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ABSTRACT

In a fluid approach, nonlinear evolution of electrostatic lower hybrid modes is studied in a cold magnetized electron-ion plasma. The background magnetic field is assumed to be constant. In the frequency range of interest $\Omega_{ci} \ll \omega \ll \Omega_{ce}$, the massive ions are treated as unmagnetized, and the electron inertia in the x-component of the momentum equation is neglected. The quasineutral plasma approximation is also relaxed. The dispersion relation for such low frequency modes reads as $\omega^2 = \omega_{pi}^2/(1 + \omega_{pe}^2/\Omega_{ce}^2)$. Spatiotemporal evolution of such modes is analyzed by employing a simple perturbation technique. Our results show that an initially excited lower hybrid mode gradually loses its coherent nature due to phase mixing and eventually breaks even at an arbitrarily low amplitude. An estimate of the phase mixing time is also given, and it is found to increase as the strength of the magnetic field is enhanced. These results will be of relevance to space plasma situations and laboratory experiments.

I. INTRODUCTION

Phase mixing[1] is an important physical process through which the neighboring oscillators constituting a plasma wave undergo a gradual loss of phase coherence, thus leading to breaking of the wave at a finite time, even if the amplitude of the wave is well below the breaking amplitude.[2–6] Phase mixing, being a collisionless damping mechanism, can transform coherent wave energy into random particle energy, thus resulting in the production of energetic electrons.[7] Furthermore, phase mixing and wave breaking phenomena are directly related to a number of other laboratory applications like plasma heating, particle energization by wake-fields, absorption of intense laser pulses by plasmas, etc.[8–12]

A spatial dependency in the wave frequency generally indicates an onset of phase mixing. An irreversible flow of energy to higher harmonics, bunching of plasma particles in space (appearance of density bursts), large electric field gradient, generation of multistream flow in the medium, etc., are common signatures of phase mixing/wave breaking.[13,14] Over the years, varieties of nonlinear processes arising due to plasma inhomogeneity,[1,15] relativistic mass variation of plasma species,[16,17] ion motion,[18,19] inhomogeneity in the background magnetic field,[20,21] etc., have been recognized as the prime sources of phase mixing. In recent years, phase mixing effects of plasma waves/oscillations have been extensively studied by several authors in different plasma contexts.[22–29]

In this paper, we report phase mixing of lower hybrid oscillations in a cold electron-ion plasma immersed in a "uniform" magnetic field. For describing such low frequency modes, we are interested in the frequency range $\Omega_{ci} \ll \omega \ll \Omega_{ce}$, where $\Omega_{ci(ce)}$ is the cyclotron frequency of ions (electrons). For our purposes, we frame basic two fluids' Maxwell's equations which describe the spatiotemporal evolution of such modes. Nevertheless, we assume that ions remain effectively unmagnetized over a lower hybrid wave period. Moreover, we neglect electron inertia in the x-component of the electron momentum equation.[30,31] A justification of taking these assumptions will be given later. An important aspect of our present problem is that we do not adopt the quasineutrality approximation, unlike previously reported studies where the quasineutrality condition is taken to study an exact coherent dynamics[32] and phase mixing effects (induced by an inhomogeneity in the background magnetic field)[21] of lower hybrid modes of frequency $\omega = \sqrt{\Omega_{ci}\Omega_{ce}}$. In the present work, we show that, when a small deviation from quasineutrality is allowed in the system, an initially excited lower hybrid mode can phase mix away and eventually break, even when the background magnetic field is homogeneous.

It should be stressed here that lower hybrid modes play an important role in the current sheet reconnection process,[33–35] and in heating methods in tokamaks.[36,37] The waves in the lower hybrid frequency range are routinely observed in most parts of the Earth's upper atmosphere like the ionosphere, magnetosheath region, etc. Moreover, plenty of space observations on these waves have also been reported.[38,39] Thus, the results of our investigation are expected to be relevant to laboratory experiments and space plasma environments.

The paper is organized as follows: In Sec. II, the basic fluid-Maxwell's equations are described and a linear mode analysis is carried out. In Sec. III, employing a simple perturbation expansion technique,[40] we have given a nonlinear analysis of our problem, which explains the occurrence of phase mixing of lower hybrid modes at arbitrary amplitudes. In Sec. IV, an estimate for the phase mixing time is presented. In Sec. V, we have summarized our work.

II. BASIC EQUATIONS

We consider a cold collisionless electron-ion plasma immersed in a uniform magnetic field which is directed along the z axis in a Cartesian coordinate system, i.e., $\mathbf{B} = B_0\hat{e}_z$, where $\hat{e}_z$ is the unit vector along the z axis. In a fluid description, the basic equations that describe electrostatic lower hybrid modes in magnetized plasmas are the continuity equations for both species

$$\frac{\partial n_j}{\partial t} + \nabla\cdot(n_j\mathbf{v}_j) = 0, \tag{1}$$

the momentum equations for both species

$$m_jn_j\left(\frac{\partial}{\partial t} + \mathbf{v}_j\cdot\nabla\right)\mathbf{v}_j = q_jn_j\left(\mathbf{E} + \frac{1}{c}\mathbf{v}_j\times\mathbf{B}\right), \tag{2}$$

and the Poisson's equation

$$\nabla\cdot\mathbf{E} = 4\pi\sum_j q_jn_j, \tag{3}$$

where n_j, $\mathbf{v}_j$, m_j, and q_j are the densities, velocities, masses, and charges of either ions or electrons. For ions, the index j equals i and so $q_i = e$. Whereas for electrons, $j = e$ and $q_e = -e$. We assume all the field variables to be functions of a single space coordinate x and time t. Thus, the velocity and wave electric fields can be represented as $\mathbf{v}_j = v_{jx}(x,t)\,\hat{e}_x + v_{jy}(x,t)\,\hat{e}_y$ and $\mathbf{E} = E(x,t)\,\hat{e}_x$, respectively. Here, $\hat{e}_x$ and $\hat{e}_y$ are the unit vectors along the x and y axes, respectively.

Next, we are concerned with the low frequency lower hybrid modes in the frequency range $\Omega_{ci} \ll \omega \ll \Omega_{ce}$, where $\Omega_{ci} = eB_0/m_ic$ and $\Omega_{ce} = eB_0/m_ec$ are the cyclotron frequencies of ions and electrons, respectively. We therefore assume that the massive ions remain effectively unmagnetized in the time scale of lower hybrid modes, and they only respond to the electric field associated with the modes. Unlike ions, lighter electrons feel the magnetic force and thus they experience a number of rotations about the magnetic field lines within one lower hybrid time period. In addition to the Larmor rotation, the $\mathbf{E}\times\mathbf{B}$-drift is also present for the electrons. A linear analysis shows that electrons' displacement is negligibly small along the x direction as compared to their displacement along the y direction. Therefore, in such a low frequency regime, we can safely neglect the electron inertia in the relevant momentum equation along the x direction.[30,31]

Under these circumstances, we rewrite the basic fluid-Maxwell's equations in normalized forms

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_iv_{ix}) = 0, \tag{4}$$

$$\left(\frac{\partial}{\partial t} + v_{ix}\frac{\partial}{\partial x}\right)v_{ix} = \Delta E, \tag{5}$$

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_ev_{ex}) = 0, \tag{6}$$

$$0 = -E - \Omega_cv_{ey}, \tag{7}$$

$$\left(\frac{\partial}{\partial t} + v_{ex}\frac{\partial}{\partial x}\right)v_{ey} = \Omega_cv_{ex}, \tag{8}$$

and

$$\frac{\partial E}{\partial x} = n_i - n_e, \tag{9}$$

where the relevant quantities are normalized as: $n_j \to n_j/n_0$, $x \to kx$, $t \to \omega_{pe}t$, $v_j \to kv_j/\omega_{pe}$, and $E \to ekE/m_e\omega_{pe}^2$. Here, $\omega_{pe} = \sqrt{4\pi n_0e^2/m_e}$ is the electron plasma frequency, $\Delta = m_e/m_i$ is the electron to ion mass ratio, and $\Omega_c = \Omega_{ce}/\omega_{pe}$ is the ratio of electron cyclotron to electron plasma frequency. The constant n_0, appearing in ω_{pe}, is the equilibrium plasma density and 'k' is the scale length of perturbations. Indeed, in order to excite lower hybrid modes, we have considered an initial plasma state prepared by giving initial perturbations to ion and electron densities as $n_i(x,0) = 1 + \delta_i\cos x$ and $n_e(x,0) = 1 + \delta_e\cos x$, respectively, where $\delta_{i(e)}$ describes the perturbation amplitude of ions (electrons).

A linear analysis of Eqs. (4)–(9) provides the dispersion relation for lower hybrid modes in normalized form as

$$\omega^2 = \frac{\Delta\Omega_c^2}{1+\Omega_c^2}. \tag{10}$$

We now discuss two limiting cases of Eq. (10).

(i) In the plasma environments like pulsars where the magnetic field is so strong that the condition $\Omega_c \gg 1$ holds. In this limit, Eq. (10) provides us $\omega^2 \approx \Delta$ which refers to the ion plasma oscillations. And, a linearized treatment of Eqs. (4)–(9) also shows $|v_{ex}| \ll |v_{ix}|$ and $v_{ey} \approx 0$. It suggests that the electrons hardly move in this case, they just provide a charge neutralizing background for the ion oscillations.

(ii) The weak magnetic field limit (for instance, ionosphere) $\Omega_c \ll 1$ gives $\omega^2 \approx \Delta\Omega_c^2$. This mode is nothing but a variant of the lower hybrid mode since its frequency lies within our frequency range of interest. Also, in such a case, one may find $v_{ex} \approx v_{ix}$ and $v_{ey} \neq 0$.

It should be emphasized here that, in the present problem, we have not used the usual "quasineutral plasma approximation," i.e., $n_i \sim n_e \sim n_0$. By taking the quasineutrality condition, a linear analysis of the full set of Eqs. (1)–(3) yields the dispersion relation in the un-normalized form as $\omega = \sqrt{\Omega_{ce}\Omega_{ci}}$. An adoption of such an approximation was shown to be instrumental in obtaining an exact space time dependent coherent wave solution.[32] Whereas, in our present case, a relaxation of quasineutrality condition rules out the possibility of obtaining an exact solution within a Lagrangian fluid approach.[41–43] As stated in the introduction, various physical situations may drive a mode to suffer phase mixing even if the amplitude of initial perturbation is kept arbitrarily small. Thus making use of this fact, in Sec. III, we perform nonlinear analysis of Eqs. (4)–(9) within a perturbative approach.

III. NONLINEAR ANALYSIS

Before employing a perturbation method, we first define the following new variables:

$$\begin{aligned} &\delta n_s = \delta n_i + \delta n_e, \;\; \delta n_d = \delta n_i - \delta n_e, \\ &V_x = v_{ix} + v_{ex}, \quad v_x = v_{ix} - v_{ex}, \end{aligned} \tag{11}$$

where $\delta n_i = n_i - 1$ and $\delta n_e = n_e - 1$. In terms of these new variables, basic equations are converted into the following forms:

$$\frac{\partial}{\partial t}\delta n_s + \frac{\partial}{\partial x}V_x = -\frac{1}{2}\frac{\partial}{\partial x}(V_x\delta n_s + v_x\delta n_d), \tag{12}$$

$$\frac{\partial}{\partial t}\delta n_d + \frac{\partial}{\partial x}v_x = -\frac{1}{2}\frac{\partial}{\partial x}(V_x\delta n_d + v_x\delta n_s), \tag{13}$$

$$\frac{\partial E}{\partial t} + \frac{\Omega_c^2}{2}(V_x - v_x) + \frac{1}{2}(V_x - v_x)\frac{\partial E}{\partial x} = 0, \tag{14}$$

$$\frac{\partial}{\partial t}(V_x + v_x) + \frac{1}{4}\frac{\partial}{\partial x}(V_x + v_x)^2 = 2\Delta E, \tag{15}$$

$$\frac{\partial E}{\partial x} = \delta n_d, \text{ and } v_{ey} = -\frac{E}{\Omega_c}. \tag{16}$$

Moreover, the initial conditions become $\delta n_s(x,0) = (\delta_i + \delta_e)\cos x$, $\delta n_d(x,0) = (\delta_i - \delta_e)\cos x$, $E(x,0) = (\delta_i - \delta_e)\sin x$, $V_x(x,0) = 0$, $v_x(x,0) = 0$. Assuming δ_i and δ_e as small quantities and their order of magnitudes to be nearly equal, the perturbative expansion for the field variables $f(x, t)$ is taken as[40]

$$f(x,t) = f^{(1)} + f^{(2)} + \cdots.$$

Inserting the above field expansion series into Eqs. (12)–(16) followed by solving them subjected to the above-mentioned initial conditions, we find the first order solutions as

$$\begin{aligned} \delta n_s^{(1)} &= b_0\cos x + \frac{a_0}{\omega^2}(\delta_i - \delta_e)\cos x\cos\omega t, \\ \delta n_d^{(1)} &= (\delta_i - \delta_e)\cos x\cos\omega t, \\ E^{(1)} &= (\delta_i - \delta_e)\sin x\cos\omega t, \\ V_x^{(1)} &= \frac{a_0}{\omega}(\delta_i - \delta_e)\sin x\sin\omega t, \\ v_x^{(1)} &= \omega(\delta_i - \delta_e)\sin x\sin\omega t, \\ \text{and} \quad v_{ey}^{(1)} &= -\frac{1}{\Omega_c}(\delta_i - \delta_e)\sin x\cos\omega t, \end{aligned} \tag{17}$$

where the constants a_0 and b_0 are

$$a_0 = \Delta\left(\frac{2 + \Omega_c^2}{1 + \Omega_c^2}\right), \text{ and } b_0 = (\delta_i + \delta_e) + \frac{a_0}{\omega^2}(\delta_e - \delta_i).$$

The first order solutions reflect the presence of the basic normal mode, that is, the lower hybrid mode of frequency ω as given in Eq. (10). This is expected since the first order analysis is nothing but the linear analysis to this problem. Furthermore, the solution of $\delta n_s^{(1)}$ contains a zero frequency DC mode. In our analysis, this zero frequency mode bears an important significance. As we take an average of $\delta n_s^{(1)}$ over a period of lower hybrid oscillation, a nonzero DC term $b_0\cos x$ is produced. An appearance of such a nonzero DC term is regarded as the onset of the phase mixing process.[18]

The first order solutions also fit with two limiting cases discussed earlier. First, if we set $\Omega_c \gg 1$ (strong magnetic field and low density), we recover ion plasma oscillations, as from Eq. (17), we readily obtain $v_{ey}^{(1)} \approx 0$ and $V_x^{(1)} \approx v_x^{(1)} \simeq \sqrt{\Delta}$ which in turn implies that $v_{ex}^{(1)} \approx 0$ and so $v_{ix}^{(1)} \gg v_{ex}^{(1)}$. It shows that electrons do not participate in the dynamics. In the opposite limit, $\Omega_c \ll 1$ (weak magnetic field and high density), a variant of the lower hybrid mode is recovered. In such a situation, we find $v_{ey}^{(1)} \neq 0$ and $V_x^{(1)} \sim 2\sqrt{\Delta}/\Omega_c$ and $v_x^{(1)} \sim \sqrt{\Delta}\,\Omega_c \approx 0$, showing $v_{ix}^{(1)} \approx v_{ex}^{(1)}$.

Next, extracting the set of second order equations from Eqs. (12)–(16) followed by inserting the first order solutions into them and letting $\delta = \delta_i - \delta_e$, we obtain the following second order solutions:

$$\begin{aligned} \delta n_s^{(2)} &= \cos 2x\left[d_1(1-\cos\omega t) + d_2(1-\cos 2\omega t) - \frac{2M_3}{\omega}t\sin\omega t\right], \\ \delta n_d^{(2)} &= \cos 2x\left(\frac{\delta^2\Delta}{2\omega^2} + B\cos\omega t + b_1 t\sin\omega t + b_2\cos 2\omega t\right), \\ E^{(2)} &= \frac{1}{2}\sin 2x\left(\frac{\delta^2\Delta}{2\omega^2} + B\cos\omega t + b_1 t\sin\omega t + b_2\cos 2\omega t\right), \\ V_x^{(2)} &= \sin 2x(M_1\sin\omega t + M_2\sin 2\omega t + M_3 t\cos\omega t), \\ v_x^{(2)} &= \sin 2x(K_1\sin\omega t + K_2\sin 2\omega t + K_3 t\cos\omega t), \\ \text{and} \quad v_{ey}^{(2)} &= -\frac{1}{2\Omega_c}\sin 2x\left(\frac{\delta^2\Delta}{2\omega^2} + B\cos\omega t + b_1 t\sin\omega t + b_2\cos 2\omega t\right), \end{aligned} \tag{18}$$

where

$$\begin{aligned} d_1 &= \frac{1}{\omega}\left(-2M_1 - \frac{a_0 b_0\delta}{2\omega} + \frac{2M_3}{\omega}\right), \\ d_2 &= \frac{1}{2\omega}\left[-2M_2 - \frac{1}{4}\left(\omega + \frac{a_0^2}{\omega^3}\right)\delta^2\right], \quad b_1 = -\frac{\delta b_0\omega^3}{4\Delta}, \\ b_2 &= \frac{\delta^2}{3\Delta}\left(a_0 + \frac{\omega^2}{\Omega_c^4} + \frac{\Delta^2}{2\omega^2}\right), \quad B = -\left(b_2 + \frac{\delta^2\Delta}{2\omega^2}\right), \\ M_1 &= \frac{1}{2\omega}\left(\Delta B + \frac{\omega^2 B}{\Omega_c^2} - \frac{2b_1\omega}{\Omega_c^2}\right) + \frac{b_1}{2\omega^2}\left(\Delta + \frac{\omega^2}{\Omega_c^2}\right), \\ M_2 &= \frac{1}{4\omega}\left(\Delta b_2 + \frac{4\omega^2 b_2}{\Omega_c^2} + \frac{\delta^2\Delta^2}{2\omega^2} - \frac{\omega^2\delta^2}{\Omega_c^4}\right), \\ M_3 &= -\frac{b_1}{2\omega}\left(\Delta + \frac{\omega^2}{\Omega_c^2}\right), \quad K_1 = \frac{1}{2}\left(B\omega - b_1 - \frac{1}{2}b_0\delta\omega\right), \\ K_2 &= \frac{1}{2}\left(2b_2\omega - \frac{\delta^2 a_0}{2\omega}\right), \quad \text{and} \quad K_3 = -\frac{1}{2}b_1\omega. \end{aligned}$$

The signature of phase-mixing is more vivid in the second order solutions. They contain DC terms along with terms like $t\sin\omega t$ and $t\cos\omega t$. An appearance of higher harmonics is also noticeable.

Next proceeding to the third order, we have found a term like $\sim A(x)t^2$ contained within the solution of $\delta n_s^{(3)}$, where the function $A(x)$ is made of spatial harmonics. Such a secular term in $\delta n_s^{(3)}$ arises due to the presence of DC terms in the first and second order solutions.[19,25] The appearance of this secular term in $\delta n_s^{(3)}$ signifies that the plasma species start to bunch up in space as time passes. It patently manifests the occurrence of phase-mixing. The reason behind the bunching of particles can be explained from our obtained solutions. From the solutions of the electric field, if the average of ∇E^2 is calculated over time scale of lower hybrid modes, a nonzero pondermotive force results in ($\nabla E^2 \neq 0$). It rearranges the plasma species in space, thereby making the plasma spatially inhomogeneous. This spatial inhomogeneity contaminates the excited lower hybrid mode so that the characteristic mode frequency becomes space dependent and thus the relevant mode gradually succumbs to phase mixing like an inevitable fate.

IV. PHASE MIXING TIME

In this section, we provide an approximate analytical expression of time the lower hybrid mode takes to get phase-mixed. In the third order, we have found a secular term in the solution of $\delta n_s^{(3)}$ (see the Appendix)

$$\delta n_s^{(3)} \approx \mu(\cos 3x - \cos x)t^2,$$
$$\text{with } \mu = \frac{3}{64}(\delta_i - \delta_e)^2 b_0 \Delta\left[\frac{\Omega_c^4 + (2+\Omega_c^2)^2}{(1+\Omega_c^2)^2}\right]. \tag{19}$$

Now from Eqs. (12)–(16), an approximate equation for δn_d can be constructed which is correct up to the fourth order and it is given as

$$\frac{\partial^2}{\partial t^2}\delta n_d + \omega^2\left(1 + \frac{1}{2}\delta n_s\right)\delta n_d \approx 0. \tag{20}$$

Inserting only the secular term of $\delta n_s^{(3)}$ into Eq. (20) and then solving it subjected to the prescribed initial conditions, we have

$$\delta n_d \approx (\delta_i - \delta_e)\cos x \cos\left[\omega\left(1 + \frac{1}{4}t^2\mu B(x)\right)t\right], \tag{21}$$

where $B(x) = \cos 3x - \cos x$. The distinct signature of phase-mixing is present in the above solution of δn_d as we observe that the mode frequency now depends on space. We can also visualize the whole process through the mode coupling phenomenon. With this aim in mind, we expand the above solution in terms of Bessel's series[44]

$$\delta n_d \approx (\delta_i - \delta_e)\cos x \sum_{\sigma=-\infty}^{+\infty}\sum_{\nu=-\infty}^{+\infty} J_\sigma\left(\frac{\mu\omega t^3}{4}\right) J_\nu \left(\frac{\mu\omega t^3}{4}\right)\cos\{\omega t + 3\sigma x + \nu(x+\pi)\}, \tag{22}$$

where J_σ and J_ν are the Bessel's functions of the first kind of order σ and ν, respectively. The above form of δn_d physically signifies that the electrostatic energy which was initially given to a primary mode flows irreversibly to its higher harmonics as time goes on. Indeed, amplitudes of higher harmonics grow at the expense of the primary mode and thus the amplitude of the primary mode starts to decay. We can term the moment as the "phase mixing time" when the amplitude of the primary mode decreases appreciably from its initial value, and it is given by

$$t_{\text{mix}} \approx \left(\frac{4}{\mu\omega}\right)^{1/3}, \tag{23}$$

which shows that, for $\delta_i \neq \delta_e$, an initially excited lower hybrid mode is destined to phase mix at some finite time.

It is to be mentioned here that the lower hybrid frequency is assumed to reside within the frequency range $\Omega_{ci} \ll \omega \ll \Omega_{ce}$. This assumption along the one which requires phase mixing time to be larger than the time period of lower hybrid oscillations puts limit on the parameter Ω_c in different regimes. In the weak magnetic field and high density case, we find $\left(\frac{3}{32}|\delta_i - \delta_e|^3\right)^{1/4} < \Omega_c \ll 1$. In such a regime, we find the dispersion relation $\omega = \sqrt{\Delta}\,\Omega_c$, signifying a variant of the lower hybrid mode. On the other extreme situation, that is, in the strong magnetic field and low density limit, the range of Ω_c is found to be $1 \ll \Omega_c \ll \frac{1}{\sqrt{\Delta}}$. In this regime, we have the dispersion relation $\omega = \sqrt{\Delta}$ for ion plasma oscillations. In addition, there is an intermediate regime ($\Omega_c \sim 1$) corresponding to lower hybrid modes having the dispersion relation as given in Eq. (10).

By setting typical values of perturbation amplitudes $\delta_e = 0.2$ and $\delta_i = 0.1$ followed by considering a proper range of Ω_c in the weak magnetic field and high density regime, in Fig. 1, we have plotted the variation of normalized phase mixing time $\omega_{pe}t_{mix}$ with Ω_c. It is clearly seen that the phase mixing time increases with the increase in the strength of the magnetic field. Likewise, Fig. 2 shows the scaling of phase mixing time for $\delta_e = 0.2$ and $\delta_i = 0.1$ corresponding to the intermediate regime. From this figure, we also find that the phase mixing time increases with Ω_c. However, the phase mixing process of the excited lower hybrid modes in the weak magnetic field and high density regime is seen

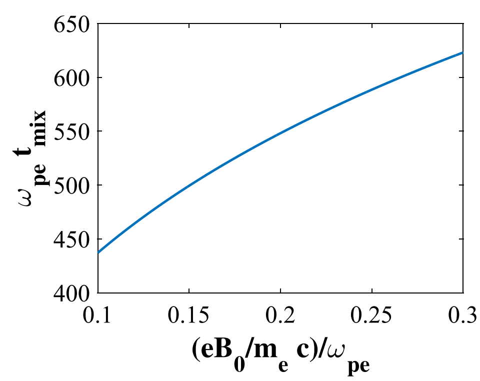

FIG. 1. Variation of normalized phase mixing time ($\omega_{pe}t_{mix}$) with the ratio of electron cyclotron to electron plasma frequency ($\Omega_c = \Omega_{ce}/\omega_{pe}$) in a weak magnetic field and a high density regime, with $\delta_e = 0.2$, $\delta_i = 0.1$, and $\Delta = 1/1836$.

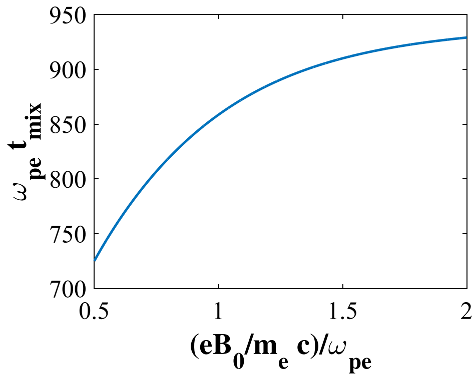

FIG. 2. Variation of normalized phase mixing time ($\omega_{pe}t_{mix}$) with the ratio of electron cyclotron to electron plasma frequency ($\Omega_c = \Omega_{ce}/\omega_{pe}$) in an intermediate regime, with $\delta_e = 0.2$, $\delta_i = 0.1$, and $\Delta = 1/1836$.

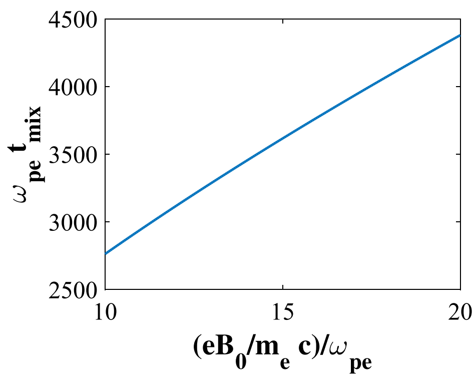

FIG. 3. Normalized phase mixing time $\omega_{pe}t_{mix}$ vs the ratio of electron cyclotron to electron plasma frequency $\Omega_c = \Omega_{ce}/\omega_{pe}$ in a strong magnetic field and a low density regime, with $\delta_e = 0$, $\delta_i = 0.1$, and $\Delta = 1/1836$. For large values of Ω_c, we recover coherent ion plasma oscillations.

to occur relatively quickly in comparison with that in the intermediate regime. In the strong magnetic field and low density regime, Fig. 3 displays the variation of $\omega_{pe}t_{mix}$ with Ω_c for $\delta_e = 0$ and $\delta_i = 0.1$. It is seen from this figure that phase mixing time increases monotonically with Ω_c. Indeed, in an idealized picture, if we set $\delta_e = 0$, one can check that for $\Omega_c \gg 1$, $b_0 \to 0$ implying $t_{mix} \to \infty$. This is expected because in this limit, we recover pure ion plasma oscillations $\left(\omega \sim \sqrt{\Delta}\right)$ where electrons do not take part in wave dynamics and the relevant mode does not phase mix quite naturally.

Admittedly, we have provided an "approximate" expression for phase mixing time through a perturbative analysis carried up to the third order. Nevertheless, at the higher order of approximation, a further improved scaling of phase mixing time may be expected and range of the parameter value may get modified accordingly.

V. SUMMARY

In summary, we have thoroughly surveyed how excited lower hybrid modes evolve with time. We have employed a fluid model to explore the underlying physical process analytically. Our nonlinear analysis shows that, when the plasma deviates marginally from its quasineutral state, an initially excited lower hybrid mode always breaks via the phase mixing phenomenon even if the value of perturbation amplitude is kept arbitrarily low. Physically, phase mixing happens due to the generation of nonzero pondermotive force which relocates electrons and ions in space so that plasma becomes spatially inhomogeneous. This in turn makes the characteristic mode frequency space dependent. An approximate expression for the phase mixing time is provided through a perurbative inspection. It is found that phase mixing time increases as the strength of the background magnetic field is increased. We note here that inclusion of other physical effects such as finite temperature,[45] collision[46,47] may further cause an increase in the phase mixing time. On the other hand, an additional spatial dependency on mode frequency may arise in relativistic[48]and inhomogeneous[49] electron-ion plasmas, thereby causing quicker mixing of phases of excited oscillations. Our investigation could also be extended to include a third particle species (positron) response[50,51] and kinetic effects.[52]

At this concluding point, we would like to add a few remarks. In this present paper, our investigation is completely focussed on a particular frequency range, namely, $\Omega_{ci} \ll \omega \ll \Omega_{ce}$. We have ignored the magnetic force on the ions and neglected the electron inertia in the x-component of electron momentum equation. If we incorporate these two factors, the linear dispersion relation reflects two normal modes which are the variants of the high frequency upper hybrid mode and the low frequency lower hybrid mode. In such a situation, we speculate lower hybrid modes to phase mix more quickly. Moreover, a nonlinear interaction between these high and low frequency modes may arise. The investigation on the phase mixing effects of these modes in magnetized plasmas is currently in progress and will be reported elsewhere.

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APPENDIX: THIRD ORDER SOLUTIONS

The third order solutions of $\delta n_s^{(3)}$ and $\delta n_d^{(3)}$ are obtained as

$$\begin{aligned}\delta n_s^{(3)} &= \mu(\cos 3x - \cos x)t^2 + P_1(x)t\sin\omega t + P_2(x)(1-\cos\omega t)\\ &\quad + P_3(x)(1-\cos 2\omega t) + P_4(x)(1-\cos 3\omega t)\\ &\quad + P_5(x)t\sin 2\omega t,\\ \delta n_d^{(3)} &= Q_0\sin\omega t + Q_1(x)t\sin\omega t + Q_2(x)(1-\cos 2\omega t)\\ &\quad + Q_3(x)\sin 2\omega t + Q_4(x)(1-\cos\omega t) + Q_5(x)\sin 3\omega t\\ &\quad + Q_6(x)(1-\cos 3\omega t) + Q_7(x)t\cos 2\omega t + Q_8(x)t\sin 2\omega t,\end{aligned}$$

where

$$Q_1(x) = -\frac{\omega\delta}{2\Delta} f_1(x)\left[\frac{\delta\Delta a_0 b_1}{2\omega^3} + \omega^2(d_1 + 2d_2) - 2b_2a_0 - (K_1 + M_1) - \frac{b_1}{2\Omega_c^2}\right]$$
$$-\frac{\omega\delta}{2\Delta} f_2(x)\left[\frac{b_0}{\delta}(\omega K_1 + K_3) + \omega(2M_2 - K_2) + \frac{\delta^2\Delta(a_0 - \omega)}{2\omega^2\Omega_c^2}\right],$$
$$Q_2(x) = -\frac{1}{6}\delta f_1(x)\left[(B + 2b_2)a_0 - 2\omega^2(d_1 - 3d_2) + \frac{b_1 a_0}{\omega} - 2M_3\right]$$
$$-\frac{1}{6}\delta f_2(x)\left[M_1 + \frac{b_0K_1}{\delta} - \frac{2b_0\omega K_2}{\delta} + \frac{M_3}{2\omega} + \frac{2\Omega_c^2}{\omega}(2K_2 - K_3)\right],$$
$$Q_3(x) = \frac{1}{\omega\Delta}\delta\left[f_1(x)\left(b_1a_0 - 2M_3\omega - \frac{b_1}{2\Omega_c^2}\right) + f_2(x)\left(-\frac{b_0K_3}{\omega\delta} + \frac{2\Delta}{\omega^2}(M_3 - K_3)\right)\right],$$
$$Q_4(x) = \frac{\delta}{4\omega^2} f_1(x)\left(2b_2a_0 - d_2\omega^2 + \frac{b_2\omega}{2\Omega_c^2}\right) + \frac{\delta}{4\omega^2} f_2(x)\left[2a_0K_2\omega - \frac{\omega(M_2 - 4K_2)}{5\Omega_c^2} + \frac{b_0\omega(4M_1 - K_1)}{\delta\Omega_c^2}\right],$$
$$Q_5(x) = \frac{\omega^2}{4\Delta}\delta\left[f_1(x)\left(2d_2\omega^2 - \frac{b_2\omega}{\Omega_c^2}\right) + f_2(x)\left(\frac{\Delta a_0\omega(M_2 - K_2)}{8\Omega_c^2}\right)\right],$$
$$Q_6(x) = \frac{1}{9\omega^2}\delta\left[f_1(x)\left(2b_1a_0 - d_2\omega^2 + \frac{b_2\omega}{8\Omega_c^2}\right) + f_2(x)\left(\frac{\omega^2(4M_2 - K_2)}{8\Omega_c^4}\right)\right],$$
$$Q_7(x) = \frac{1}{4\omega}\delta\left[f_1(x)\left(4K_3 + d_1a_0\omega - \frac{M_3}{8}\right) + f_2(x)\left(M_3 - \frac{3a_0(M_3 - 4K_3)}{\delta\Omega_c^2}\right)\right],$$
$$Q_8(x) = \frac{\omega}{2\Delta}\delta\left[f_1(x)\{2(d_1 + 3d_2)a_0 - 4M_3\omega\} + f_2(x)\left(\frac{2(M_3 + K_3)}{\Delta\Omega_c^2} - \frac{b_0K_3\omega}{5\delta}\right)\right],$$
$$Q_0 = -\frac{1}{\omega}[2Q_3 + 3Q_5 + Q_7], \quad P_1(x) = -\frac{4Q_1}{\omega} + \frac{\Delta}{2\omega^2}Q_0 - \frac{5\Delta a_0M_3}{9\omega^2}f_1(x) + \frac{4\delta b_2\Delta}{\omega^2}f_2(x),$$

$$P_2(x) = \frac{1}{2\omega}\left(-2\Delta Q_1 + \frac{b_1\delta\omega^2}{8\Omega_c^4}f_1(x) + \frac{\delta b_2a_0}{2\omega}f_2(x)\right) + \frac{4b_0M_1}{3\omega^2}(f_1(x) + 2f_2(x)),$$
$$P_3(x) = \frac{1}{2\omega}\left[-2\left(\Delta + \frac{4\omega^2}{\Omega_c^2}\right)Q_2 - \frac{3\delta}{8}\left(\omega K_3 + \frac{a_0M_3}{8\omega^3}f_1(x)\right)\right] + \frac{\delta^3\Delta}{2\omega^2}f_2(x),$$
$$P_4(x) = -\frac{1}{2\omega}\left(2a_0\Delta + \frac{2\omega^2}{\Omega_c^2}\right)Q_6 - \frac{\delta a_0d_2}{2\omega}f_1(x) + \frac{\Omega_c^2\delta B}{9\omega^4}f_2(x),$$
$$P_5(x) = -\frac{\Delta}{2\omega^2}\left(2Q_8 - \frac{Q_7\omega^2}{1 + \Omega_c^2}\right) - \frac{4\delta a_0(B + 2K_3 - K1)}{\Omega_c^2}f_1(x) + \frac{\delta\Delta(b_2 - 2M_2 - 4M_1)}{2\omega^2}f_2(x),$$

with

$$f_1(x) = \cos x\cos 2x - 2\sin x\sin 2x, \quad \text{and}$$
$$f_2(x) = 2\cos x\cos 2x - \sin x\sin 2x.$$

Other relevant quantities in the third order can be obtained from the solutions of $\delta n_s^{(3)}$ and $\delta n_d^{(3)}$.

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