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Onset of turbulence induced by electron nonthermality in a complex plasma in presence of positively charged dust grains

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In this paper onset of turbulence has been detected from the study of non linear dust acoustic wave propagation in a complex plasma considering electrons nonthermal and equilibrium dust charge positive. Dust grains are charged by secondary electron emission process. Our analysis shows that increase in electron nonthermality makes the grain charging process faster by reducing the magnitude of the nonadiabaticity induced pseudo viscosity. Consequently nature of dust charge variation changes from nonadiabatic to adiabatic one. For further increase of electron nonthermality, this pseudo viscosity becomes negative and hence generates a turbulent grain charging behaviour. This turbulent grain charging phenomenon is exclusively the outcome of this nonlinear study which was not found in linear analysis.
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I. INTRODUCTION

Dusty plasma has become an emerging field of plasma research since last three decades. Most of the space and astrophysical plasmas contain huge charged dust grains. Charging of these dust grains is one of the most important phenomena in physics of dusty plasma. Nanometer to micrometer sized dust grains are charged in dusty plasma by different mechanisms like plasma current, secondary electron emission, photo electron emission etc. If dust grains are charged by the plasma current equilibrium dust charge becomes negative.[1] Positive equilibrium dust charge may exist when some electrons emit from dust surface. In case of dust charging by secondary electron emission process three equilibrium dust charge states exist out of which two are stable and one is unstable. Between these two stable equilibrium dust charge states one is positive and the other is negative.[1–3] The positive equilibrium dust charge state exists at high temperature when secondary electron emission from dust surface is frequent and dominates over the process of collection of electrons and ions by dust grains. Consequently the secondary electron yield which is the ratio of the emitted electrons to incident electrons possesses high value. On the other hand negative equilibrium dust charge state exists at low temperature when secondary electron emission from dust surface is rare and the process of collection of electrons and ions by the dust grains dominates over the secondary electron emission from the dust surface and hence value of the secondary electron yield is low.[3] In this paper we are interested in the grain charging by secondary electron emission process with positive equilibrium dust charge state.

Due to heavy mass of dust grains, dust plasma frequency is very small. The dust charging frequency is high compared to the dust plasma frequency which makes the ratio of the dust plasma

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frequency to the dust charging frequency small. The dust charging frequency is directly proportional to the dust radius and electron number density but inversely proportional to the square root of the electron temperature which have been found from our expression of dust charging frequency in the present text. Thus dust charging process is slow in a high temperature low density plasma containing small dust grains and fast in a low temperature high density plasma containing larger dust grains. In a slow process, dust charging frequency is low enough (but higher than dust plasma frequency) and hence ratio of the dust plasma frequency to the dust charging frequency is a small nonzero finite number. Such a dust charging process is called non adiabatic process[4,5] This is a possible phenomenon in solar corona due to very high coronal temperature of the order of 10^6 K.[28,29] This process induces a viscous like effect or pseudo viscosity in dusty plasma which produces dust acoustic or dust ion acoustic shock waves. On the other hand if the grain charging process is fast enough, the dust charging frequency becomes very high. The ratio of the dust plasma frequency to the dust charging frequency then reduces to zero value. This process is known as adiabatic dust charging process. It is a possible phenomenon in ionosphere because of low ionospheric temperature of the order 10^3 K.[29] In this case the pseudo viscosity disappears and generates dust acoustic or dust ion acoustic soliton instead of shock.[6–10] This pseudo viscosity is a viscous like effect in the dusty plasma medium induced by the nonadiabaticity of dust charge variation. It is not the classical molecular fluid viscosity generated by the friction among the fluid layers but behaves like fluid viscosity. Analogous to the property of fluid viscosity, this pseudo viscosity generates shock wave so long it is positive. Negative viscosity is macroscopic viscosity or eddy viscosity which develops turbulence.[11]

All the wave phenomena mentioned above are outcome of deterministic model. A grain charging process may be so fast the deterministic behaviour may lost and the situation may be turbulent. Onset of this turbulence may be detected from a nonlinear deterministic model when non adiabaticity induced viscosity becomes negative[11] but such deterministic formulation will no longer remain valid once the turbulence is developed.

Study of the effects of adiabatic and nonadiabatic grain charging behaviour on nonlinear wave propagation in dusty plasma is important as they give rise distinct physical situation. It was studied by several authors assuming the dust charging by plasma current and hence equilibrium dust charge negative.[4–10] Charging of dust grains by secondary electron emission process in this study was first reported by us considering negative[12] as well as positive[13] equilibrium dust charges and both electrons and ions thermal. Presence of nonthermal ions for the study of dust acoustic soliton and shock wave propagation in presence of secondary electron emission has also been recently reported.[14] In all these investigations non adiabaticity induced pseudo viscosity was positive. Our objective of this paper is to investigate whether electron nonthermality can reduce the pseudo viscosity to a negative value to make the system turbulent.

The Sun is a giant particle accelerator. During solar flares, magnetic field energy stored in the corona is suddenly released and transferred to local heating of the coronal plasma, mass motions (e.g. jets) and the generation of energetic particles, i.e. electrons, protons and heavy ions. Basically, a flare occurs as a local enhancement of the emission of electromagnetic radiation from the radio up to the γ-ray range on the Sun.

That indicates the production of energetic electrons during flares. NASA's RHESSI mission investigated electron acceleration processes by means of imaging spectroscopy.[25] A substantial part of the energy released during a flare is carried by these energetic electrons. In this way 10^{36} electrons are accelerated up to energies beyond 30 keV which is one of the open questions in solar physics.[26] These accelerated electrons are non-thermal, possessing kinetic energies several times the thermal energy of the surrounding plasma. They interact with the ambient solar atmosphere, releasing part of their energy as bremsstrahlung X-ray photons. The remaining energy contributes to heating of the atmosphere, driving the plasma temperature to tens of millions of degrees – well above the pre-flare values.[27] Though protons and heavy ions are also accelerated by this mechanism their acceleration can be neglected in comparison with the electron's acceleration.

Existence of nonthermal electrons were detected by several satellite observations in space plasmas and solar environment.[15–17] Nonlinear dust acoustic wave propagation in presence of such nonthermal electrons were studied by several authors considering the grain charging by plasma current.[18–22]

No one considered the presence of secondary electrons in their study. Not also they obtained any turbulent effect. If in presence of nonthermal electrons dust grains are charged by secondary electron emission process, due to nonthermality of incident electrons emitted secondaries may also be nonthermal. Thus inclusion of electron nonthermality is important in the study of nonlinear dust acoustic wave propagation when dust grains are charged by secondary electron emission mechanism. We have considered this effect in our present investigation for nonadiabatic dust charge variation assuming the presence of inertia less nonthermal primary and secondary electrons, thermal ions and positive equilibrium dust charge generated by frequent secondary electron emission from dust grains.

Present analysis shows that for both weak and strong nonadiabaticity increasing grain charge number reduces the pseudo viscosity from positive to zero value to make the process adiabatic. Moreover magnitude of this positive pseudo viscosity is lower at higher electron nonthermality. This implies increasing electron nonthermality reduces nonadiabaticity of dust charging behaviour and consequently dust charging becomes faster. For further increase in electron nonthermality, this pseudo viscosity becomes negative at both weak and strong nonadiabaticity. This negative pseudo viscosity implies that the system has become turbulent.[11] This interesting result is the outcome of the nonlinear theory **of** dust acoustic wave propagation obtained by considering electron nonthermality in presence of positive equilibrium dust charge generated by secondary electron emission process. Its linear theory was studied earlier showing the existence of purely growing dust acoustic mode but turbulence was not found there.[23] Our investigation also shows that at weak nonadiabaticity increasing electron nonthermality increases oscillation of oscillatory dust acoustic shock and at strong nonadiabaticity it reduces monotonicity of monotonic dust acoustic shock. This increase in oscillation or decrease in monotonicity is very fast even at very small increase in electron nonthermality. This also indicates the onset of turbulence.

II. FORMULATION OF THE PROBLEM

Dusty plasma under our consideration consists of nonthermal primary and secondary electrons, thermal ions and positively charged dust grains generated by frequent secondary electron emission from dust surface. Since primary electrons are nonthermal, secondary electrons have also considered nonthermal. We are investigating here the nonlinear behaviour of low frequency dust acoustic waves, so electrons and ions are both considered inertialess, only dust grains are inertial. All of them satisfy the quasineutrality condition,

$$n_{io} + z_{d0} n_{d0} = n_{eo} + n_{so} \tag{1}$$

where n_{io}, n_{eo}, n_{so} and n_{d0} are equilibrium number densities of ions, primary electrons, secondary electrons and dust grains respectively and z_{d0} is the number of charges on dust grains in equilibrium. This quasi-neutrality condition implies that the number density ratio $\frac{n_{i0}}{n_{e0}}$ is less than $(1+\frac{n_{s0}}{n_{e0}})$.

The nonthermal electrons satisfies the velocity distribution[24]

$$F_e(v_e) = F_e(v_x, v_y, v_z) = \frac{n_{e0}}{1+3b}\left(\frac{1}{2\pi v_{te}^2}\right)^{3/2}\left[1 + 4b\left(\frac{1}{2}\frac{v_x^2}{v_{te}^2} + \Phi\right)^2\right]\exp\left(-\frac{v_x^2+v_y^2+v_z^2}{2v_{te}^2} - \Phi\right) \tag{2}$$

where $v_{te} = \sqrt{\frac{T_e}{m_e}}$ is the electron thermal velocity, $\Phi = \frac{\mathrm{e}\phi}{\mathrm{T_e}}$ is the normalized plasma potential, T_e is the electron temperature, m_e is the electron mass and v_x, v_y, v_z are x, y, and z components of electron velocity. Here b is the electron nonthermal parameter which measures the deviation of the distribution function of the nonthermal electrons from the distribution function of the thermal electrons. The thermal (Boltzmann) distribution is restored if we put $b = 0$ in (2). This implies for $b \neq 0$ maximum entropy of the system is not reached but for $b = 0$ maximum entropy is reached. This nonthermal distribution function (2) after velocity space integration gives the dimensionless number densities of primary and secondary electrons in the form,

$$N_e = \left[1 + \frac{4b}{1+3b}(-\Phi + \Phi^2)\right]\exp(\Phi) \tag{3}$$

$$N_s = \left[1 + \frac{4b}{1+3b}(-\frac{\Phi}{\sigma_s} + \frac{\Phi^2}{\sigma_s^2})\right] \exp\left(\frac{\Phi}{\sigma_s}\right) \tag{4}$$

The number density of Boltzmann distributed ions being,

$$N_i = \exp\left(-\Phi/\sigma_i\right) \tag{5}$$

where $\sigma_i = \frac{T_i}{T_e}$ and $\sigma_s = \frac{T_s}{T_e}$ are dimensionless temperature ratios, T_i and T_s are ion and secondary electron temperatures respectively.

The dimensionless form of continuity and momentum equations for inertial dust grains are

$$\frac{\partial N_d}{\partial T} + \frac{\partial}{\partial X}(N_d V_d) = 0 \tag{6}$$

$$\frac{\partial V_d}{\partial T} + V_d \frac{\partial V_d}{\partial X} = -\frac{Q_d}{\alpha_d}\frac{\partial \Phi}{\partial X} \tag{7}$$

where $\alpha_d = \frac{(1-\delta_i+\delta_s)}{(1+\frac{\delta_i}{\sigma_i}+\frac{\delta_s}{\sigma_s})}$, $\delta_i = \frac{n_{i0}}{n_{e0}}$ and $\delta_s = \frac{n_{s0}}{n_{e0}}$ are number density ratios and the variable dust charge Q_d obeys the dimensionless grain charging equation[4,5,12–14]

$$(\frac{\omega_{pd}}{\nu_d})(\frac{\partial Q_d}{\partial T} + V_d \frac{\partial Q_d}{\partial X}) = \frac{1}{\nu_d}(\frac{\bar{I}_i + \bar{I}_e + \bar{I}_e^s}{z_{do} e}) \tag{8}$$

Here ν_d is the grain charging frequency, $\bar{I}_i$, $\bar{I}_e$ are the ion and electron current flowing to and $\bar{I}_i^s$ is the secondary electron current flowing out of the dust grain.

The Poisson equation satisfied by the nondimensional electric potential Φ is,

$$\frac{\partial^2 \Phi}{\partial X^2} = -\frac{1}{(1+\frac{\delta_i}{\sigma_i}+\frac{\delta_s}{\sigma_s})}\left[\delta_i \exp\left(-\frac{\Phi}{\sigma_i}\right) - \left\{1 + \frac{4b}{1+3b}(-\Phi+\Phi^2)\right\}\exp(\Phi)\right.$$
$$\left. -\delta_s\left\{1 + \frac{4b}{1+3b}(-\frac{\Phi}{\sigma_s}+\frac{\Phi^2}{\sigma_s^2})\right\}\exp\left(\frac{\Phi}{\sigma_s}\right) + (1-\delta_i+\delta_s)Q_d N_d\right] \tag{9}$$

Here the ion, primary electron, secondary electron, and dust number densities n_i, n_e, n_s and n_d, dust fluid velocity u_d, electrostatic potential energy $e\phi$, dust charge q_d and the independent space variable x and time variable t are nondimensionalized in the following way,

$$N_i = n_i/n_{i0};\quad N_e = n_e/n_{e0};\quad N_s = n_s/n_{s0};\quad N_d = n_d/n_{d0};\quad V_d = u_d/c_d;$$
$$\Phi = \frac{e\phi}{T_e};\quad Q_d = q_d/ez_{d0}X = x/\lambda_d;\quad T = \omega_{pd}t;, \tag{10}$$

where $\omega_{pd} = \left(\frac{4\pi n_{d0} z_{d0}^2 e^2}{m_d}\right)^{1/2}$ is the dust plasma frequency, $c_d = \sqrt{\frac{z_{d0}T_{eff}}{m_d}}$ is the dust acoustic speed, $\lambda_D = \left(\frac{T_{eff}}{4\pi z_{d0} n_{d0} e^2}\right)^{1/2}$ is the dusty plasma Debye length, m_d is the dust mass and z_{d0} is the number of charges on dust grains in equilibrium. The effective temperature T_{eff} is defined by

$$\frac{1}{T_{eff}} = \frac{1}{z_{do} n_{do}}(\frac{n_{io}}{T_i} + \frac{n_{eo}}{T_e} + \frac{n_{so}}{T_s}) \tag{11}$$

The expressions for nonthermal primary electron and secondary electron currents $\bar{I}_e$, $\bar{I}_e^s$ for positively charged dust grains have been calculated from the nonthermal electron velocity distribution (2) in the form,[23]

$$\bar{I_e} = -\pi r_0^2 e\sqrt{\frac{8T_e}{\pi m_e}}\frac{n_{e0}}{1+3b}\left[\left\{(1+\frac{24b}{5}) - 16b\Phi + 4b\Phi^2\right\} + zQ_d(1+\frac{8b}{5} - \frac{8b\Phi}{3} + 4b\Phi^2)\right]\exp(\Phi) \tag{12}$$

$$\bar{I}_e^s = 3.7\delta_M \pi r_0^2 e\sqrt{\frac{8T_e}{\pi m_e}} n_{e0}\left[1 + \frac{4b}{1+3b}(-\Phi+\Phi^2)\right]\exp(\Phi)(1+\frac{zQ_d}{\sigma_s})\exp(zQ_d - \frac{zQ_d}{\sigma_s})F_{5,B'}\left(\frac{E_M}{4T_e}\right) \tag{13}$$

where $z = z_{d0}e^2/r_0T_e$, r_0 is the grain radius and the function $F_{5,B}(x)$ is given by,[1]

$$F_{5,B}(x) = x^2 \int_B^\infty u^5 \exp\left[-(xu^2+u)\right] du, \text{with}\, x = \frac{E_M}{4T_e} \text{ and } B = \sqrt{\frac{\frac{eq_d}{r_0T_e}}{x}} \tag{13a}$$

The current expression for Boltzmann distributed ion flow is,[1]

$$\bar{I}_i = \pi r_0{}^2 e\sqrt{\frac{8T_i}{\pi m_i}} n_{i0} \exp\left(-\frac{\Phi}{\sigma_i}\right) \exp\left(-\frac{zQ_d}{\sigma_i}\right) \tag{14}$$

Here m_i is the ion mass and δ_M is the maximum yield of secondary electrons which occurs when the impinging electrons have the maximum kinetic energy E_M.

The grain charging frequency, which is the rate at which dust grains are charged has been calculated in the form,

$$\nu_d = -\frac{\partial(\bar{I}_i + \bar{I}_e + \bar{I}_e^s)}{\partial Q_d}\Bigg|_{Q_d = z_d e} = \frac{r_0}{\sqrt{2\pi}} \frac{\omega_{pe}^2}{V_{te}} \frac{(5+8b)}{5(1+3b)} \left[1 + \frac{5(1+3b)}{(5+8b)} \frac{\delta_i}{\alpha_{2s}\sqrt{m_e^i \sigma_i}} \exp(-\frac{z}{\sigma_i})\right] \tag{15}$$

$$\text{where } m_e^i = \frac{m_i}{m_e} \text{ and } \alpha_{2s} = 1 - 3.7\delta_M \frac{5(1+3b)}{(5+8b)} \exp(z - \frac{z}{\sigma_s})\{\frac{1}{\sigma_s} + (1+\frac{z}{\sigma_s})(1-\frac{1}{\sigma_s})\} F_{5,B'}\left(\frac{E_M}{4T_e}\right) \tag{15a}$$

From (15) it is clear that ν_d is directly proportional to the grain radius r_0 and electron density n_{e0} whereas inversely proportional to the square root of electron temperature T_e. Thus larger grains at high electron density are quickly charged than the smaller grains at low electron density. Moreover grain charging is frequent at low electron temperature.

Here grain charge number z is not arbitrary. It satisfies the quasineutrality condition (1). The equilibrium current balance equation $\bar{I}_i + \bar{I}_e + \bar{I}_e^s = 0$ gives

$$\delta_i = \frac{n_{i0}}{n_{e0}} = \sqrt{\frac{m_e^i}{\sigma_i}} \left[\frac{(5+24b) + z(5+8b)}{5(1+3b)}\right] \alpha_{1s} \exp(\frac{z}{\sigma_i}), \tag{16}$$

$$\text{with } \alpha_{1s} = 1 - 3.7\delta_M \exp(z - \frac{z}{\sigma_s}) F_{5,B'}\left(\frac{E_M}{4T_e}\right)(1+\frac{z}{\sigma_s})\left[\frac{5(1+3b)}{(5+24b)+z(5+8b)}\right] \tag{16a}$$

Thus to satisfy the quasineutrality condition (1), the normalized grain charge number z(=$z_{d0}e^2/r_0T_e$) must satisfy the condition,

$$\sqrt{\frac{m_e^i}{\sigma_i}} \left[\frac{(5+24b)+z(5+8b)}{5(1+3b)}\right] \alpha_{1s} \exp(\frac{z}{\sigma_i}) < (1 + \frac{n_{so}}{n_{eo}}). \tag{17}$$

III. REDUCTIVE PERTURBATION ANALYSIS

For the study of small amplitude dust acoustic waves in presence of nonthermal primary and secondary electrons, Boltzmann distributed ions and positively charged dust grains, we employ the reductive perturbation technique, using the stretched coordinates $\xi = \varepsilon^{1/2}(X - \lambda T)$ and $\tau = \varepsilon^{3/2}T$ where ε is a small parameter and λ is the wave velocity normalized by c_d. The variables N_d,V_d,Φ and Q_d are then expanded as,

$$\begin{aligned} N_d &= 1 + \varepsilon N_{d1} + \varepsilon^2 N_{d2} + \ldots \\ V_d &= \varepsilon V_{d1} + \varepsilon^2 V_{d2} + \ldots \\ \Phi &= \varepsilon\Phi_1 + \varepsilon^2\Phi_2 + \ldots \\ Q_d &= 1 + \varepsilon Q_{d1} + \varepsilon^2 Q_{d2} + \varepsilon^3 Q_{d3} + \ldots \end{aligned} \tag{18}$$

Here first term in the right hand side of Q_d is 1 as the equilibrium dust charge is positive, i.e. $q_{do} = +z_{do}e$.

Substituting these expansions into equations (3)–(9) with (12)–(14) and collecting the terms of different powers of ε we obtain,

$$\lambda N_{d1} = V_{d1},\ V_{d1} = \frac{\Phi_1}{\lambda\alpha_d},\ \mathrm{N}_{d1} = \frac{\Phi_1}{\lambda^2\alpha_d}, \Phi_1 = \alpha_a(\mathrm{N}_{d1} + \mathrm{Q}_{d1}) \tag{19}$$

$$\frac{\partial N_{d1}}{\partial\tau} - \lambda\frac{\partial N_{d2}}{\partial\xi} + \frac{\partial}{\partial\xi}(N_{d1}V_{d1}) + \frac{\partial V_{d2}}{\partial\xi} = 0 \tag{20}$$

$$\frac{\partial V_{d1}}{\partial\tau} - \lambda\frac{\partial V_{d2}}{\partial\xi} + V_{d1}\frac{\partial V_{d1}}{\partial\xi} = -\frac{1}{\alpha_d}\left(\frac{\partial\Phi_2}{\partial\xi} + Q_{d1}\frac{\partial\Phi_1}{\partial\xi}\right) \tag{21}$$

$$\frac{\partial^2\Phi_1}{\partial\xi^2} = P\Phi_2 - \alpha_d N_{d2} - \alpha_d Q_{d2} - R\Phi_1^2 \tag{22}$$

where $\alpha_a = \dfrac{1-\delta_i+\delta_s}{\left[\left(\frac{1-b}{1+3b}\right) + \left(\frac{1-b}{1+3b}\right)\left(\frac{\delta_s}{\sigma_s}\right) + \frac{\delta_i}{\sigma_i}\right]}$, $P = \dfrac{\left[\frac{\delta_i}{\sigma_i} + \left(\frac{1-b}{1+3b}\right) + \left(\frac{1-b}{1+3b}\right)\left(\frac{\delta_s}{\sigma_s}\right)\right]}{\left[1 + \frac{\delta_i}{\sigma_i} + \frac{\delta_s}{\sigma_s}\right]}$,

$$\mathrm{R} = \left[\frac{1}{\lambda^2}\left(\frac{1}{\alpha_a} - \frac{1}{\alpha_d\lambda^2}\right) + \frac{1}{2}\frac{\left(\frac{\delta_i}{\sigma_i^2} - \frac{\delta_s}{\sigma_s^2} - 1\right)}{\left(\frac{\delta_i}{\sigma_i} + \frac{\delta_s}{\sigma_s} + 1\right)}\right]$$

Since nonadiabatic dust charge variation is a comparatively slow grain charging process, in this case grain charging time is large and grain charging frequency is low. Thus ratio of the dust plasma frequency to dust charging frequency is small but finite. It may be assumed that, $\frac{\omega_{pd}}{\nu_d} = \nu\sqrt{\varepsilon}$ where ν is of order unity[4,5] and ε is a small positive quantity. This gives first and second order dust charge fluctuation

$$Q_{d1} = \beta_d\Phi_1;\ Q_{d2} = \beta_d\Phi_2 + \gamma_d\Phi_1^2 + \frac{\lambda\nu\beta_d}{\sigma_i\beta_a}\left(\delta_i\exp\left(-\frac{z}{\sigma_i}\right) + \alpha_{2s}\sqrt{m_e^i\sigma_i}\frac{(5+8b)}{5(1+3b)}\right)\frac{\partial\Phi_1}{\partial\xi} \tag{23}$$

Eliminating all the second-order terms of equation (20)–(23) we get the KdV-Burger equation,

$$\frac{\partial\Phi_1}{\partial\tau} + A\Phi_1\frac{\partial\Phi_1}{\partial\xi} + B\frac{\partial^3\Phi_1}{\partial\xi^3} = \mu\frac{\partial^2\Phi_1}{\partial\xi^2} \tag{24}$$

where coefficient of nonlinearity $A = B\left[\frac{3P}{\lambda^2\alpha_d} + 2\alpha_d\gamma_d + \frac{\left(\frac{\delta_i}{\sigma_i^2} - \frac{\delta_s}{\sigma_s^2} - 1\right)}{\left(\frac{\delta_i}{\sigma_i} + \frac{\delta_s}{\sigma_s} + 1\right)}\right]$, coefficient of dispersion $B = \frac{\lambda^3}{2} = \frac{1}{2}(P - \alpha_d\beta_d)^{-3/2}$ and the nonadiabaticity induced Burger coefficient

$$\mu = -\frac{\lambda^4\nu\beta_d}{2\sigma_i\beta_a}\left[\exp\left(-\frac{z}{\sigma_i}\right) + \frac{\alpha_{2s}\sqrt{m_e^i\sigma_i}}{\delta_i}\frac{(5+8b)}{5(1+3b)}\right] \tag{25}$$

all of which depend upon the of electron nothermality parameter b and maximum secondary electron yield δ_M. Here $\lambda = \frac{1}{\sqrt{P-\alpha_d\beta_d}}$, provided $P>\alpha_d\beta_d$ is the normalized phase velocity of the dust acoustic wave. The expressions of β_d and γ_d are too long and hence given in Appendix.

If we put $\mu = 0$ in (24) we get the K-dV equation,

$$\frac{\partial\Phi_1}{\partial\tau} + A\Phi_1\frac{\partial\Phi_1}{\partial\xi} + B\frac{\partial^3\Phi_1}{\partial\xi^3} = 0 \tag{26}$$

which is the governing equation of nonlinear dust acoustic waves when dust charge variation is adiabatic and hence is a faster process.[4,5,12–14]

Transformation to the wave frame $\eta = \xi - M\tau$, gives shock solution of equation (24) which have been plotted in figure 4 for weak nonadiabaticity and in figure 5 for strong nonadiabaticity.

The travelling wave solution of equation (26) for adiabatic dust charge variation with the same transformation gives,

$$\Phi_1 = \Phi_{1m} \sec h^2 \left[\frac{(\xi - M\tau)}{w} \right] \tag{27}$$

which represents dust acoustic soliton that propagates with amplitude $\Phi_{1m} = \frac{3M}{A}$ and width $w = 2\sqrt{\frac{B}{M}}$, where M is the Mach number.

The expression of the pseudo viscosity μ in (25) involves β_d whose expression has been provided in the appendix. This pseudo viscosity may be positive, zero or negative depending on the magnitude and sign of β_d. This β_d again depends on the magnitude of electron nonthermal parameter b. Hence μ ultimately depends on b. The grain charging process is nonadiabatic if μ is positive, adiabatic if μ is zero and turbulent if μ is negative. The negative μ corresponds to macroscopic or eddy viscosity which makes the grain charging and consequently the whole system turbulent. After development of turbulence this proposed model will no longer valid. This interesting phenomenon is the outcome of this nonlinear theory which we did not found in our linear analysis.[23]

IV. NUMERICAL ESTIMATION

For numerical estimation we have considered the values $\kappa T_e \approx 2eV$, $\kappa T_s \approx 3eV$, $E_M = 650eV$ and $\delta_M \approx 24$. As proposed by Meyer-Vernet,[1] E_M lies in the range 400 eV-1500 eV and maximum secondary electron yield δ_M of positively charged MgO material dusts lies in the range 22 ~ 24. The range $0.20 < z < 0.25$ of z has been fixed from Figure 1 satisfying the quasineutrality condition (1). Figures 2 and 3 have been plotted for the coefficient of Berger term μ versus z for weak nonadiabaticity(ν=0.5) and strong nonadiabaticity(ν=5) respectively taking b = 0.007, 0.009, 0.02. These two figures show that μ is higher for ν = 5.0 than for ν = 0.5. We have considered these two values of ν because they show two distinct behaviour of dust acoustic shock which is clear from figures 4 and 5. For ν=0.5 (weak nonadiabaticity) we get oscillatory dust acoustic shock (figure 4) which indicates higher dispersion than dissipation whereas for strong nonadiabaticity (ν=5) we get monotonic dust acoustic shock (figure 5) which indicates stronger dissipation than dispersion. The smallness parameter ε of the order of 10^{-6} has been calculated from the expression $\frac{\omega_{pd}}{\nu_d} = \nu\sqrt{\varepsilon}$.

An interesting behaviour is observed from figures 2 and 3. They show that for both ν = 0.5 and ν = 5.0 the pseudo viscosity μ reduces to zero with increasing z for b = 0.007 and 0.009. The same figures also show that magnitude of μ is lower for higher b values but it maintains positive

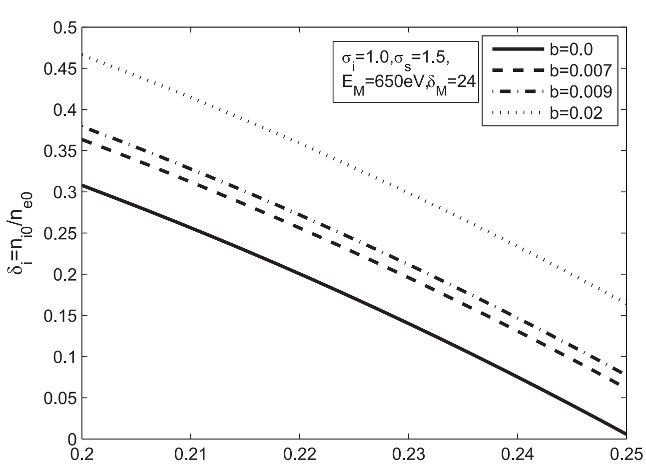

FIG. 1. Plot of $\delta_i(=\frac{n_{i0}}{n_{e0}})$ versus $z\left(=\frac{z_{do}e^2}{r_0 T_e}\right)$ for equilibrium dust charge positive.

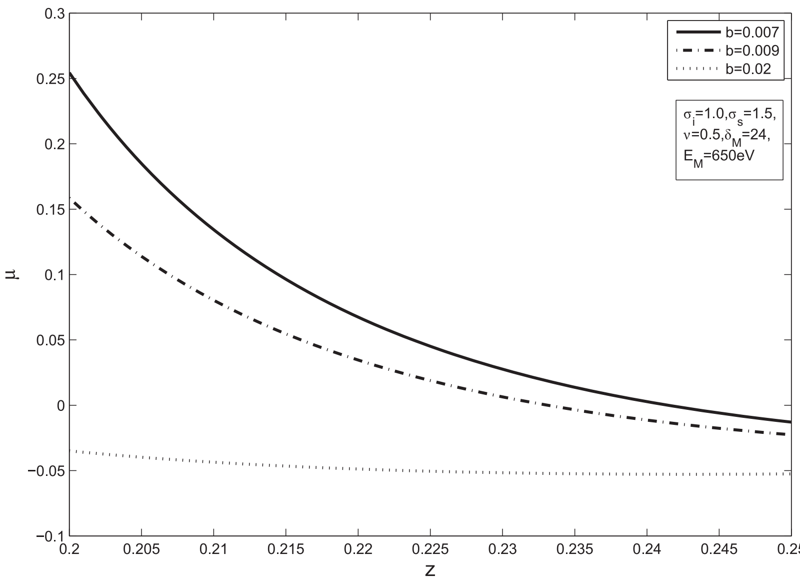

FIG. 2. Plot of the coefficient of Berger's term μ versus z for different b at δ_M = 24 and ν = 0.5 in case of nonadiabatic dust charge variation.

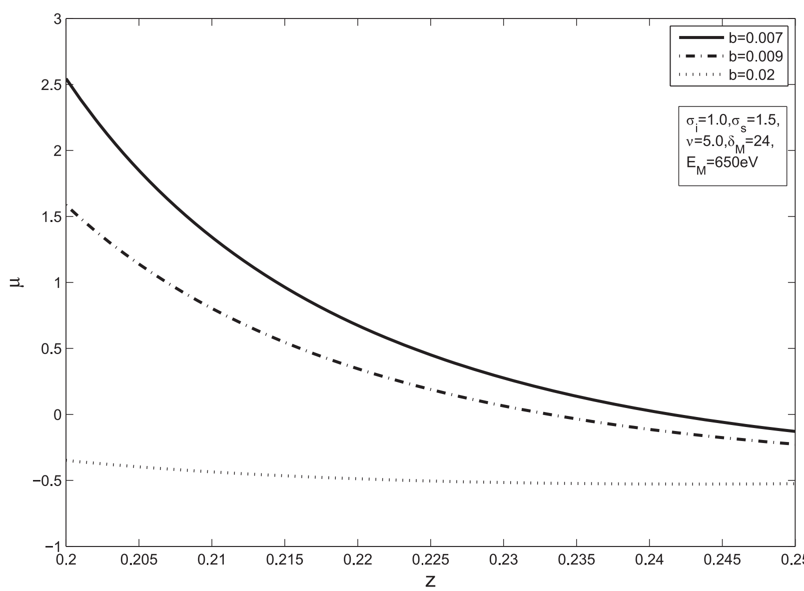

FIG. 3. Plot of the coefficient of Berger's term μ versus z for different b at δ_M=24 and ν=5.0 in case of nonadiabatic dust charge variation.

sign for b=0.007 and 0.009. This μ becomes negative for all z when b = 0.02. This implies as the electron nonthermality increases from b = 0.007, the grain charging process looses nonadiabaticity and transfers from slow to a fast one. Interestingly the grain charging process and consequently the total system becomes turbulent when b = 0.02 as μ is then negative.[11]

This onset of turbulence is also clear from figures 4 and 5 which show that increasing electron nonthermality increases oscillation of dust acoustic shock for weak nonadiabaticity(ν = 0.5) and reduces monotonicity of monotonic dust acoustic shock at strong nonadiabaticity(ν = 5). This increases in oscillation or decrease in monotonicity is very fast even at very small increase in electron nonthermal parameter.

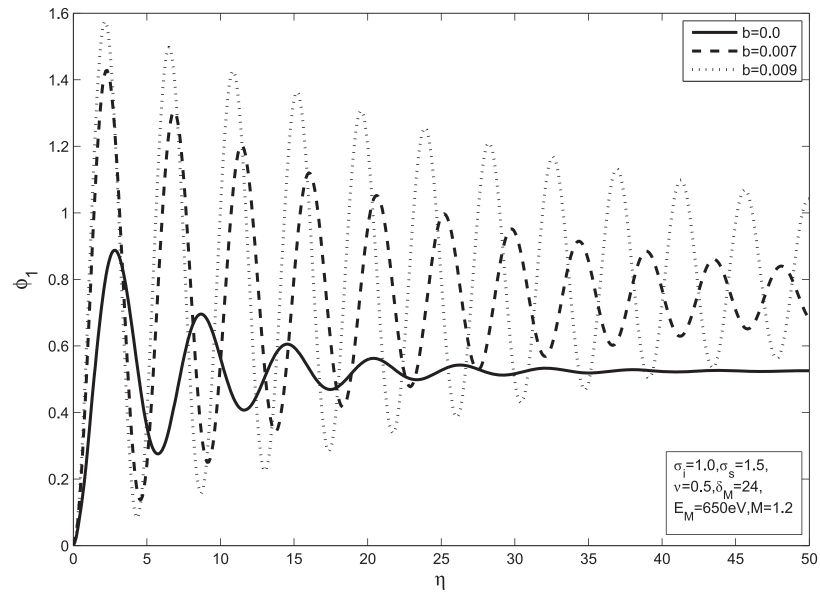

FIG. 4. Oscillatory shock wave for different b at δ_M=24 and ν=0.5 in case of nonadiabatic dust charge variation.

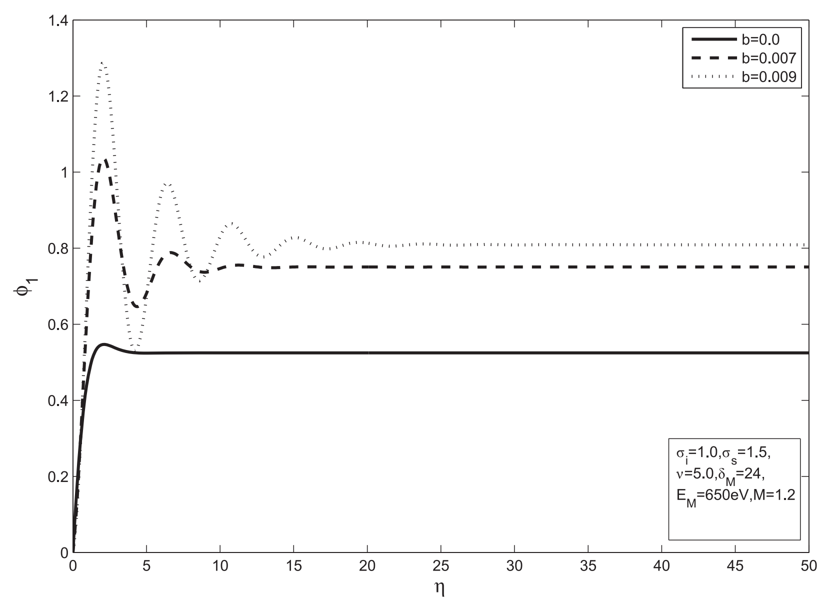

FIG. 5. Monotonic shock wave for different b at δ_M=24 and ν=5 in case of nonadiabatic dust charge variation.

V. CONCLUSION

In this paper we have investigated the grain charging behaviour induced by electron nonthermality on nonlinear dust acoustic wave propagation in a complex plasma when equilibrium dust charge generated by secondary electron emission is positive. We started our investigation considering a slow nonadiabatic dust charge variation which induces a viscous like effect called pseudo viscosity in the dusty plasma medium. Our investigation shows that very small increase in electron nonthermality reduces the magnitude of this pseudo viscosity which becomes negative after crossing zero value. This negative pseudo viscosity develops turbulence. Thus a slow nonadiabatic grain charging process may lead to turbulence even at very weak electron nonthermality $O(10^{-2})$.

APPENDIX

The expressions of β_d and γ_d are calculated in the following form.

$$\beta_d = \frac{\beta_b}{z\beta_a};\ \beta_b = \beta_{b1} + \beta_{b2};$$

$$\beta_a = \frac{z\delta_i}{\sigma_i^2} - \frac{\delta_i}{\sigma_i} - \sqrt{\frac{m_e^i}{\sigma_i}}\left(\frac{5+8b}{5(1+3b)}\right) + 3.7\delta_M F_{5,B'}\sqrt{\frac{m_e^i}{\sigma_i}}\,[\frac{1}{\sigma_s}\exp(z-\frac{z}{\sigma_s}) + (1+z)\{(1-\frac{1}{\sigma_s}) + z(1-\frac{1}{\sigma_s})^2\}];$$

$$\beta_{b1} = \frac{\delta_i}{\sigma_i} - \frac{z\delta_i}{\sigma_i^2} + \sqrt{\frac{m_e^i}{\sigma_i}}\left(\frac{1}{1+3b}\right)\left\{\left(1-\frac{8b}{15}\right) + z\left(1-\frac{16b}{15}\right)\right\}$$

$$\beta_{b2} = -3.7\delta_M F_{5,B'}\sqrt{\frac{m_e^i}{\sigma_i}}(1+\frac{z}{\sigma_s})\left(\frac{1-b}{1+3b}\right)\{1+(z-\frac{z}{\sigma_s})\},$$

$$\text{and } \gamma_d = \frac{\gamma_c}{z\beta_a},\ \gamma_c = (\gamma_{c11}+\gamma_{c12}) + (\gamma_{c21}+\gamma_{c22})(z\beta_d) + (\gamma_{c3})(z\beta_d)^2$$

$$\gamma_{c11} = \frac{1}{2}\left[\sqrt{\frac{m_e^i}{\sigma_i}}\left(\frac{1}{1+3b}\right)\left\{\left(1+\frac{32b}{5}\right) + z\left(1+\frac{64b}{15}\right)\right\} - \frac{\delta_i}{\sigma_i^2}\right];$$

$$\gamma_{c12} = \frac{1}{2}[-3.7\delta_M F_{5,B'}\sqrt{\frac{m_e^i}{\sigma_i}}(1+\frac{z}{\sigma_s})\exp(z-\frac{z}{\sigma_s})]$$

$$\gamma_{c21} = \left[\sqrt{\frac{m_e^i}{\sigma_i}}\left(\frac{1}{1+3b}\right)\left(1-\frac{16b}{5}\right) - \frac{\delta_i}{\sigma_i^2}\right]$$

$$\gamma_{c22} = -3.7\delta_M F_{5,B'}\sqrt{\frac{m_e^i}{\sigma_i}}\left\{\left(\frac{1-b}{1+3b}\right)\frac{1}{\sigma_s}\exp(z-\frac{z}{\sigma_s}) + (1+\frac{z}{\sigma_s})(1-\frac{1}{\sigma_s})\right\}$$

$$\gamma_{c3} = [-\frac{\delta_i}{2\sigma_i^2} - 3.7\delta_M F_{5,B'}\sqrt{\frac{m_e^i}{\sigma_i}}\{\frac{1}{\sigma_s}(1-\frac{1}{\sigma_s}) + (1-\frac{1}{\sigma_s})^2(\frac{1+3z/\sigma_s}{2})\}];$$

All the notations are defined in the main body of the paper.

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