=-\frac{WK_1(W)}{K_0(W)} \qquad (4)$$

The fundamental modal field in the cladding is expressed as

$$\Psi(R)\approx K_0(WR). \qquad R>1 \qquad (5)$$

We have shown [10] in the appendix how $\psi(R)$ is found by the Chebyshev technique employing the linear relation of $K_1(W)/K_0(W)$ with $1/W$ for values of W appropriate for single-mode behaviour of the fiber.

The normalised group delay (b_1) can be expressed in terms of the modal field ψ and the normalised propagation constant b (= W^2/V^2) as [6, 7],

$$b_1=\frac{d}{dV}(Vb)=b+\frac{2}{V^2}\frac{\int_0^\infty R\left(\frac{d\psi}{dR}\right)^2 dR}{\int_0^\infty R\psi^2 dR}. \qquad (6)$$

It may be interesting to note that [5]

$$\frac{\int_0^\infty R\left(\frac{d\psi}{dR}\right)^2 dR}{\int_0^\infty R\psi^2 dR}=\frac{2}{w_d^2} \qquad (7)$$

where w_d is the normalised Petermann II spot size. Using the value of ψ given by (A8) of appendix in (6), we get [11, 12]

$$b_1=\frac{W_c^2}{V^2}+\frac{2}{V^2}\frac{\left(4T_1+T_2W_c\left(W_c+2T_5-W_cT_5^2\right)\right)}{T_3+2T_4+T_2\left(T_5^2-1\right)} \qquad (8)$$

where W_c is the value of W obtained by the present Chebyshev technique and

$$\begin{aligned}
T_1&=A_2^2/2+A_4^2+3A_6^2/2\\
&\quad+4A_2A_4/3+3A_2A_6/2+12A_4A_6/5\\
T_2&=\left(1+A_2+A_4+A_6\right)^2\\
T_3&=1+A_2^2/3+A_4^2/5+A_6^2/7\\
T_4&=A_2/2+A_4/3+A_6/4\\
&\quad+A_2A_4/4+A_2A_6/5+A_4A_6/6\\
T_5&=1.034623+0.3890323/W_c.
\end{aligned} \qquad (9)$$

The normalised modal dispersion parameter (b_2) is $V\frac{d^2}{dV^2}(Vb)$ and is expressed by [6, 7]

$$b_2=2\frac{d}{dV}\left(\frac{1}{V}\frac{\int_0^\infty R\left(\frac{d\Psi}{dR}\right)^2 dR}{\int_0^\infty R\Psi^2 dR}\right). \qquad (10)$$

Again, using the value of ψ in (10) we can express b_2 as

$$b_2=2\frac{d}{dV}\left(\frac{1}{V}\left(\frac{4T_1+T_2W_c\left(W_c+2T_5-W_cT_5^2\right)}{T_3+2T_4+T_2\left(T_5^2-1\right)}\right)\right). \qquad (11)$$

The first derivative with respect to V in (11) is carried out numerically in order to calculate the normalised dispersion parameter.

3 Results and discussions

In order to check the validity of our formulation for normalised group delay and dispersion parameter, we compare our results with the exact results in case of step index fiber and also with the available exact numerical results for the parabolic index fiber as well. In Table 1 we have presented the values of b_1 found by our technique along with the available exact values [5] for a step index fiber. It is seen that for V = 1.4, the error is maximum with a value of 0.91% and for V = 2.4, the error is minimum with that of 0.09%. In Fig. 1, the solid curves represent the variations of b_1 and b_2 with normalised frequency V for a step index fiber [13] whereas the values obtained by our model in the single-mode range are represented by crosses. It is clearly seen from Fig. 1 that the values predicted by our model agree excellently with the exact ones. Similarly, for parabolic index fiber the variation of b_1 and b_2 are plotted against V in Fig. 2, where our values, the exact numerical ones [1] and those based on Gaussian approximation for the field [1] are presented by crosses, solid and dashed lines respectively. We see that Gaussian approximation which is taken to be sufficiently reliable for parabolic index fiber is not suita-

Table 1: Values of normalised group delay b_1 for step index fiber

V	b_1 (Found)	b_1 (Exact) [5]	Error
1.4	0.764	0.771	0.91%
1.6	0.917	0.913	0.44%
1.8	1.012	1.006	0.60%
2.0	1.070	1.065	0.47%
2.2	1.105	1.102	0.27%
2.4	1.125	1.124	0.09%

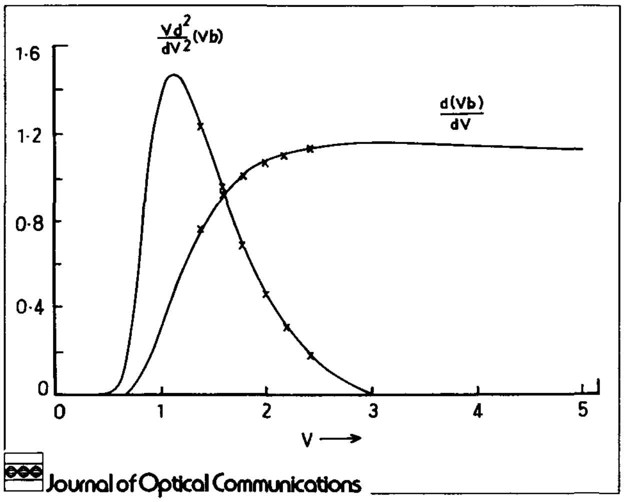

Fig. 1 : d(Vb)/dV and V d²(Vb)/dV² versus V for the fundamental mode of weakly-guiding step index fiber (x our theoretical results based on Chebyshev technique; ——— theoretical results [13])

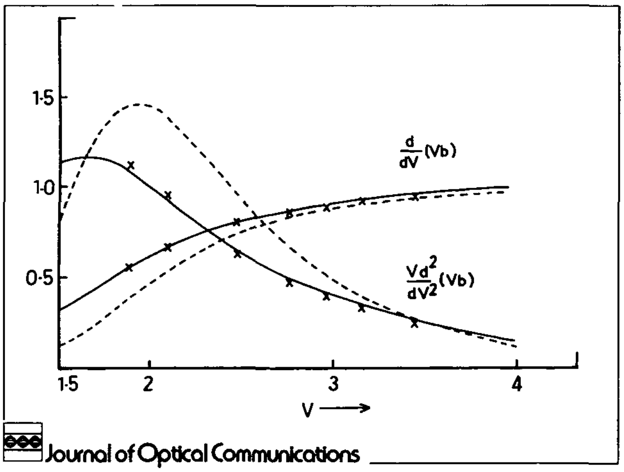

Fig. 2: d(Vb)/dV and V d²(Vb)/dV² versus V for the fundamental mode of weakly-guiding parabolic index fiber (x our theoretical results based on Chebyshev technique; ——— theoretical results [1]; - - - - - theoretical results based on Gaussian approximation [1])

ble as to predict the normalised group delay and modal dispersion over the single-mode region of practical interest. On the other hand, our simple method based on Chebyshev technique predicts results which agree nicely with the exact numerical ones in case of parabolic index fiber.

Further, it may be relevant to mention that we have not presented any data in the low V region, namely for $V < 1.4$ for step index and $V < 1.9$ for parabolic index fibers, since our method is based on a linear approximation of $K_1(W)/K_0(W)$ with $1/W$ in the interval $0.6 \le W \le 2.5$. However, observing the excellent predictions of our model for intermediate and high values of V up to respective cutoff values, if one feels interested to extend the analysis in the low V region for single-mode fibers, one has formulate the linear relationship of $K_1(W)/K_0(W)$ with $1/W$ for a few intervals of W where $W < 0.6$. Such study will be reported shortly.

4 Conclusions

We present a simplified analysis to investigate the normalised group delay and modal dispersion parameter for single-mode graded index fibers. The analysis is based on a linear formulation of $K_1(W)/K_0(W)$ with $1/W$ over a long and practical range of W values appropriate for single-mode guidance in graded index fibers and also approximation of the fundamental modal field by Chebyshev power series. With examples of step and parabolic index fibers, it is shown that the results predicted by the proposed method agree excellently with the available results justifying the accuracy of our model.

5 Acknowledgement

The financial assistance of Council of Scientific and Industrial Research, India is gratefully acknowledged.

6 Appendix

Remembering that $\psi(R)$ in case of fundamental mode is an even function of R, $\psi(0)$ is nonzero and $\psi'(0)$ is zero, we can approximate $\psi(R)$ in terms of a Chebyshev power series as [14, 15]

$$\Psi(R) = \sum_{j=0}^{j=M-1} a_{2j} R^{2j}. \tag{A1}$$

The Chebyshev points are given by [15]

$$R_m = \cos\left(\frac{2m-1}{2M-1}\frac{\Pi}{2}\right); \quad m = 1, 2 \ldots (M-1). \tag{A2}$$

For simplicity of calculations we take M = 4 [10] whereby we get

$$\Psi(R) = \sum_{j=0}^{j=3} a_{2j} R^{2j} \tag{A3}$$

and the corresponding Chebyshev points for M = 4 are obtained from (A2) as

$$R_1 = 0.4338, \quad R_2 = 0.7818 \text{ and } R_3 = 0.9749. \tag{A4}$$

The three values of R in (A4) and expression for ψ in (A3) are employed in (3) to get three equations as [10]

$$\begin{aligned} & a_0\left(V^2\left(1-f(R_i)\right)-W^2\right) \\ & +a_2\left(4+R_i^2\left(V^2\left(1-f(R_i)\right)-W^2\right)\right) \\ & +a_4\left(16R_i^2+R_i^4\left(V^2\left(1-f(R_i)\right)-W^2\right)\right) \\ & +a_6\left(36R_i^4+R_i^6\left(V^2\left(1-f(R_i)\right)-W^2\right)\right)=0 \end{aligned}$$

$$i = 1, 2, 3 \text{ imply the three equations.} \tag{A5}$$

By least square fitting procedure for the interval $0.6 \le W \le 2.5$ we get [10]

$$\frac{K_1(W)}{K_0(W)} = 1.034623 + 0.3890323 / W. \quad (A6)$$

Using (A3) in (4) together with (A6) we get

$$\begin{aligned} & a_0(1.034623W + 0.3890323) \\ & +a_2(1.034623W + 2.3890323) \\ & +a_4(1.034623W + 4.3890323) \\ & +a_6(1.034623W + 6.3890323) = 0. \end{aligned} \quad (A7)$$

Clearly W for a given value of V can be found from the condition of the four equations given by (A5) and (A7) being conformable for solution [10]. Further, using any three of the equations given by (A5) and (A7) we get the constants a_{2j} ($j = 1, 2, 3$) in terms of a_0. Accordingly, we obtain the field both in the core and cladding by Chebyshev technique as

$$\begin{aligned} \psi(R) &= a_0\left(1 + A_2R^2 + A_4R^4 + A_6R^6\right), & R \le 1 \\ &= a_0\left(1 + A_2 + A_4 + A_6\right)\frac{K_0(W_cR)}{K_0(W_c)}, & R > 1 \end{aligned} \quad (A8)$$

where $A_{2j} = a_{2j}/a_0$; $j = 1, 2, 3$ and W_c is the value of the normalised propagation constant obtained by the Chebyshev technique.

References

[1] P. Sansonetti: Electron. Lett. 18(1982) 3, 136–138
[2] P. Sansonetti: Electron. Lett. 18 (1982) 11, 647–648
[3] C. Pask: Electron. Lett. 20 (1984) 2, 144–145
[4] A. H. Liang: Applied Opt. 30 (1991) 34, 5017–5018
[5] E.-G. Neumann: "Single Mode Fibres Fundamentals", Vol 57 (Springer-Verlag, 1988)
[6] P. K. Mishra et al.: Opt. Quantum Electron. 16 (1984) 1, 287–296
[7] R. Tewari, S. I. Hosain, K. Thyagarjan: Opt. Commun. 48 (1983) 3, 176–180
[8] D. Marcuse : J. Opt. Soc. Amer. 68 (1978) 1, 103–109
[9] A. Sharma, A. K. Ghatak: Opt. Commun. 36 (1981) 1, 22–24
[10] S. Gangopadhyay et al.: "Novel Method for Studying Single-Mode Fibers involving Chebyshev Technique", accepted for publication in J. Opt. Comm.
[11] M. Abramowitz, I. A. Stegun: "Handbook of Mathematical Functions", Dover Publications,1972
[12] I. S. Gradshteyn, I. M. Ryzhik: "Table of Integrals, Series and Products", Academic Press, Inc. (London), 1980
[13] M. J. Adams: "An Introduction to Optical Waveguides", John Wiley and Sons Ltd., New York, 1981
[14] J. Shijun: Electron. Lett. 23 (1987) 10, 534–535
[15] P. Y. P. Chen: Electron. Lett. 18 (1982) 24, 1048–1049