J. Opt. Commun. 18 (1997) 2, 75–78

Journal of
Optical Communications

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Novel Method for Studying Single-mode Fibers involving Chebysev Technique

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Summary

A simple numerical method based on Chebysev technique is proposed to calculate the propagation characteristics of single mode fibers having arbitrary index profiles. The method also formulates a linear relation of $K_1(W)/K_0(W)$ with 1/W. The method predicts the normalised fiber parameters and field functictions in the core and cladding as well for step and parabolic index fibers excellently.

1 Introduction

Propagation characteristics of single-mode fibers which process information through a single guided mode can be predicted if one can prescribe a suitable functional form for the fundamental mode. It should be simple as well for practical applications. From the knowledge of the modal field, one can calculate appropriate spot sizes which are related to bending loss, splice loss, modal dispersion etc [1–3]. However, to know the modal field one has to adopt either variational or rigorous numerical methods which mostly require complicated and involved calculations [4]. The variational method has some limitations in low V region, although it is still receiving attention for approximating the fundamental mode [5, 6].

In the recent past, an approximate numerical method based on Chebysev technique has been suggested [7, 8] to calculate LP_{11} mode cut-off frequency of single-mode fibers having arbitrary index profiles. The method approximates the first higher order-mode in terms of a Chebysev power-series and involves calculation at suitable Chebysev points efficiently and conveniently. This leads us to examine whether the same technique can be applied to investigate the propagation characteristics involving the fundamental mode. In this context, a look at the boundary condition reveals that if the term $WK_1(W)/K_0(W)$ is suitably simplified, the method can work effectively for LP_{01} mode as well. Therefore, it is felt that one should study the variation of $K_1(W)/K_0(W)$ with respect to 1/W; from the data and graph given in [9], it seems plausible that one can prescribe a linear relation of $K_1(W)/K_0(W)$ with 1/W over a range of W values appropriate for a single mode region. Accordingly, a linear relation of $K_1(W)/K_0(W)$ with 1/W is obtained. In this communication. we report a novel method based on Chebysev technique and the above relation an the kingpin in order to study the propagation characteristics of single-mode fibers.

2 Analysis

The refractive index profile in case of a weakly guiding fiber can be written as

$$n^2(R) = n_1^2[1 - 2\delta f(R)], \quad 0 \le R \le 1$$
$$= n_2^2, \quad R > 1 \qquad (1)$$

where R = r/a, a = core radius, $\delta = (n_1^2 - n_2^2)/2n_1^2$, n_1 and n_2 being respectively maximum value of the refractive index of the core and refractive index of the cladding. Here, f(R) defines the shape of the profile and for a graded index fiber it is given by:

$$f(R) = R^q, \quad R \le 1$$
$$= 0, \quad R > 1 \qquad (2)$$

where q is the profile exponent and it is ∞ for step index fiber and 2 for parabolic index fiber.

In order to get the field ψ(R) in the core and normalised propagation constant W, one has to solve the following scalar wave equation:

$$\frac{d^2\psi}{dR^2} + \frac{1}{R}\frac{d\psi}{dR} + \left[V^2(1 - f(R)) - W^2\right]\psi = 0, \quad R \le 1 \qquad (3)$$

together with the boundary condition

$$\left(\frac{1}{\psi}\frac{d\psi}{dR}\right)_{R=1} = -W\frac{K_1(W)}{K_0(W)} \qquad (4)$$

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Received 23 August 1995

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whereas the field in the cladding is given by

$$\psi(R) \sim K_0(WR), \quad R > 1 \tag{5}$$

As stated earlier, we apply the least-square fitting procedure to obtain the following relation for $K_1(W)/K_0(W)$

$$\frac{K_1(W)}{K_0(W)} = 1.034623 + \frac{0.3890323}{W} \tag{6}$$

for the interval $0.6 \le W \le 2.5$, a considerable range pertaining to single-mode region of a fiber having step and graded index profiles. Keeping in mind that $\psi(R)$ is an even function of R and $\psi(0)$ is nonzero, we approximate the fundamental mode $\psi(R)$ in terms of a Chebysev power series as

$$\psi(R) = \sum_{j=0}^{j=3} a_{2j} R^{2j}. \tag{7}$$

As regards choice of three Chebysev points, we shall choose Chebysev points given by [8]:

$$R_m = \mathrm{Cos}\left(\frac{2m-1}{2M-1}\frac{\pi}{2}\right); \quad m = 1,2\ldots(M-1). \tag{8}$$

Here we take Chebysev points given by $R_1 = 0.4338$, $R_2 = 0.7818$ and $R_3 = 0.9749$ corresponding to $M = 4$. These three values of R along with (7) being used in (3) gives three equations

$$\begin{aligned} &a_0\left[V^2(1-f(R_i)) - W^2\right] \\ &+a_2\left[4 + R_i^2\left\{V^2(1-f(R_i)) - W^2\right\}\right] \\ &+a_4\left[16R_i^2 + R_i^4\left\{V^2(1-f(R_i)) - W^2\right\}\right] \\ &+a_6\left[36R_i^4 + R_i^6\left\{V^2(1-f(R_i)) - W^2\right\}\right] = 0 \end{aligned} \tag{9}$$

$i = 1, 2, 3$ implying the three equations.

Using (7) in (4) together with (6) we get

$$\begin{aligned} &a_0[1.034623W + 0.3890323] \\ &+a_2[1.034623W + 2.3890323] \\ &+a_4[1.034623W + 4.3890323] \\ &+a_6[1.034623W + 6.3890323] = 0 \end{aligned} \tag{10}$$

a_0, a_2, a_4, a_6 given by three equations of (9) and (10) will be conformable for solutions if

$$\begin{vmatrix} A_1 & B_1 & C_1 & D_1 \\ A_2 & B_2 & C_2 & D_2 \\ A_3 & B_3 & C_3 & D_3 \\ A_4 & B_4 & C_4 & D_4 \end{vmatrix} = 0 \tag{11}$$

where

$$\begin{aligned} &A_i = V^2(1-f(R_i)) - W^2; \\ &B_i = 4 + R_i^2\left[V^2(1-f(R_i)) - W^2\right]; \\ &C_i = 16R_i^2 + R_i^4\left[V^2(1-f(R_i)) - W^2\right]; \\ &D_i = 36R_i^4 + R_i^6\left[V^2(1-f(R_i)) - W^2\right]; \\ &i = 1,2,3, \end{aligned} \tag{12}$$

and

$$\begin{aligned} &A_4 = 1.034623W + 0.3890323; \\ &B_4 = 2 + A_4; \; C_4 = 4 + A_4; \; D_4 = 6 + A_4. \end{aligned} \tag{13}$$

Now to determine W for a given value of V one has to solve (11). Further, choosing any three of the four equations given by (9) and (10) one can find ψ in terms of a_0.Thus, normalisation of other coefficients in terms of a_0 enables one to get the field both in the core and cladding by Chebysev technique:

$$\begin{aligned} \psi(R) &= 1 + a_2'R^2 + a_4'R^4 + a_6'R^6, \quad R \le 1 \\ &= \frac{1 + a_2' + a_4' + a_6'}{K_0(W_c)} K_0(W_c R), \quad R > 1 \end{aligned} \tag{14}$$

where $a_{2j}' = a_{2j}/a_0$; $j = 1, 2, 3$, and W_c is the value of W obtained by the present Chebysev method.

The above values are to be compared with the analytical fields for step index fiber given by

$$\begin{aligned} \psi(R) &= J_0(UR), \quad R \le 1 \\ &= \frac{J_0(U)}{K_0(W)} K_0(WR), \quad R > 1 \end{aligned} \tag{15}$$

and for graded index fibre with the exact numerical field values. U, V and W are exact analytical or numerical values of the normalised fibre parameters, U being given by $(V^2 - W^2)^{1/2}$.

3 Results and discussions

In order to verify the validity of the proposed Chebysev technique, we first confine our attention to predict the value of normalised fiber parameter W for any particular value of V. In this connection, we choose the step index fiber as the test case. In Table 1 we present the values of W obtained by the present method as Wc and compare it with exact values shown as W [10–12] for different prescribed values of V. It is found that the values of W predicted by the proposed method are extremely accurate within an accuracy of 0.25%. Then we test the accuracy of the method in the evaluation of the modal field. In Figures 1 (a) and (b) the variation of the modal field ψ with normalised radius R are depicted. Figure 1(a) shows $\psi(R)$ as obtained from Chebysev method (solid curve) along with simulated data (as crosses) from equation (15) for V=1.4. Figure 1 (b) represents the same variations for V=2.4, near cut-off. Both the figures show that the Chebysev approximation gives the modal field identically coinciding with the analytical values. Further the method is applied to parabolic index fiber. In Table 2 we present the values of W_c and W for some typical values of V. The method predicts the values of W very accurately within an order of 1.28% [11, 12] In Figures 2(a),(b) and (c) $\psi(R)$ are represented for V = 2.0, 3.0 and 3.5, respectively, in the same way as in the case of step index profile. We see that the method describes accurately within 2% for lower V and 4% for higher V (3.5), near cut-off. It may be relevant to mention that the Chebysev technique [7] predicted the value of LP_{11} mode cut-off frequency for step and parabolic index fibre within an accuracy of the order of 0.17% and 1.17% respectively. In this regard this method seems to be quite consistent with [7] in predicting propagation cha-

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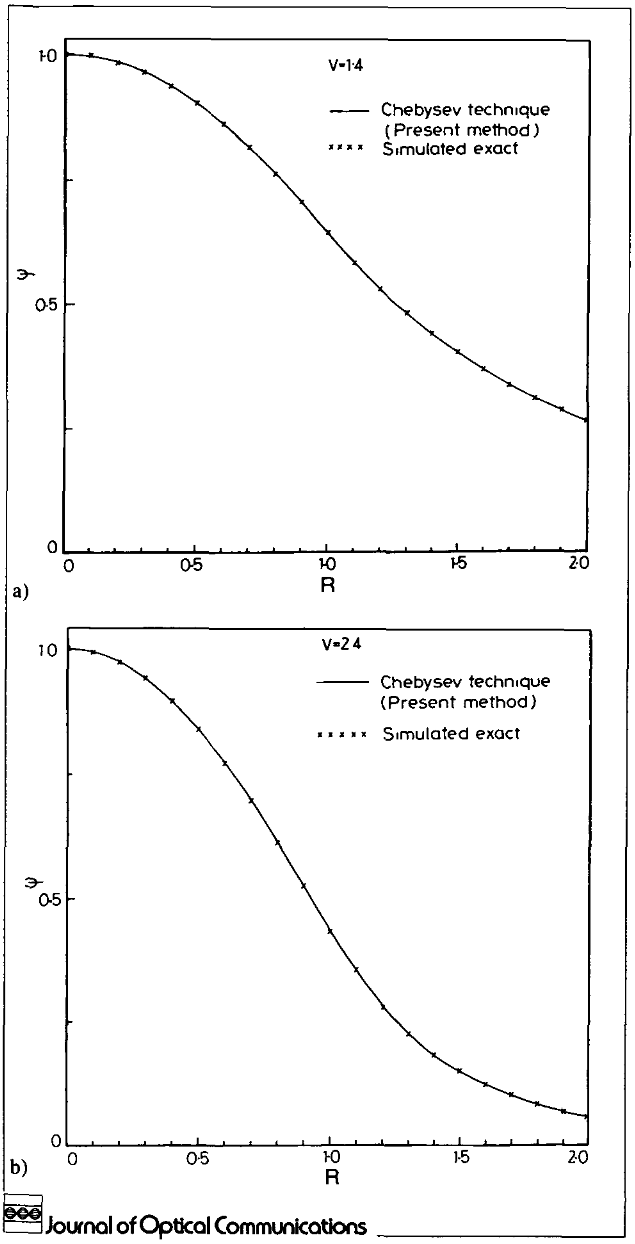

Figure 1 : The fundamental modal field ψ as a function of normalised radius R for a step index fiber with (a) V = 1.4 and (b) V = 2.4

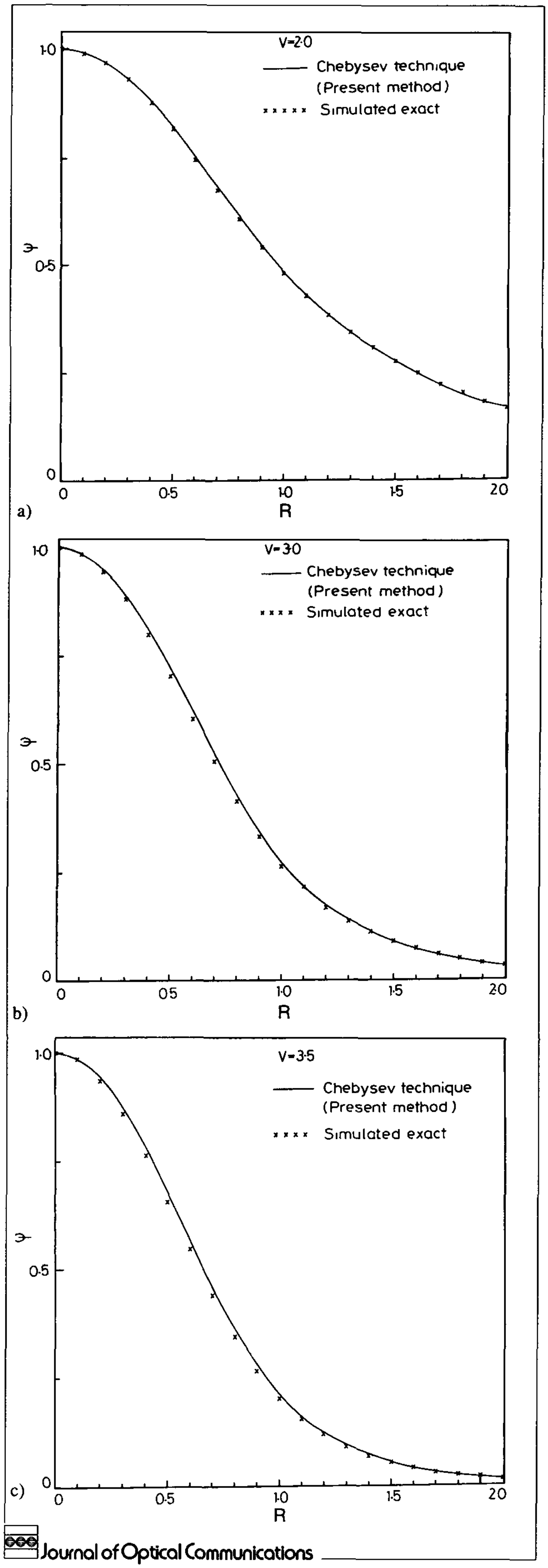

Figure 2 : The fundamental modal field ψ as a function of normalised radius R for a parabolic index fiber with (a) V = 2.0, (b) V = 3.0 and (c) V = 3.5

racteristics of single-mode fibers having step index and parabolic profile. In all the above calculations we take the value of M to be equal to 4. We have also checked that for the sake of simplicity if we take M equal to 3 and retain the first three terms in the expression for ψ in (7), the accuracy in the value of W in step index profile is still seen to be within 0.3%, whereas for parabolic index profile it rises to 7%. Introduction of next higher order term a_6r^6 minimises the error in step index profile and specially for parabolic index profile the error is minimised a lot having its maximum value 1.2%; as the method is first proposed for single-mode graded index fiber, inclusion of more number of terms in ψ is expected to lead to more accuracy for W and ψ with a little more computational effort. However, the proposal of the linear relation of $K_1(W)/K_0(W)$ with 1/W saves more computational effort and gives desired results quite accurately.

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Table 1 : For step index fiber

V	W_c	W	Error
1.4	0.60447	0.60600	0.252%
1.5	0.71837	0.71873	0.050%
2.0	1.29184	1.29031	0.118%
2.4	1.74757	1.74700	0.033%
2.6	1.97331	1.97400	0.035%

Table 2: For parabolic fiber

V	W_c	W	Error
2.0	0.77387	0.77871	0.552%
2.5	1.27778	1.28728	0.738%
3.0	1.78884	1.80767	1.042%
3.5	2.29866	2.32850	1.281%

4 Conclusion

A linear variation of $K_1(W)/K_0(W)$ with respect to $1/W$ is proposed over a long and practical range of W values. With this relation a method based on Chebysev technique is developed to predict accurately fiber parameters and field in the core as well as in the cladding for single-mode fibers having arbitrary index profile. It is also seen that judicious choice of the number of terms in the Chebysev power series is to be made depending on the degree of accuracy required.

5 Acknowledgement

The financial support of Council of Scientific and Industrial Research, India is gratefully acknowledged. Three authors (M. S., S. K. M. and G. D.) are thankful to University Grants Commission, India for award of fellowships.

References

[1] P. Sansonetti : Electron. Lett. 18 (1982) 11, 647–648
[2] C. Pask : Electron. Lett. 20 (1984) 2, 144–145
[3] A. H. Liang : Applied Opt. 30 (1991) 34, 5017–5018
[4] E.-G.Neumann : "Single Mode Fibres Fundamentals"; Vol. 57 (Springer-Verlag,1988)
[5] A. Ankiewicz, G. D. Peng : IEEE J. of Quant. Electron. 27 (1991) 5, 1123–1188
[6] M. J. Holmes, D. M. Spirit , F. P. Payne : J. of Lightwave Tech. 12 (1994) 2, 193–201
[7] J. Shijun : Electron. Lett. 23 (1987) 10, 534–535
[8] P. Y. P. Chen : Electron. Lett. 18 (1982) 24, 1048–1049
[9] M. Abramowitz, I. A. Stegun: "Handbook of Mathematical Functions"; Dover Publications, 1972
[10] A. W. Snyder, J. D. Love: "Optical Wave Guide Theory"; Chapman and Hall,1983
[11] P. K. Mishra et al.: Opt. Acta. 31 (1984) 9, 1041–1044
[12] E. K. Sharma, A. Sharma, I. C. Goyal: IEEE Trans. Micro. Th. Tech. MTT 30 (1982) 10, 1472–1477

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