PLANE SYMMETRIC SUPERGRAVITY SOLUTIONS

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Received 28 February 1981
Revised manuscript received 30 April 1982

Plane symmetric supergravity solutions given in a recent paper are generalised.

In a recent paper Dereli and Tucker [1] have shown that for a plane symmetric metric

$$ds^2 = e^{2\mu}[-(dx^0)^2 + (dx^3)^2] + e^{2\nu}[(dx^1)^2 + (dx^2)^2], \tag{1}$$

and an ansatz

$$T = 16\pi G \chi \wedge \chi^\dagger,$$

$$\chi = e^{-\mu/2} f(v)\, d\bar{z}\, \phi_1\, 2^{-1/2}(1 - i\hat{e}_3), \tag{2}$$

where

$$u = 2^{-1/2}(x^0 + x^3), \quad v = 2^{-1/2}(x^3 - x^0),$$

$$z = 2^{-1/2}(x^1 + ix^2), \quad \bar{z} = 2^{-1/2}(x^1 - ix^2). \tag{3}$$

T is the torsion, G is Newton's gravitational constant, χ is a Majorana spinor valued one-form, f is an arbitrary complex function of v and ϕ_1, is a complex odd Grassman constant, the field equations for supergravity for spin 3/2 gravitino reduce to

$$\mu_{uv} + \nu_{uv} + \nu_u \nu_v = 0, \tag{4a}$$

$$\nu_{uu} + \nu_u^2 - 2\mu_u \nu_u = 0, \tag{4b}$$

$$\nu_{uv} + 2\nu_u \nu_v = 0, \tag{4c}$$

$$\nu_{vv} + \nu_v^2 - 2\mu_v \nu_v = e^{-2\nu}\lambda(v), \tag{4d}$$

$$\lambda(v) = \left[\frac{\bar{f}(v)}{|f(v)|}\frac{d}{dv}\left(-i\frac{f(v)}{|f(v)|}\right)\right]\frac{|f|^2}{\sqrt{2}}\phi_1\phi_1^*. \tag{4e}$$

Some particular solutions of (4) have been given by Dereli and Tucker [1]. Present note gives the complete solution as follows.

(4c) can be rewritten as

$$(e^{2\nu}\nu_u)_v = 0,$$

i.e.

$$(e^{2\nu})_{uv} = 0,$$

i.e.

$$e^{2\nu} = U(u) + V(v), \tag{5}$$

where $U(u)$ and $V(v)$ are arbitrary functions of u and v, respectively. Putting (5) into (4b)

$$\mu = -\tfrac{1}{4}\ln[U(u) + V(v)] + Y(v) + \tfrac{1}{2}\ln(U_u V_v), \tag{6}$$

where $Y(v)$ is an arbitrary function of v.

Putting (5) and (6) into (4d) one gets after a little calculation

$$(dV/dv)\, dY(v)/dv = -\lambda(v). \tag{7}$$

It is now easy to see that (5), (6) and (7) together satisfies eqs. (4a)–(4d).

Hence the complete solution of (4) is given as

$$\nu = \tfrac{1}{2}\ln[U(u) + V(v)],$$

$$\mu = -\tfrac{1}{4}\ln[U(u) + V(v)] + Y(v) + \tfrac{1}{2}\ln(U_u V_v),$$

$$\lambda(v) = \left[\frac{d}{dv}\left(-\frac{i f(v)}{|f(v)|}\right)\right]\frac{|f|^2}{\sqrt{2}}(\phi_1\phi_1^*)\frac{f(v)}{|f(v)|}, \tag{8}$$

$$Y(v) = -\int \frac{\lambda(v)}{V_v}\, dv,$$

where $U(u)$ and $V(v)$ are arbitrary functions, which gives a spin $\frac{3}{2}$ gravitino supergravity solution for a plane symmetric metric [1].

[1] T. Dereli and R.W. Tucker, Phys. Lett. 97B (1980) 396.

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