

Polarization-based Compensation of Astigmatism

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Abstract:

One of the approaches for aberration compensation of an imaging system is to introduce a suitable phase mask at the aperture plane of an imaging system. The present study utilizes this principle for compensation of astigmatism. A suitable polarization mask used on the aperture plane together with a polarizer-retarder combination at the input of the imaging system provides the compensating polarization induced phase steps at different quadrants of the apertures masked by different polarizers. The aberrant phase can be considerably compensated by proper choice of the polarization mask and suitable selection of the polarization parameters involved. The results presented here bear out our theoretical expectation.

Introduction:

For a specific point object, wave aberration for a given imaging system is a measure of the asphericity of the imaging wavefront with respect to a reference spherical wavefront centred on the principal ray at the exit pupil and converging to the Gaussian image point. As a logical consequence, one of the approaches to aberration compensation is the introduction of an appropriately designed phase mask on the exit pupil of the imaging system so that the phase distribution near the mask just about compensates for the aberrant phase distribution. The nature of the aberration to be compensated for, decides the nature of the phase distribution over the phase mask. Off-axial aberration like astigmatism would therefore require a circularly asymmetric phase mask for proper compensation. Different technologies have been proposed and implemented to introduce a phase profile which for complete compensation should be a complex conjugate of that

introduced by the aberrated system. For example, a partial compensation of specific eye aberrations, third order spherical and coma, has been performed with radially symmetric lenses¹⁻². Compensation of the wave aberration, not restricted to specific Seidel terms, has been reported with the use of deformable mirrors³⁻⁵ and also liquid-crystal spatial light modulators⁶

However, it is not always essential that the compensating phase is introduced in terms of a varying optical path difference over the compensating phase plate, as seen in the 'figured' correcting plate of Schmidt camera devices⁷. By using suitably designed polarization masks at the aperture plane, and by the use of a polarizer-retarder combination at the input of the imaging system, it is possible to introduce a space varying polarization phase over the aperture plane. The present study utilizes polarization phase for compensation of astigmatism.

If an elliptically polarized beam passes through a linear polarizer, the transmitted beam will have an associated phase that depends on the state of polarization of the original beam and also on the orientation of the linear polarizer. This provides a simple but useful technique for introducing a phase in a beam of light or a phase difference between two beams derived from an original beam. The phase introduced by utilizing this method is termed as polarization phase⁸⁻¹⁶. The input beam, elliptically polarized in general, is passed through a lens masked by linear polarizers placed in different zones of its aperture. The polarizers at the different masked zones select the components of the elliptically polarized input beam along their respective transmission axes. These components in general differ in phase and amplitude. This effectively implies that the polarization mask introduces a different phase steps and attenuation factors in different zones of the aperture. If the mask is circularly symmetric the phase introduced is also circularly symmetric. The concept of this polarization induced phase using circularly symmetric polarization mask is utilized in focal shift application¹⁷ and for the partial compensation of a spherically aberrated imaging system¹⁸. Moreover, the versatility of this technique lies in the fact that the polarization phase introduced can be controlled by variation of the polarization parameters involved.

In the presence of off-axial aberrations¹⁹⁻²⁰ like astigmatism, the imaging wavefront exhibit circular asymmetry with respect to the optical axis of the imaging system. This

necessitates the introduction of a compensating polarization phase distribution which is also circularly asymmetric. For compensation of astigmatism, each quadrant of the aperture is masked by suitably oriented linear polarizers. The quadrant masked aperture can be rotated as a whole for best compensation.

With increasing astigmatic behaviour of the imaging system the sagittal and tangential foci will be further removed from each other. The separation between these two planes serves as the measure of astigmatism. Midway between these planes, i.e, corresponding to the defocus term, $w_{20} = -w_{22}/2$, the intensity spread is found to be minimum and has four fold symmetry. The transverse plane passing through $w_{20} = -w_{22}/2$ is referred to as the plane of minimum aberration variance for astigmatism²¹.

For simplification of the analysis, effects due to non-normal incidence of polarized light waves on polarization devices and optical components²²⁻²⁶ have been neglected and as such no effort has been made towards evaluation of the polarization aberration involved. Although this approximation will not radically influence the final observations, methods for incorporating these effects in the analysis, using specific system operators, may be utilized, if and when required^{27,28}.

In what follows, the PSF of a quadrant masked aperture in presence of pre-specified astigmatism is plotted at the plane corresponding to the minimum-aberration-variance (circle of least confusion) for astigmatism.. The PSF of a clear aperture lens in presence of the same amount of astigmatism is also plotted for comparison and indicates an obvious improvement of image quality due to polarization masking.

Proposed system:

As shown in Fig.1, the input to the imaging system under consideration is a polarizer-quarter waveplate combination (PC), which renders the imaging beam elliptically polarized in general. F and S represent the fast and the slow axes of the compensator. The

orientation of the transmission axis of the polarizer P and that of the fast axis of the compensator C with the reference abscissa are θ and ϕ respectively. These parameters determine the nature of the vibrational ellipse of the polarized light incident on the mask M. The circular aperture consists of polarization-masked quadrants, with diametrically opposite quadrants masked by identically oriented polarizers. This is followed by an analyzer P(γ) before the image plane S. The polarization masked aperture is shown in Fig. 2

Mathematical Formulation:

With reference to Fig.3a , the general expression for intensity point spread function for an aperture bounded by radii R_1 and R_2 and azimuths ϕ_1 and ϕ_2 is given by²⁹

$$I(v, \theta') = \left| \int_{R_1}^{R_2} \int_{\phi_1}^{\phi_2} T(\rho, \theta) \exp[ikW(\rho, \theta)] \exp[-iv\rho \cos(\theta - \theta')] \rho d\rho d\theta \right|^2 \quad [1]$$

where $w(\rho, \theta)$ is the aberration function, (ρ, θ) is the exit pupil plane coordinates in polar form, (v, θ') the polar coordinates of the image plane and k is the propagation constant. In the present analysis, however, we shall consider apertures for which $R_1=0$ and R_2 normalized to unity as shown in Fig.3b.

The imaging beam is elliptically polarized and is represented by the Jones vector

$$\epsilon_i = \begin{pmatrix} a \\ be^{i\delta} \end{pmatrix} \quad [2]$$

where a and b are the amplitudes of the x and y components of the beam respectively and δ is the phase difference between these two components. The input beam is passed

through N number of sector apertures, each masked by a polarizer $P(\alpha_p)$, where α_p is the angle made by the transmission axis of the p -th aperture with the reference axis. Then the general expression for the intensity point spread function is modified to,

$$I(v, \theta') = \left| \sum_{p=1}^N T_p \exp(i\Delta_p) \int_0^{\phi_2} \int_{\phi_1}^{\phi_2} \exp[ikW(\rho, \theta)] \exp[-iv\rho \cos(\theta - \theta')] \rho d\rho d\theta \right|^2 \quad [3]$$

where the suffix p denotes the parameters for the p -th aperture. The azimuths ϕ_1 and ϕ_2 are given by

$$\phi_1 = (p-1) \frac{2\pi}{N}$$

and

$$\phi_2 = p \frac{2\pi}{N}$$

Δ_p and T_p are the polarization phase and the attenuation given by ^{12,14}

$$\Delta_p = \tan^{-1} \frac{b \sin \alpha_p \sin \delta}{a \cos \alpha_p + b \sin \alpha_p \cos \delta} \quad [5]$$

$$T_p = [a^2 \cos^2 \alpha_p + b^2 \sin^2 \alpha_p + ab \sin 2\alpha \cos \delta]^{1/2} \quad [6]$$

Finally, an analyzer $P(\gamma)$ at the output, where γ is the angle made by the transmission axis of the analyzer with reference axis, allows the contributions from the different sectors to be coherently superposed and the polarization attenuation T_p in Eq.[6] is modified to,

$$T_p' = T_p \cos(\gamma - \alpha_p) \quad [7]$$

In presence of astigmatism and defocus, the aberration function³⁰ may be written as

$$w(\rho, \theta) = w_{20}\rho^2 + w_{22}\rho^2 \cos^2 \theta \quad [8]$$

where w_{20} and w_{22} are the coefficients for the defocus and the astigmatism respectively. Petzval curvature of the lens system is neglected in this calculation.

$$\text{Putting } \rho^2 = m \quad [9]$$

The amplitude point spread function is then written as

$$I(v, \theta') = \sum_{p=1}^N T_p' \exp(i\Delta_p) \int_0^1 \int_{\theta_1}^{\theta_2} \exp\{ikm (w_{20} + w_{22} \cos^2 \theta)\} \exp\{-ivm^{1/2} \cos(\theta - \theta')\} d\theta dm \quad [10]$$

It is difficult to find an analytic solution for the expression given by Eq.[10] and as such a numerical solution has been attempted and rigorously tested for trivial cases and predictable solutions for the IPSF for specific values of the polarization parameters involved.

Computation:

An astigmatic imaging wavefront may be characterized by two different radii of curvature along the sagittal and meridional directions. A possible approach for morphological transformation of the astigmatic wavefront to an anastigmatic wavefront is the introduction of a polarization phase between the sagittal and the meridional direction. This justifies the choice of the mask as shown in Fig. 2 . For this configuration, it can be observed that each quadrant has a phase difference given by Eq.(5) with its adjacent quadrant, while diametrically opposite quadrants have identical phases. The computations are carried out with this particular orientation of the polarization mask.

For an imaging system suitably compensated for astigmatism, the intensity distribution on the plane of minimum aberration variance (in absence of aberration other than astigmatism) resembles closely to that of a diffraction limited system. The results presented are therefore all at this defocus plane, $w_{20} = -w_{22}/2$.

Fig. 4 shows the light intensity distribution for a clear aperture diffraction limited imaging system. Fig. 5(a) shows the diffraction pattern for an unmasked lens for $w_{22} = 3\lambda$. In presence of the compensating mask with $\alpha_1 = 0^\circ$, $\alpha_2 = 90^\circ$ and input beam parameters $a/b = 1$, $\delta = 90^\circ$, the intensity distribution is shown in Fig. 5(b) with the analyzer kept at 135° . Comparison of Fig. 5(b) with Fig.4 and Fig.5(a) indicate that appreciable compensation has been achieved and the central intensity patch closely resembles that of a diffraction limited imaging system. The side lobes still exist, but is modified and much reduced in intensity. Fig. 6(a) shows the intensity distribution for an unmasked lens with $w_{22} = 5\lambda$. Keeping all the polarization parameters identical as in Fig. 5(b), the intensity distribution for the masked lens with $w_{22} = 5\lambda$ is shown in Fig 6(b). Fig 7(a) and Fig 8(a) show the diffraction pattern for unmasked lens with $w_{22} = 7\lambda$ and $w_{22} = 9\lambda$ respectively. Fig

7(b) and Fig 8(b) show the compensation for these magnitudes of astigmatism with the compensating mask (masking parameters same as in Fig 5(b) and 6(b)) . A study of the Fig.5 to Fig.8 indicate the tolerance of the system in terms of astigmatism compensation for the given polarization parameters for coefficient of astigmatism $w_{22}=3\lambda, 5\lambda, 7\lambda$ and 9λ .

The degree of compensation depends on the orientation of the mask, input beam parameter δ and the orientation of the analyzer, γ . The sensitivity of the intensity point spread function to the orientation of the mask can be observed from Fig 9. The mask used in Fig 5(b) is now rotated through 90° , all other parameters remaining same. The quality of the image deteriorates. So Fig 9 conclusively shows the obvious that the orientation of the mask is important in determining the degree of compensation. The response of the mask in Fig 5(b) is studied with the input beam parameter δ changed to 180° . It is found that the deviation of the diffraction pattern for this mask from that of a diffraction limited lens system is much pronounced which signifies that degree of compensation depends on the input beam parameter δ . This is shown in Fig 10. That the IPSF is sensitive to the orientation of the analyzer (γ) can be observed from Fig 11. Here, we have used the same mask as in Fig 5(b). Only in this case, the analyzer is kept at 60° . The deterioration of the quality of the image is obvious from the Fig11. Hence the orientation of the analyzer is another parameter which also determines the extent of compensation achievable.

Conclusion

A method for compensating astigmatism using sector shaped polarization mask has been proposed. The astigmatic wavefront is partially compensated by the polarization phase introduced by the mask. Since the polarization phase can be controlled by changing the polarization state of the input beam or by reorientating the masking polarizers and the analyzer, we have a number of degrees of freedom that can be utilized for compensating the aberrated wavefront. The image formed by the polarization masked aperture shows

that the distorting effects of astigmatism is partly eliminated and the polarization parameters can be adjusted *in-situ* to achieve best compensation.

Although the principle is demonstrated using optical polarization devices, use of liquid crystal variable phase plates can also be employed to achieve identical results. Use of such electro-optic devices that require no mechanical alignment may be useful for compensation in adaptive optics system.

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Figure Captions:

Figure 1: Schematic diagram of the proposed optical set-up.

P-polariser, L-focusing lens, C-compensator, A-analyser, M-polarization mask, S- screen

Figure 2: Proposed Polarization Mask

Figure 3a: Aperture bounded by radii R_1 and R_2 and azimuths φ_1 and φ_2

Figure 3b: Sector aperture bounded by radii $R_1=0$ and $R_2=1$, and azimuths φ_1 and φ_2 .

Figure 4: IPSF for a diffraction limited imaging system

Figure 5(a) : Image of an off-axial point source formed by a clear aperture lens in presence of astigmatism $w_{22}=3\lambda$ at $w_{20}=-w_{22}/2$

(b): Image of an off-axial point source formed by a compensating mask, with $\alpha_1=0^\circ$, $\alpha_2=90^\circ$, $a/b=1$, $\delta=90^\circ$, $\gamma=135^\circ$ with $w_{22}=3\lambda$

Figure 6(a): Image of an off-axial point source formed by a clear aperture lens in presence of astigmatism $w_{22}=5\lambda$ at $w_{20}=-w_{22}/2$

6(b) : Image of an off-axial point source formed by a compensating mask, with $\alpha_1=0^\circ$, $\alpha_2=90^\circ$, $a/b=1$, $\delta=90^\circ$, $\gamma=135^\circ$ with $w_{22}=5\lambda$

Figure 7(a) : Image of an off-axial point source formed by a clear aperture lens in presence of astigmatism $w_{22}=7\lambda$ at $w_{20}=-w_{22}/2$

7(b) : Image of an off-axial point source formed by a compensating mask, with $\alpha_1=0^\circ$, $\alpha_2=90^\circ$, $a/b=1$, $\delta=90^\circ$, $\gamma=135^\circ$ with $w_{22}=7\lambda$

Figure 8(a) : Image of an off-axial point source formed by a clear aperture lens in presence of astigmatism $w_{22}=9\lambda$ at $w_{20}=-w_{22}/2$

8(b) : Image of an off-axial point source formed by a compensating mask, with $\alpha_1=0^\circ$, $\alpha_2=90^\circ$, $a/b=1$, $\delta=90^\circ$, $\gamma=135^\circ$ with $w_{22}=9\lambda$

Figure 9 : Image of an off-axial point source formed by a compensating mask, with $\alpha_1=0^\circ$, $\alpha_2=90^\circ$, $a/b=1$, $\delta=90^\circ$, $\gamma=135^\circ$ with $w_{22}=3\lambda$, mask is rotated through 90° from that shown in Fig.(2)

Figure 10: Image of an off-axial point source formed by a compensating mask, with $\alpha_1=0^\circ$, $\alpha_2=90^\circ$, $a/b=1$, $\delta=180^\circ$, $\gamma=135^\circ$ with $w_{22}=3\lambda$

Figure 11: Image of an off-axial point source formed by a compensating mask, with $\alpha_1=0^\circ$, $\alpha_2=90^\circ$, $a/b=1$, $\delta=90^\circ$, $\gamma=60^\circ$ with $w_{22}=3\lambda$.

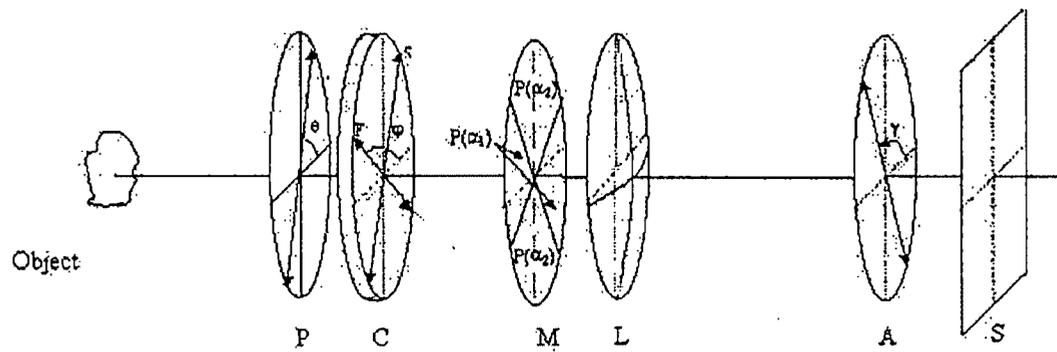


Fig 1

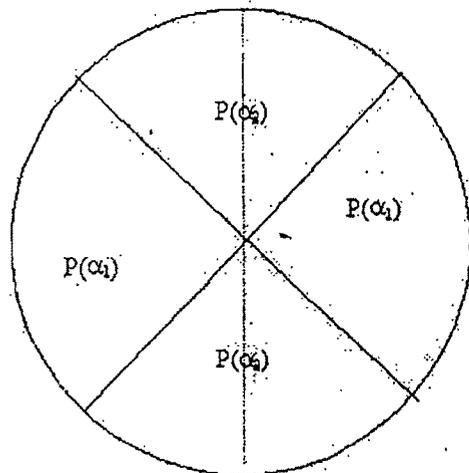


Fig 2

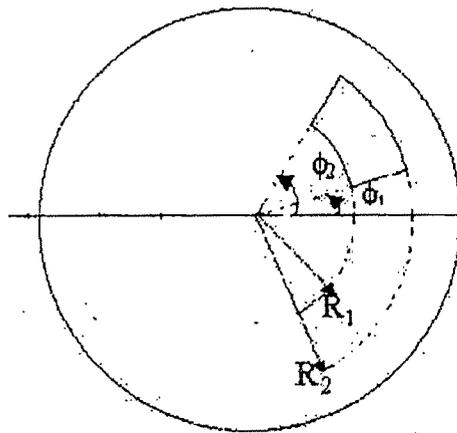


Fig.3.2(a)

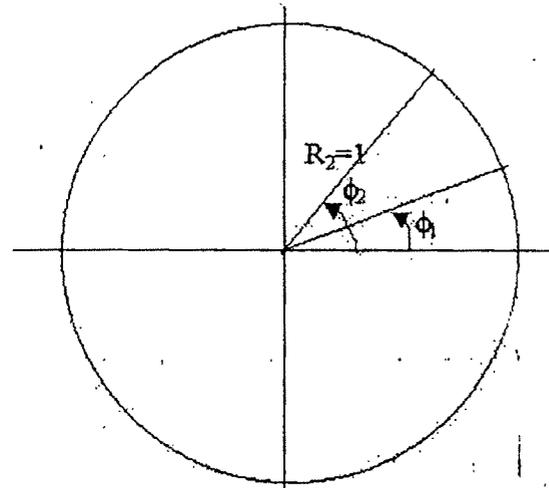


Fig 3.2(b)

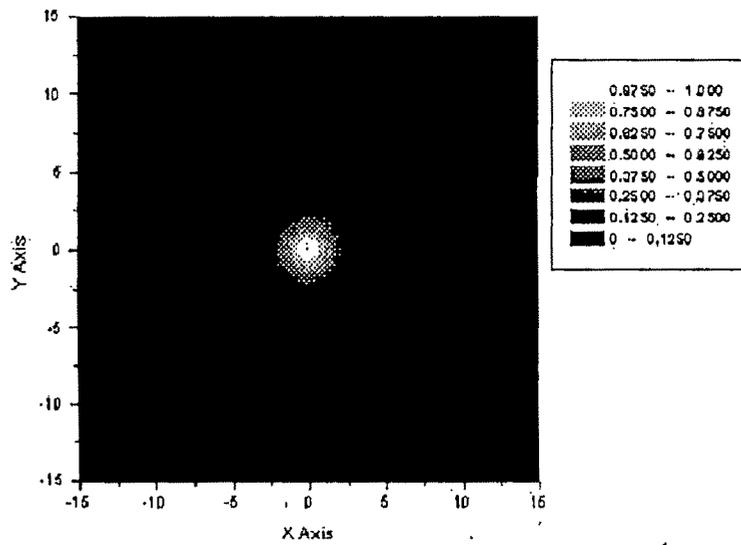
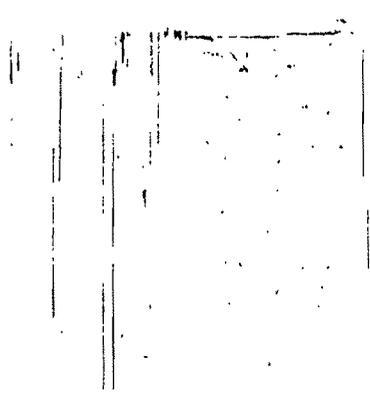


Fig 4



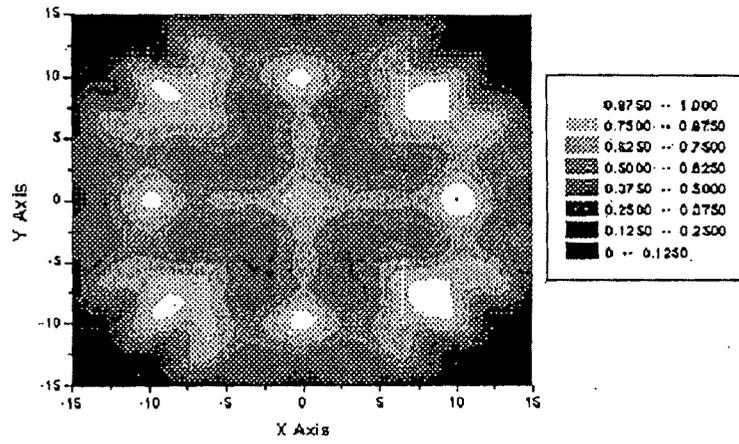


Fig 5(a)

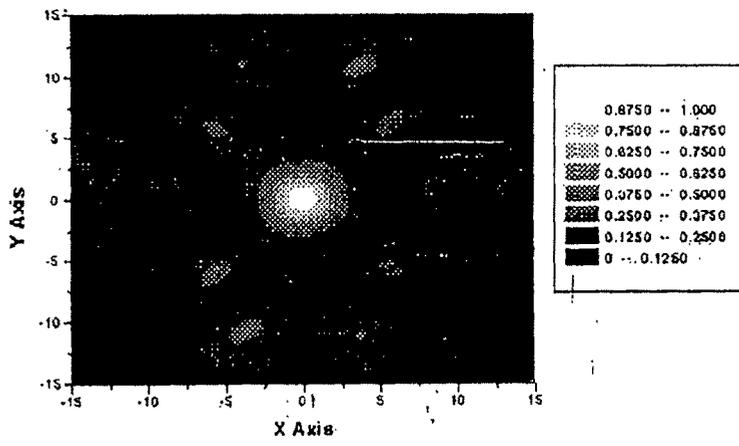
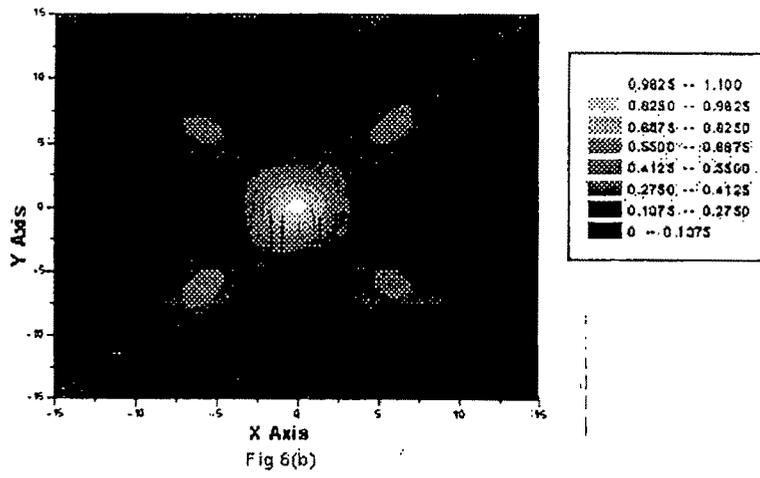
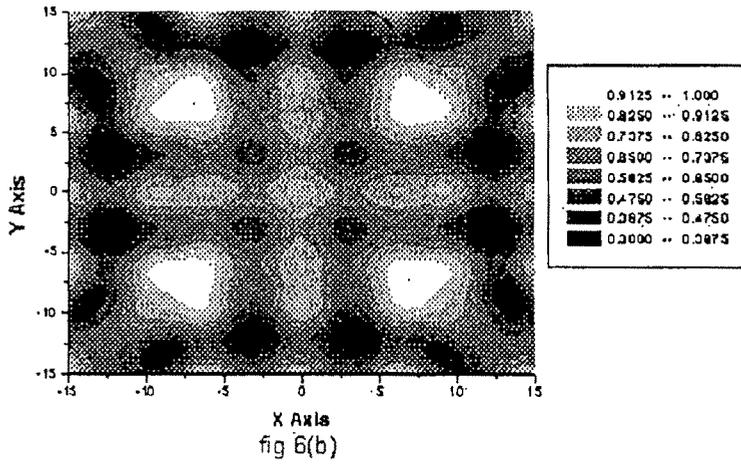
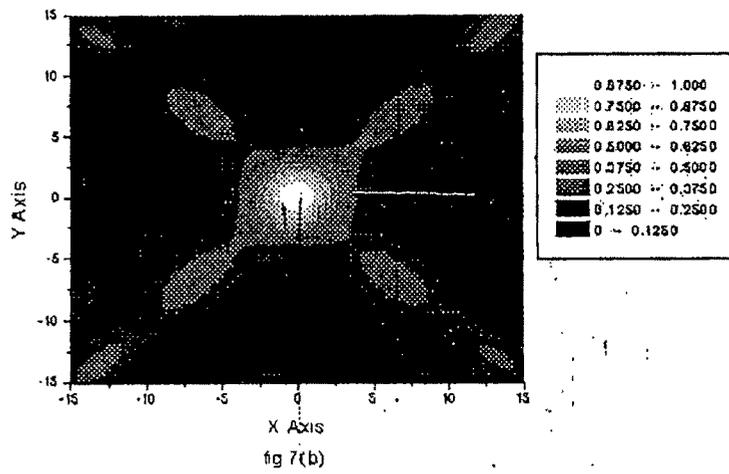
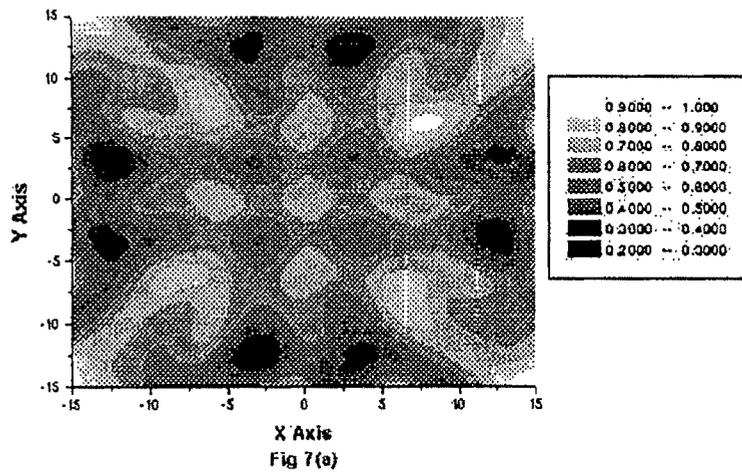
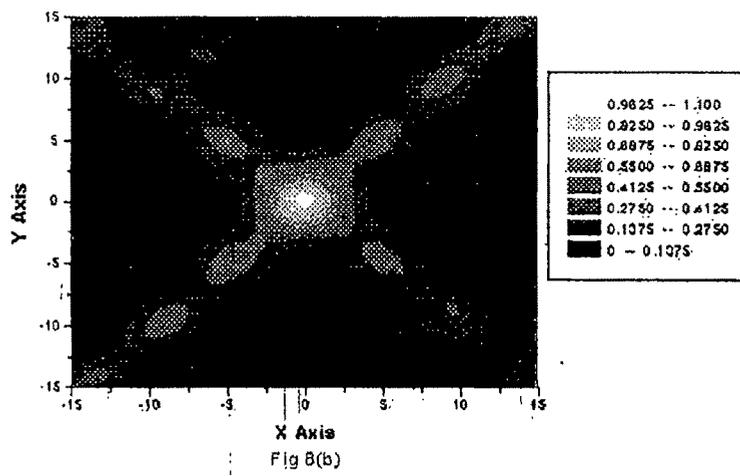
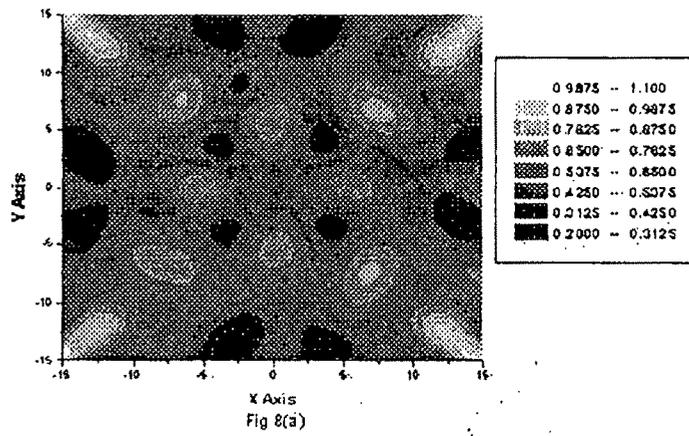


Fig5(b)







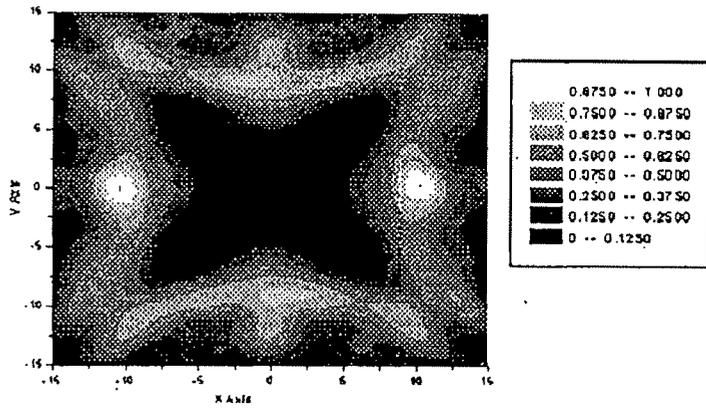


fig 9

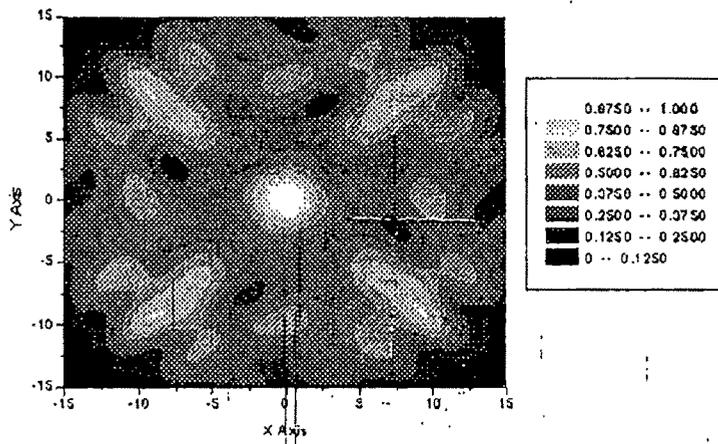


fig 10

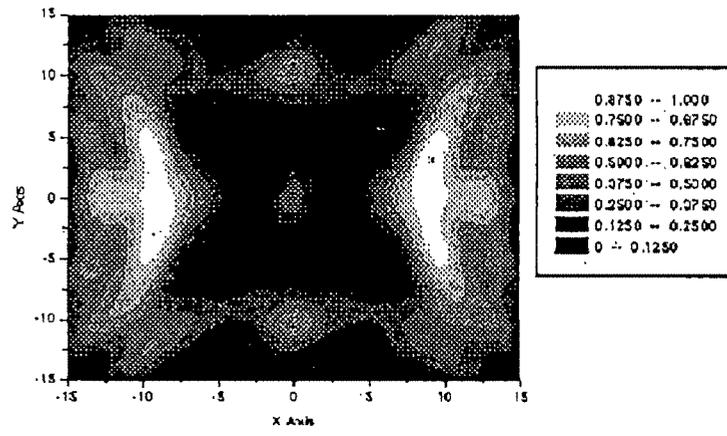


Fig 11