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Nonsteady dust charge variation induced ion acoustic monotonic shock structure in dusty plasma: Roles of ionization, ion loss, and collision

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The effects of nonsteady dust charge variation, ionization, ion loss, and collision on finite amplitude nonlinear ion acoustic wave are investigated in a complex (dusty) plasma. The dynamics of the nonlinear wave is governed by a Burger equation with linear damping or growth term. The anomalous dissipation originating from the nonsteady dust charge variation is responsible for the Burger term, whereas the other dissipative mechanisms (ionization, ion loss, and collision) are responsible for the linear damping (collision and ion loss are the dominant) or growth term (ionization is the dominant). Analytical approximate time evolution solution reveals that the wave possesses monotonic shock structure with exponentially growing or decaying wave amplitude. Its implications in gas discharge laboratory plasma are discussed. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2363171]

I. INTRODUCTION

A complex (dusty) plasma is a highly dissipative plasma medium consisting of several charged components (electrons, ions, and dust grains) and the neutral component. The laboratory gas discharge weakly ionized complex (dusty) plasma is a strongly nonequilibrium plasma medium, where ionization, recombination, and particle loss (electron-ion loss due to attachment on the dust grains) processes play an essential role to define a stationary state in this plasma medium.[1,2] It was found by Akhiezer *et al.*[3] and Johnson *et al.*[4] that ionization causes instability of ion acoustic wave in a dust free low density gas discharge plasma. Later it was found that in the presence of a significant background pressure of neutrals dust ion acoustic wave (DIAW) (Ref. 5) in the short wavelength limit and dust acoustic wave (DAW) (Ref. 6) in the long wavelength limit are often excited due to ionization[7–9] in a gas discharge weakly ionized dusty plasma.

Dust grains immersed in a plasma can exhibit self-consistent charge variations in response to the surrounding plasma oscillations and thus become a time dependent dynamical variable like density, velocity, potential, etc. As a consequence of the nonsteady dust charge variations, there arises an anomalous dissipation which causes collisionless, non-Landau damping of the linear plasma modes,[10] whereas in the nonlinear regime this anomalous dissipation causes a new kind of shock wave in dusty plasma.[11–14] It was found that in the small amplitude limit for small but finite value of $\omega_{\rm ch}(=\omega_{pi}/\nu_{\rm ch})$ (ω_{pi} is the ion oscillation frequency and $\nu_{\rm ch}$ is the dust charging frequency),[13] the unsteady dust charge variations induced anomalous dissipation causes generation of an ion acoustic shock wave described by the Korteweg-de Vries Burger equation in a dusty plasma. This occurs in the absence of ionization, collision, and particle losses. On the other hand it has been seen recently that if the dust charge variation effect is ignored, the balance between nonlinearity and dispersion leads to the generation of a weakly dissipative ion acoustic solitary wave governed by a modified form of the Korteweg-de Vries (KdV) equation[15] when effects due to ionization, collisions, and particle losses are taken into account.

However, in this paper the nonlinear propagation characteristics of DIAW are investigated incorporating the effects of nonsteady dust charge variations, ionization, collision, and particle loss. In contrast to the assumption $\omega_{\rm ch}\ll 1$, but $\neq 0$,[13] here it is assumed that $\omega_{\rm ch}$ is finite. Also it is assumed that the frequencies of ionization, ion loss, and ion-neutral collision are small compared to ion oscillation frequency (ω_{pi}).

The manuscript is organized in the following manner. Formulation of the problem including the basic assumptions and equations are described in Sec. II. The nonlinear evolution equation using the reductive perturbation technique is derived in Sec. III. The analytical solution of the nonlinear equation is derived in Sec. IV. Section V contains the numerical solution related to gas discharge dusty plasma. The findings of the present investigation are summarized in Sec. VI.

II. FORMULATION OF THE PROBLEM: BASIC ASSUMPTIONS AND EQUATIONS

The following basic assumptions are made that will help to formulate the physical problem and to find the explicit final results:

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(a) The neutrals and also dust grains are immobile, i.e., the dynamics of both the neutrals and dust grains are neglected as the mass per unit volume of both are much larger than that of the plasma. The dust grains are negatively charged and the charge varies continuously with time.

(b) At far upstream, i.e., at $x \to -\infty$, there is a neutral fluid flow velocity v_0 where the plasma is assumed to be in its equilibrium state defined by $\phi=0$; $n_e=n_{e0}$, $n_i=n_{i0}$, $n_d=n_{d0}$, and $q_d=-z_d e$ so that the plasma is quasineutral

$$n_{e0}+z_d n_{d0}=n_{i0}. \tag{1}$$

(c) It is assumed that electron collisional mean free path is much greater than the wave characteristic length (here ion Debye length λ_{D_i}) so that electrons are Boltzmann distributed. Hence the electron number density is given by

$$n_e=n_{e0}\exp(\Phi), \tag{2}$$

where $\Phi=e\phi/T_e$, T_e is the electron temperature.

(d) The dust grains are negatively charged due to plasma currents [electron current (I_e) and ion current (I_i)]. The normalized expressions for I_e and I_i for spherical dust grains with radius r_0 are[16]

$$I_e=-\pi r_0^2 e\sqrt{\frac{8T_e}{\pi m_e}}n_{e0}\exp[\Phi+z(-1+Q)], \tag{3}$$

$$I_i=\pi r_0^2 e\sqrt{\frac{8T_i}{\pi m_i}}n_{i0}N\left[\left(\chi(y)+\frac{z\psi(y)}{\sigma_i}\right)-\frac{z\psi(y)}{\sigma_i}Q\right], \tag{4}$$

where T_i is the ion temperature, $N=n_i/n_{i0}$, $\sigma_i=T_i/T_e$, $Q=Q_d/z_d e$, and $z=z_d e^2/4\pi\epsilon_0 r_0 T_e$ is the nondimensional dusty plasma parameter. The charge on the dust grain is $q_d=-z_d e+Q_d$, Q_d is the fluctuating dust charge. Also

$$\chi(y)=\frac{\sqrt{\pi}}{4}\frac{(1+2y^2)}{y}\,\mathrm{erf}(y)+\frac{1}{2}e^{-y^2};\quad \psi(y)=\frac{\sqrt{\pi}}{2}\frac{\mathrm{erf}(y)}{y}, \tag{5}$$

where $\mathrm{erf}(y)=2/\sqrt{\pi}\int_0^y e^{-p^2}dp$ is the error function, $y=v_0/V_{ti}$ and $V_{ti}(=\sqrt{T_i/m_i})$ is the ion thermal velocity.

(e) The new ions are created through ionization of neutral gas by fast electrons. The ion creation term $Q_i=\sigma(\Phi)n_n\Psi$,[7] where $\sigma(\Phi)$=ionization cross section, Ψ=the flux of ionizing electrons (whose density is much smaller than the density of the thermal electrons). The normalized ion creation term $\bar{Q}_i(=Q_i/n_{i0}\omega_{pi})$ is given by[15]

$$\bar{Q}_i=\bar{Q}_{i0}\left[1+\frac{\Delta\sigma}{\sigma_0}\Phi+\frac{1}{2\sigma_0}\left(\frac{d^2\sigma}{d\Phi^2}\right)_0\Phi^2+\cdots\right], \tag{6}$$

where $\Delta\sigma=(d\sigma/d\Phi)_0$, σ_0 and $Q_{i0}=n_n\sigma_0\Psi$ are the values of σ and Q_i at $\Phi=0$.

(f) The ions are lost from the ion fluid because of the attachment on the dust grains within the plasma (the ions colliding with the dust grains are lost from the ion fluid). The ion flux onto the dust grains is I_i/e and therefore the ion loss term (the number of ion loss per unit volume per time) $Q_l=I_i n_{d0}/e$. Thus with the help of (4) the normalized ion loss term $\bar{Q}_l(=Q_l/n_{i0}\omega_{pi})$ can be estimated as

$$\bar{Q}_l=\frac{I_i}{en_{i0}\omega_{pi}}n_{d0}=\bar{\nu}_L N\left[1-\frac{z\psi(y)Q}{z\psi(y)+\sigma_i\chi(y)}\right];\quad \bar{\nu}_L=\frac{\nu_L}{\omega_{pi}}. \tag{7}$$

The ion loss frequency $\nu_L=(n_{d0}/en_{i0})I_{i0}$ can be expressed as

$$\nu_L=\frac{r_0}{\sqrt{2\pi}}\frac{\omega_{pi}^2}{V_{ti}}\frac{(z\psi(y)+\sigma_i\chi(y))(\delta-1)}{z\delta};\quad \delta=\frac{n_{i0}}{n_{e0}}. \tag{8}$$

On the basis of the above assumptions, the nonlinear dynamics of the one-dimensional DIAW in a complex plasma with relative ion fluid velocity $V=V_i-V_0$ (V_i and V_0 are the normalized ion fluid and background neutral fluid flow velocities) is governed by the following normalized equations:

$$\partial_T N+\partial_X(NV)=\bar{Q}_i-\bar{Q}_l, \tag{9}$$

$$N(\partial_T V+V\partial_X V)=-N\partial_X\Phi-\sigma_i\partial_X N-\bar{\nu}_{in}NV-\bar{Q}_i V, \tag{10}$$

$$\delta\partial_X^2\Phi=\exp(\Phi)-\delta N-(\delta-1)(-1+Q). \tag{11}$$

Normalizations are defined as follows: $T=\omega_{pi}t$, $X=x/\lambda_{D_i}$, $V(V_i,V_0)=v(v_i,v_0)/c_i$, $\lambda_{D_i}(=\sqrt{\epsilon_0 T_e/n_{i0}e^2})$ is the ion Debye length, $\omega_{pi}(=\sqrt{n_{i0}e^2/\epsilon_0 m_i})$ is the ion plasma frequency, and $c_i(=\sqrt{T_e/m_i})$ is the ion acoustic speed, respectively. In the ion momentum conservation equation (10), $\bar{\nu}_{in}$ is the normalized (normalized by ω_{pi}) ion-neutral collision frequency. The ion continuity equation (9) shows that at equilibrium $\bar{Q}_{i0}=\bar{Q}_{l0}\Rightarrow\bar{Q}_{i0}=\bar{\nu}_L$.

The normalized dust charge variable Q is determined by the following normalized (OML) dust charging equation

$$\omega_{\mathrm{ch}}\partial_T Q=\frac{(I_e+I_i)}{\nu_{\mathrm{ch}}z_d e}=\beta_{\mathrm{ch}}\left[N\left(1-\frac{z\psi(y)Q}{z\psi(y)+\sigma_i\chi(y)}\right)-\exp(\Phi+zQ)\right], \tag{12}$$

where $\omega_{\mathrm{ch}}=\omega_{pi}/\nu_{\mathrm{ch}}$, $\beta_{\mathrm{ch}}=z\psi(y)+\sigma_i\chi(y)/z((1+z)\psi(y)+\sigma_i\chi(y))$ and $\nu_{\mathrm{ch}}(=r_0/\sqrt{2\pi}(\omega_{pi}^2/V_{ti})z\psi(y)+\sigma_i\chi(y)/z\beta_{\mathrm{ch}})$ is the dust charging frequency.

III. MODIFIED FORM OF THE BURGER EQUATION

To derive the nonlinear evolution equation using the reductive perturbation technique, the independent variables are stretched as

$$\xi=\epsilon(X-\Lambda T);\quad \tau=\epsilon^2 T, \tag{13}$$

where $\Lambda=V_{\mathrm{ph}}-V_0$, V_{ph} is the normalized phase velocity of the linear DIAW and ϵ is a small parameter characterizing the strength of the nonlinearity. The dependent variables are expanded as

$$f=f^{(0)}+\epsilon f^{(1)}+\epsilon^2 f^{(2)}+\cdots, \tag{14}$$

where $f=N,V,\Phi,Q$; $f^{(0)}=1$ for $f=N$ and $f^{(0)}=0$ for $f=\Phi,Q,V$.

To make the nonlinear perturbation consistent with (13) and (14), the following assumptions and scalings are made (later in the numerical section, the validity of these physical assumptions are justified on the basis of plasma parameters relevant to laboratory experiment):

(i) The ratio of the ion oscillation frequency (ω_{pi}) to dust charging frequency ($\nu_{\rm ch}$) is large but finite, i.e., $\omega_{\rm ch}=\omega_{pi}/\nu_{\rm ch}$ is finite.

(ii) The ion loss frequency (ν_L) is small compared to the ion oscillation frequency, i.e., $\bar{\nu}_L=\nu_L/\omega_{pi}\ll 1$ and hence it is assumed that $\bar{\nu}_L\sim O(\epsilon^2)$.

(iii) The ion-neutral collision frequency (ν_{in}) is small compared to ion oscillation frequency, i.e., $\bar{\nu}_{in}=\nu_{in}/\omega_{pi}\ll 1$ and hence it is assumed that $\bar{\nu}_{in}\sim O(\epsilon^2)$.

Introducing (13) and (14) into Eqs. (2)–(12) and using the assumptions (i), (ii), (iii), the following relations to lowest order in ϵ are obtained:

$$V^{(1)}=\Lambda N^{(1)};\quad \Phi^{(1)}=\Lambda V^{(1)}-\sigma_i N^{(1)},$$
$$\Phi^{(1)}=\delta N^{(1)}+(\delta-1)Q^{(1)};\quad Q^{(1)}=\beta_{\rm ch}(N^{(1)}-\Phi^{(1)}). \tag{15}$$

The above set of Eqs. (15) self-consistently yields the following linear wave phase velocity:

$$\Lambda=-\sqrt{\sigma_i+\frac{\delta+\beta_{\rm ch}(\delta-1)}{1+\beta_{\rm ch}(\delta-1)}}\Rightarrow V_{\rm ph}=V_0 - \sqrt{\sigma_i+\frac{\delta+\beta_{\rm ch}(\delta-1)}{1+\beta_{\rm ch}(\delta-1)}}. \tag{16}$$

Here, the minus sign is considered because of the fact that for the generation of the shock wave, the plasma flow V_0 at far upstream is greater than the wave phase velocity $V_{\rm ph}$. Finally, for the nontrivial solution, the usual perturbational analysis yields the following nonlinear Burger equation with a linear damping or growth term in the lowest order fluctuation:

$$\partial_\tau N^{(1)}+\alpha N^{(1)}\partial_\xi N^{(1)}+\Gamma N^{(1)}=\mu_{\rm ch}\partial_\xi^2 N^{(1)}, \tag{17}$$

where

$$\mu_{\rm ch}=\frac{1}{2}\frac{\omega_{\rm ch}\beta_{\rm ch}(\delta-1)^2}{(1+\beta_{\rm ch}(\delta-1))^2}, \tag{18}$$

$$\alpha=\frac{1}{2\Lambda}\left[2\sigma_i+\frac{\delta+\beta_{\rm ch}(\delta-1)}{1+\beta_{\rm ch}(\delta-1)}\left(3-\frac{\delta+\beta_{\rm ch}(\delta-1)}{1+\beta_{\rm ch}(\delta-1)}\right)+\alpha_{cv}\right], \tag{19}$$

where

$$\alpha_{cv}=\frac{2z\beta_{\rm ch}^2(\delta-1)}{(1+\beta_{\rm ch}(\delta-1))^3}\left[\delta+\left(1-\frac{z}{2}\right)\beta_{\rm ch}(\delta-1)+\frac{\psi(y)(1+\beta_{\rm ch}(\delta-1))}{z\psi(y)+\sigma_i\chi(y)}\right] \tag{20}$$

and

$$\Gamma=\frac{\bar{\nu}_{\rm in}+\gamma_{\rm loss}-\gamma_{\rm ion}}{2}, \tag{21}$$

where

$$\gamma_{\rm loss}=\bar{\nu}_L\left[2+\frac{z\psi(y)\beta_{\rm ch}(\delta-1)}{(z\psi(y)+\sigma_i\chi(y))(1+\beta_{\rm ch}(\delta-1))}\right];$$
$$\gamma_{\rm ion}=\bar{\nu}_L\frac{\Delta\sigma}{\sigma_0}\frac{(\delta+\beta_{\rm ch}(\delta-1))}{(1+\beta_{\rm ch}(\delta-1))}. \tag{22}$$

Expression (18) shows that the Burger term in (17) is proportional to $\omega_{\rm ch}$ ($=\omega_{pd}/\nu_{\rm ch}$ and $\delta\neq 1$), arises due to the nonsteady dust charge variations [Eq. (12)]. Also for $\delta=1$, $\mu_{\rm ch}=0$. This is expected because of the fact that in the absence of dust grains, i.e., for the effectively two component electron-ion plasma there is no possibility of dust charge variations. Also for $\omega_{\rm ch}(=\omega_{pi}/\nu_{\rm ch})\approx 0\Rightarrow\mu_{\rm ch}\approx 0$ (no Burger term). Hence the Burger term which is responsible for the generation of shock wave originates from the nonsteady dust charge variation under the assumption that $\omega_{\rm ch}(=\omega_{pi}/\nu_{\rm ch})$ is finite.

The presence of the term α_{cv} in the expression of coefficient of nonlinearity α [Eq. (19)] shows that the dust charge variations modify the nonlinearity of the ion acoustic wave in dusty plasma as $\alpha_{cv}\propto\beta_{\rm ch}$ [Eq. (20): as Eqs. (12) and (15) show that $Q^{(1)}\propto\beta_{\rm ch}$]. In the absence of dust grains it vanishes ($\delta=1$ implies no dust charge variations $\Rightarrow\alpha_{cv}=0$).

Also the expression for Γ [Eq. (21)] shows that the third term in the LHS of the modified form of the Burger equation (17) represents the effects of collision, ion loss, and ionization. In the absence of collision, ion loss, and ionization, $\Gamma=0$ and we recover the Burger equation for dust ion acoustic shock wave.

Hence, Eq. (17) shows that the finite amplitude nonlinear ion acoustic wave in complex plasma is governed by a modified form of Burger equation and collision, ion loss, and ionization are responsible for the modification.

IV. ANALYTICAL APPROXIMATE TIME EVOLUTION SOLUTION

The presence of the Burger term in Eq. (17) implies the possibility of the existence of a shock wave provided at far upstream the plasma flow V_0 is greater than the wave phase velocity $V_{\rm ph}$. In the absence of collision, ion loss, and ionization, i.e., for $\Gamma=0$, Eq. (17) reduces to the well-known Burger equation and one can easily find the stationary wave solution of the Burger equation [Eq. (17) with $\Gamma=0$ and $\alpha=-|\alpha|$] in the following form:

$$N^{(1)}=N\left(1+\tanh\frac{\eta}{L_w}\right);\quad L_w=\frac{2\mu_{\rm ch}}{|\alpha|N}, \tag{23}$$

where $\eta=\xi+V_f\tau$, $V_f=N|\alpha|$ is the shock velocity, N is the shock amplitude, and L_w is the shock width.

To find the effects of collision, ion loss, and ionization, i.e., for $\Gamma\neq 0$, we assume the solution of the modified form of the Burger equation (17) in the following slow time (τ) dependent form:

$$N^{(1)}=N(\tau)(1+\tanh\beta(\tau)\eta(\tau));\quad \eta(\tau)=\xi+\theta(\tau), \tag{24}$$

where $d\theta/d\tau$=shock velocity. To avoid the secular terms (terms proportional to ξ), we assume that the time (τ) variations of $\beta(\tau)$ is negligible so that $d\beta(\tau)/d\tau\approx 0$. Finally sub-

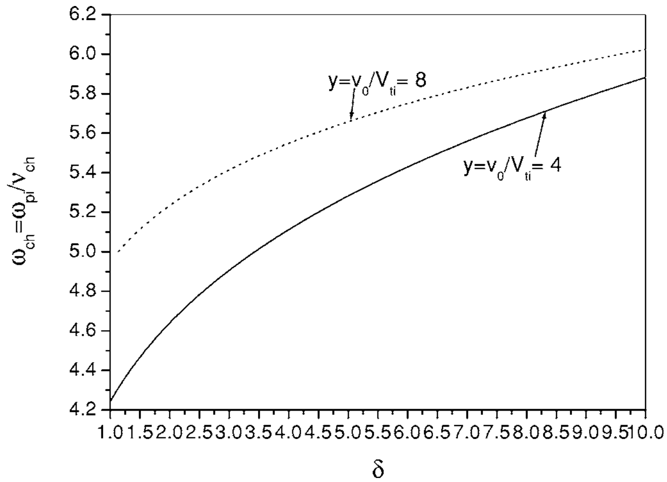

FIG. 1. Variations of the ratio of ion oscillation frequency to dust charging frequency $\omega_{ch}=\omega_{pi}/\nu_{ch}$ with ion-electron density ratio $\delta=n_{i0}/n_{e0}$. The solid curve is for $y=4$ and the dotted curve is for $y=8$.

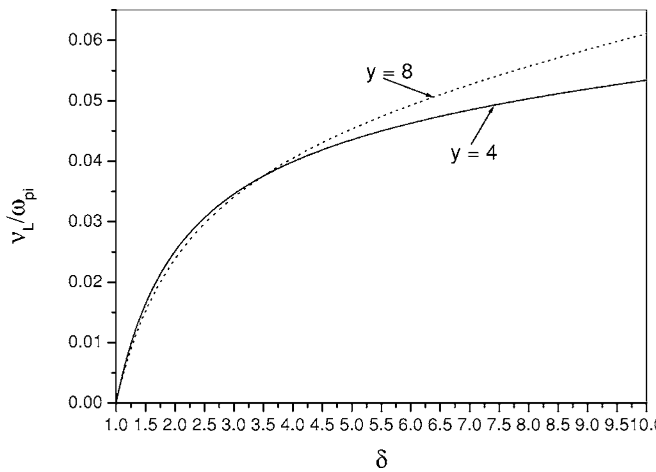

FIG. 2. Variations of the ratio of ion loss frequency to ion oscillation frequency $\bar{\nu}_L=\nu_L/\omega_{pi}$ with δ. The different curves are labelled as Fig. 1.

stituting this solution in Eq. (17) and equating the coefficients of different powers of tanh $\beta(\tau)\eta(\tau)$ to zero, we obtain

$$N(\tau)=N_0e^{-\Gamma\tau};\quad \theta(\tau)=\frac{|\alpha|N_0}{\Gamma}(1-e^{-\Gamma\tau}), \tag{25}$$

where $\beta(\tau)=|\alpha|N(\tau)/2\mu_{ch}$ and $N_0=N(\tau=0)$ is the initial amplitude/intensity of the shock. Also taking the limit $\Gamma\to 0$ of (25) one can recover the solution (23), i.e., the solution of the Burger equation. The solution (24) together with (25) of the modified form of the Burger equation [Eq. (17)] shows that the shock amplitude $N(\tau)$ and shock velocity $d\theta/d\tau(=|\alpha|N(\tau))$ decreases (increases) exponentially with time τ according to $\Gamma>0(<0)$. On the other hand the shock width $L_w(=\beta(\tau)^{-1}=2\mu_{ch}/|\alpha|N(\tau))$ behaves conversely and hence the product of the shock amplitude and shock width $[N(\tau)L_w=2\mu_{ch}/|\alpha|$ is time (τ) independent] is constant as shock propagates from the upstream to the downstream side.

In the expression of Γ [Eq. (21)], the contributions of the ion-neutral collision $(\bar{\nu}_{in})$, and ion loss (γ_{loss}) are positive thus leading to wave damping, whereas, the ionization effect (γ_{ion}) leads to wave growth as it is negative. However, the combined action of all the dissipative mechanisms reveal the following criteria for growth $(\Gamma<0)$ or damping $(\Gamma>0)$:

$$\Gamma<>0\Rightarrow \bar{\nu}_{in}+\bar{\nu}_L\left[2+\frac{z\psi(y)\beta_{ch}(\delta-1)}{(1+\beta_{ch}(\delta-1))(z\psi(y)+\sigma_i\chi(y))}\right] <> \bar{\nu}_L\frac{\Delta\sigma}{\sigma_0}\left(\frac{\delta+\beta_{ch}(\delta-1)}{1+\beta_{ch}(\delta-1)}\right). \tag{26}$$

This shows that the shock wave becomes damped when collision and ion loss are the dominant dissipative processes and it becomes unstable when the production process (ionization) is the dominant dissipative process.

V. NUMERICAL ANALYSIS AND DISCUSSIONS

For numerical computations, we use the following representative plasma parameters related to gas discharge laboratory plasma:[4,7] ion mass $m_i\sim 6.69\times 10^{-26}$ kg, ion density $n_{i0}\sim 10^{17}$ m^{-3}, ion temperature $T_i\sim 0.3$ eV, neutral atom density $n_n\sim 1.12\times 10^{22}$ m^{-3}, neutral atom temperature $T_n\sim 0.03$ eV, electron temperature $T_e\sim 3$ eV, and dust grain radius $r_0\sim 5$ μm. The ion-electron number density $\delta(=n_{i0}/n_{e0})$ as a function of nondimensional dusty plasma parameter z is given by the following equilibrium current balance equation

$$\delta=\frac{\sqrt{\sigma_i}}{z\psi(y)+\sigma_i\chi(y)}\sqrt{\frac{m_i}{m_e}}e^{-z};\quad z=\frac{z_de^2}{4\pi\epsilon_0 r_0T_e}, \tag{27}$$

where $\psi(y)\sim 0.222$, $\chi(y)\sim 3.66$ for $y=v_0/V_{ti}=4$ and $\psi(y)\sim 0.111$, $\chi(y)\sim 7.15$ for $y=8$. Solving this equation $\delta=\delta(z)$, the values of ω_{ch} and $\bar{\nu}_L$ are drawn in Figs. 1 and 2 for different δ and y. Also for the above mentioned plasma parameters $\bar{\nu}_{in}(=\nu_{in}/\omega_{pi})\sim 7.2\times 10^{-2}$. This value of $\bar{\nu}_{in}$ and the values of the ω_{ch}, $\bar{\nu}_L$ (Figs. 1 and 2) justify assumptions and scalings of (i), (ii), and (iii) in Sec. II, on the basis of which the reductive perturbation analysis is performed.

Figures 1 and 2 show that with the increase of $y=v_0/V_{ti}$ (i.e., with the increase of plasma flow) and δ [i.e., with the decrease of z by Eq. (27)] both ω_{ch} and $\bar{\nu}_L$ increases.

The variations of damping or growth term Γ [Eq. (21)] with δ are depicted in Fig. 3. Both curves (solid and dotted) show that for $\Delta\sigma/\sigma_0=5$, collision and ion loss dominate over ionization $(\Gamma>0)$ for lower values of δ (higher values of z) but for higher values of δ (lower values of z) the latter dominates over the former $(\Gamma<0)$. The comparative study between the two curves (solid and dotted) in Fig. 3 show that in the case of $\Gamma<0$, i.e., in the case of growth, the growth rate is large for higher values of y (dotted curve) than the lower values (solid curve).

In the case of $\Gamma>0$, i.e., when ion-neutral collision and ion loss due to attachment on the dust grains are the dominant processes, the dynamical behavior of the approximate solution [Eq. (24)] of the modified form of the Burger equa-

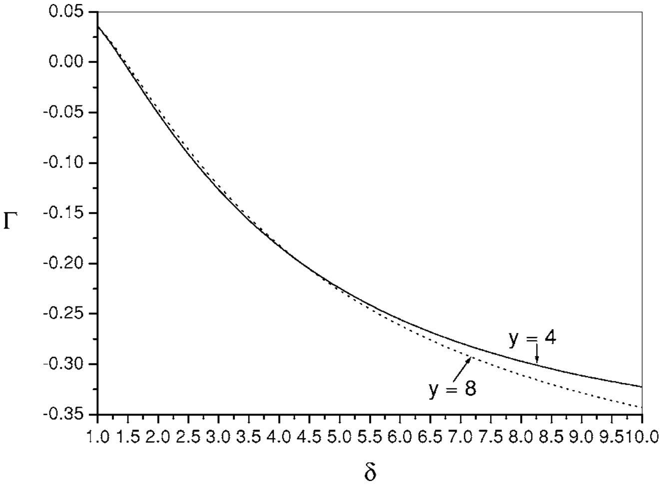

FIG. 3. Variations of Γ [Eq. (21)] with δ for relative change of ionization cross section $\Delta\sigma/\sigma_0=5$. The different curves are labelled as Fig. 1.

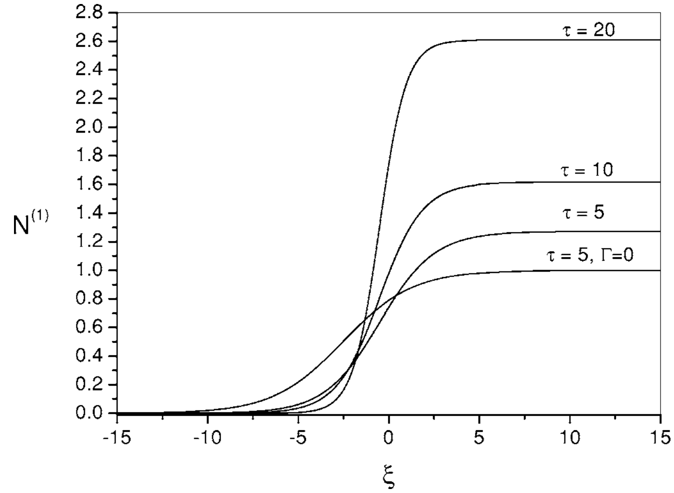

FIG. 5. Monotonic shock structure of ion number density $N^{(1)}$ [Eq. (24)] for $\Gamma<0$ [Eq. (26)]. The plasma parameters are: $\delta\sim 2$, $y=8$, $\Gamma\sim -0.048$, and initial amplitude $N_0=0.5$.

tion [Eq. (17)] is shown in Fig. 4. It is seen that the shock amplitude $N(\tau)$ decays with time τ and the damped shock moves with a gradually diminishing velocity. As a result, it will propagate only a finite distance $L\sim|\alpha|N_0/\Gamma\sim 1.04\times 10^3$ μm before it dies out [$\tau\rightarrow\infty$].

On the other hand, in the case of $\Gamma<0$, i.e., when ionization is the dominant process, the dynamical behavior represented by the approximate solution [Eq. (24)] is exhibited in Fig. 5. The figure shows that the shock amplitude $N(\tau)$ grows with time τ and the shock moves with a gradually increasing velocity. However, for large τ the assumption of the weak time variation is violated as $dN/d\tau$ being proportional to $\Gamma N(\tau)$ is no longer small as $N(\tau)$ becomes large so that solution (24) is invalidated for $\Gamma<0$ and $\tau\rightarrow\infty$.

VI. SUMMARY

The results obtained in this investigation can be summarized as follows:

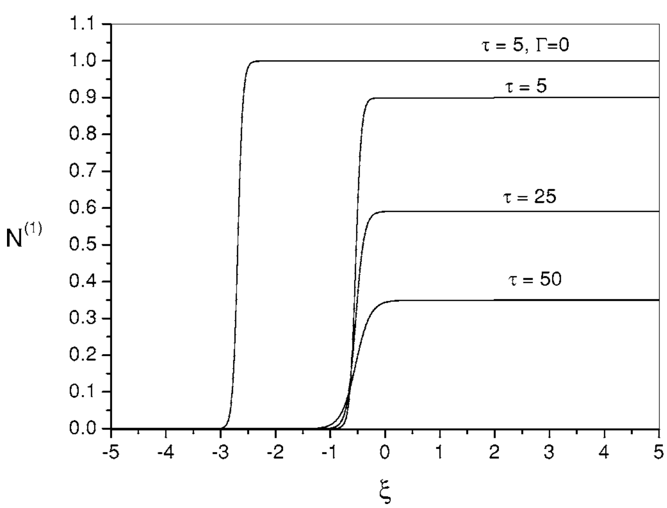

FIG. 4. Monotonic shock structure of ion number density $N^{(1)}$ [Eq. (24)] for $\Gamma>0$ [Eq. (26)]. The plasma parameters are: $\delta\sim 1.2$, $y=8$, $\Gamma\sim 0.021$, and initial amplitude $N_0=0.5$.

(a) In the absence of ionization, collision, and ion loss, the anomalous dissipation originating from nonsteady dust charge variation is found to cause dust ion acoustic shock wave governed by the Burger equation [Eq. (17) with $\Gamma=0$] for finite values of $\omega_{ch}(=\omega_{pi}/\nu_{ch})$, whereas for small but finite values of ω_{ch}, the shock wave is governed by the KdV Burger equation.[13]
(b) The presence of ionization, collision, and ion loss is found to modify the nonlinear propagation characteristics of DIAW for which the nonlinear wave is governed by the Burger equation with a linear damping or growth term [Eq. (17)].
(c) The growth rate ($\Gamma<0$) increases with an increase of ion-electron density ratio δ and neutral fluid flow velocity V_0 (Fig. 3: dotted curve $y=v_0/V_{ti}=8$), whereas damping ($\Gamma>0$) rate decreases with the increase of $\delta(=n_{i0}/n_{e0})$ (Fig. 3).
(d) The analytical [Eq. (24) together with (25)] and numerical solutions (Figs. 4 and 5) of the modified form of the Burger equation show that the nonlinear dust ion acoustic wave possesses compressive (with increasing ion density) monotonic shock structure (Figs. 4 and 5) and the shock amplitude as well as shock velocity may decay (Fig. 4: collision and ion loss are the dominant processes) or may grow (Fig. 5: ionization is the dominant process) with time due to the simultaneous effects of ionization, ion loss, and ion-neutral collisions.
(e) In case of damping the shock wave move at most a finite distance $L\sim 1.04\times 10^3$ μm $\gg\lambda_{D_i}\sim 40$ μm before it dies out. Hence the shock moves a sufficiently long distance to be observed in the laboratory.
(f) In the case of growth, after a sufficiently long time, it is impossible to observe shock structure because of the ionization instability.
(g) Finally, the ion acoustic monotonic shock structure in a dusty plasma in the absence of ionization, collision, and ion loss was observed in a Q machine.[12] Hence the results of the present investigation may be helpful for

understanding the formation of ion acoustic shock wave in dusty laboratory plasma. Also our results will be useful for investigating the nonlinear features of the ion acoustic wave in a weakly ionized charge varying complex (dusty) gas discharge laboratory plasma.

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