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Nonlinear properties of small amplitude dust ion acoustic solitary waves

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In this paper some nonlinear characteristics of small amplitude dust ion acoustic solitary wave in three component dusty plasma consisting of electrons, ions, and dust grains have been studied. Simultaneously, the charge fluctuation dynamics of the dust grains under the assumption that the dust charging time scale is much smaller than the dust hydrodynamic time scale has been considered here. The ion dust collision has also been incorporated. It has been seen that a damped Korteweg–de Vries (KdV) equation governs the nonlinear dust ion acoustic wave. The damping arises due to ion dust collision, under the assumption that the ion hydrodynamical time scale is much smaller than that of the ion dust collision. Numerical investigations reveal that the dust ion acoustic wave admits only a positive potential, i.e., compressive soliton.
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I. INTRODUCTION

Recently, there has been much interest in studying the properties of dusty plasmas, which are observed in planetary rings, asteroid zones, cometary tails, and magnetosphere, as well as in the lower part of the Earth's ionosphere.[1–4]

In general, dust particles have no consequence on high-frequency oscillations, except on the damping factors,[5] since the response time of heavy mass dust particles to high-frequency oscillation is very large. Therefore, dusty plasmas exhibit a number of low-frequency excitations. Rao *et al.*[6] have presented the theory of dust acoustic waves in an unmagnetized dusty plasma. Shukla and Silin[7] have shown the existence of the dust ion acoustic wave in unmagnetized dusty plasma. Many authors[8–13] have experimentally observed both the dust acoustic and the dust ion acoustic waves. Several authors[14–18] have studied the propagation characteristics of low-frequency linear dust acoustic and dust ion acoustic waves in dusty plasma. In all the works, the charges on the dust grains have been assumed to be fixed. In fact, the charges on the dust grains are not fixed, because the imbalance of electron current and ion current flowing through the grain surface causes charge fluctuation. Nejoh[19] pointed out that dust charge variation plays an important role in the study of collective effects of the dusty plasma. Due to the charge fluctuation of the dust grains, the wave becomes dampened.[20–22]

Also the experimental works[9,10,13] reveal that both the dust acoustic and dust ion acoustic wave can be highly nonlinear. The nonlinear dust acoustic wave admits either positive or negative electrostatic potentials.[23–28]

Recently Ma *et al.*[29] and Xie *et al.*[30] have studied the dust acoustic soliton, considering grain charge variation both by the Sagdeev potential and reductive perturbation method.

Shukla and Silin[7] have studied the linear property of the dust ion acoustic wave by considering a fixed charge on the dust grain. In this paper, the nonlinear properties of small amplitude dust ion acoustic solitary wave with grain charge variation have been studied by the reductive perturbation technique. As predicted in some experimental works,[9,31] in this paper it has been assumed that the dust charging time scale is much smaller than the dust hydrodynamical time scale. Recent experimental work of Nakamura *et al.*[13] shows that the ion dust collision plays an important role for the nonlinear propagation of ion acoustic waves in dusty plasma. Ion dust collisions are also considered here under the assumption that the ion hydrodynamical time scale is much smaller than that of the ion dust collisions. Thus in linear analysis the phase velocity of the dust ion acoustic wave is modified by the grain charge variation. However, the nonlinear propagation characteristics of the dust ion acoustic wave are governed by the damped Korteweg–de Vries (KdV) equation. It is seen that the coefficient of the nonlinear term in the KdV equation describing the dust ion acoustic solitary wave becomes minimum at some critical value of the electron ion number density ratio and this wave admits only positive potentials.

The basic equations describing the model have been presented in Sec. II. Section III contains the reductive perturbation analysis and the damped KdV equation, which governs the small amplitude dust ion acoustic solitary wave. Numerical results and their graphical representation are given in Sec. IV. Section V contains the general discussion.

II. BASIC EQUATIONS

A collisional, nonrelativistic unmagnetized dusty plasma consisting of electrons, ions, and charged dust grains have been considered. Variation of grain charge has also been considered. In this situation the charge neutrality condition becomes

$$n_{e0}+z_d n_{d0}=z_i n_{i0}, \tag{1}$$

where $n_{j0}(j=e,i,d)$ is the equilibrium number density of

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the jth species and $z_j(j=d,i)$ is the number of charge on the jth particles. We have considered a singly ionized plasma system for which $z_i=1$.

Our objective here is to study the features of a low-frequency electrostatic dust ion acoustic solitary wave in a collisional, unmagnetized dusty plasma with phase velocity $v_{td}, v_{ti} \ll \omega/k \ll v_{te}$. This suggests that the electrons are inertialess Boltzmann distributed. Therefore, in this situation, the dynamics of dust ion acoustic oscillations is governed by the following one-dimensional continuity and momentum fluid equations and Poisson's equations:

$$\frac{\partial N_d}{\partial T}+\frac{\partial}{\partial X}(N_d V_d)=0, \quad (2)$$

$$\frac{\partial V_d}{\partial T}+V_d\frac{\partial V_d}{\partial X}=-\mu_d(\delta Q-1)\frac{\partial \Phi}{\partial X}, \quad (3)$$

$$\frac{\partial N_i}{\partial T}+\frac{\partial}{\partial X}(N_i V_i)=0, \quad (4)$$

$$\frac{\partial V_i}{\partial T}+V_i\frac{\partial V_i}{\partial X}=-\frac{\partial \Phi}{\partial X}-\frac{\sigma}{N_i}\frac{\partial N_i}{\partial X}-\nu_{id}V_i, \quad (5)$$

$$\frac{\partial^2 \Phi}{\partial X^2}=-\frac{1}{z_i n_{i0}}[z_i n_{i0} N_i+z_d n_{d0} N_d(\delta Q-1)-n_e], \quad (6)$$

$$n_e=n_{e0}\exp(\Phi), \quad (7)$$

$\mu_d=z_d m_i/z_i m_d$ and $\delta=n_{e0}/z_i n_{i0}$, where $m_i(m_d)$ is the ion (dust) mass and $n_{i0}(n_{e0})$ is the equilibrium ion (electron) number density, z_i (z_d) is the number of charge on the ion (dust). $(\delta Q-1)[\delta Q=\delta Q_d/z_d e]$ is the charge on the dust grain normalized by the equilibrium charge $z_d e$ of the dust grain as the charge on the dust grain $Q_d=-z_d e+\delta Q_d$, δQ_d is the variation of the charge. N_i and N_d are the ion and dust number density normalized by n_{i0} and n_{d0}, respectively. The electrostatic potential Φ is normalized by T_e/e. ν_{id} is the ion dust collision frequency normalized by ion plasma frequency ω_{pi}.

In fact, the dust ion acoustic wave is the ion acoustic wave in dusty plasma media with an increasing phase velocity of the order of $(z_i n_{i0}/n_{e0})^{1/2} c_i$, where c_i is the ion acoustic speed. Thus the propagation characteristics of the dust ion acoustic wave are similar to those of the ion acoustic wave in electron ion plasma without the dust grains. The only difference is that the phase velocity of the dust ion acoustic wave increases according to $z_i n_{i0}/n_{e0}$. Therefore, to study the dust ion acoustic solitary wave, the velocities V_d and V_i of dust grains and ions are normalized by the ion acoustic velocity $c_i=\sqrt{z_i T_e/m_i}$. The time scale T and the space scale X are normalized by ω_{pi}^{-1} and λ_{Di}, respectively, where $\omega_{pi}=\sqrt{n_{i0}z_i^2 e^2/\varepsilon_0 m_i}$ is the ion plasma frequency and $\lambda_{Di}=\sqrt{\varepsilon_0 T_e/z_i n_{i0} e^2}$ is the ion Debye length.

Now to determine the normalized charge variable δQ, we consider the orbital motion limited current[32] and charge balance equation. In normalized form it reads as

$$\frac{\omega_{pd}}{\nu_d}\left(\frac{\partial \delta Q}{\partial T}+V_d\frac{\partial \delta Q}{\partial X}\right)=\sqrt{\mu_d(1-\delta)}\frac{1}{\nu_d z_d e}(I_e+I_i), \quad (8)$$

where

$$I_e=-\pi a^2 e\sqrt{\frac{8T_e}{\pi m_e}}n_{e0}\exp(\Phi)\exp(-z+z\delta Q), \quad (9a)$$

$$I_i=\pi a^2 e\sqrt{\frac{8T_i}{\pi m_i}}n_{i0}N_i\left[\left(1+\frac{z}{\sigma}\right)-\frac{z}{\sigma}\delta Q\right], \quad (9b)$$

$\sigma=T_i/T_e$ and $z=z_d e^2/4\pi\varepsilon_0 a T_e$, $4\pi\varepsilon_0 a$ is the capacitance of the spherical dust grain of radius a.

ν_d is the dust charging frequency given by Ref. 33,

$$\nu_d=\frac{a}{\sqrt{2\pi}}\frac{\omega_{pi}^2}{\nu_{ti}}(1+\sigma+z). \quad (10)$$

Also $\omega_{pd}/\nu_d=\tau_{ch}/\tau_d$,

where τ_{ch} is the charging time

$$\approx(d\delta Q/dT)^{-1} \quad (11a)$$

and τ_d is the dust hydrodynamical time

$$\approx(\omega_{pd}=\sqrt{n_{d0}z_d^2 e^2/\varepsilon_0 m_d})^{-1}. \quad (11b)$$

III. REDUCTIVE PERTURBATION ANALYSIS

In order to study the dust ion acoustic solitary wave, we use the reductive perturbation technique[34] to obtain a KdV equation that governs the behavior of small amplitude dust ion acoustic solitary wave. The independent variables are stretched as

$$\xi=\varepsilon^{1/2}(X-\lambda T), \quad \tau=\varepsilon^{3/2}T, \quad (12)$$

where λ is the velocity of linear dust ion acoustic wave and ε is a small parameter characterizing the strength of the nonlinearity.

The dependent variables are expanded as

$$N_d=1+\varepsilon N_d^{(1)}+\varepsilon^2 N_d^{(2)}+\cdots, \quad (13)$$

$$N_i=1+\varepsilon N_i^{(1)}+\varepsilon^2 N_i^{(2)}+\cdots, \quad (14)$$

$$V_d=\varepsilon V_d^{(1)}+\varepsilon^2 V_d^{(2)}+\cdots, \quad (15)$$

$$V_i=\varepsilon V_i^{(1)}+\varepsilon^2 V_i^{(2)}+\cdots, \quad (16)$$

$$\Phi=\varepsilon \Phi^{(1)}+\varepsilon^2 \Phi^{(2)}+\cdots, \quad (17)$$

$$\delta Q=\varepsilon\,\delta Q^{(1)}+\varepsilon^2\,\delta Q^{(2)}+\cdots, \quad (18)$$

for $\tau_d \approx 2-10$ ms,[10,29] $\tau_{ch}\approx 95-10$ ns,[29,31] $\omega_{pd}/\nu_d=\tau_{ch}/\tau_d \approx 10^{-4}-10^{-5}$ implying $\tau_{ch}\ll\tau_d(\omega_{pd}\ll\nu_d)$ justifying the scaling

$$\omega_{pd}/\nu_d=\tau_{ch}/\tau_d\approx\mu\varepsilon^{3/2},$$

μ is a finite quantity of the order of unity. (19)

Also to make the nonlinear perturbation consistent, we assume that ν_{id} is small and is proportional to $\varepsilon^{3/2}$ (Refs. 35 and 36) under the assumption that the ion hydrodynamic time scale is much smaller than the ion dust collision time scale.

We take $\nu_{id}=\bar{\nu}\varepsilon^{3/2}$, $\bar{\nu}$ is a finite quantity of the order of unity. (20)

The boundary conditions are as follows:

As $|X|\to\infty$, both $N_d, N_i\to 1$; Φ, V_d, $V_i\to 0$. (21)

Now introducing (12)–(20) into Eqs. (2)–(9b) and equating the terms in lowest powers ε, we obtain the following relations:

$$V_d^{(1)}=\lambda N_d^{(1)}, \tag{22}$$

$$\lambda V_d^{(1)}=-\mu_d\Phi^{(1)}, \tag{23}$$

$$V_i^{(1)}=\lambda N_i^{(1)}, \tag{24}$$

$$\lambda V_i^{(1)}=\Phi^{(1)}+\sigma N_i^{(1)}, \tag{25}$$

$$N_i^{(1)}=(1-\delta)N_d^{(1)}+\delta\Phi^{(1)}-(1-\delta)\,\delta Q^{(1)}, \tag{26}$$

$$\delta Q^{(1)}=\beta_d N_i^{(1)}-\beta_d\Phi^{(1)}, \tag{27}$$

$$\beta_d=\frac{(\sigma+z)}{z(1+z+\sigma)}. \tag{28}$$

From (22) and (23) we obtain

$$N_d^{(1)}=-\mu_d\frac{\Phi^{(1)}}{\lambda^2}. \tag{29}$$

From (24) and (25) we obtain

$$N_i^{(1)}=\frac{\Phi^{(1)}}{\lambda^2-\sigma}. \tag{30}$$

From (27) and (30) we obtain

$$\delta Q^{(1)}=\frac{\beta_d}{\lambda^2-\sigma}(1+\sigma-\lambda^2)\Phi^{(1)}. \tag{31}$$

Finally from (26) and using (29)–(31) we obtain the following quadratic equation in λ:

$$\lambda^2=\frac{\{1+\mu_d(1-\delta)+(\sigma+1)(1-\delta)\beta_d+\delta\sigma\}\pm\sqrt{\{1+\mu_d(1-\delta)+(\sigma+1)(1-\delta)\beta_d+\delta\sigma\}^2-4\sigma\mu_d(1-\delta)\{\delta+\beta_d(1-\delta)\}}}{2\{\delta+\beta_d(1-\delta)\}}. \tag{32}$$

This equation yields the phase velocity λ of the linear dust ion acoustic wave in three component dusty plasma with dust charge variation.

The terms containing β_d occur here due to charge variation of the dust grains.

For fixed charge on the dust grain, $\beta_d=0$ and hence Eq. (32) becomes as follows:

$$\lambda^2=\frac{\{1+\mu_d(1-\delta)+\delta\sigma\}\pm\sqrt{\{1+\mu_d(1-\delta)+\delta\sigma\}^2-4\,\delta\sigma\mu_d(1-\delta)}}{2\delta}. \tag{33}$$

Again on the assumption that $\sigma\ll 1$ and $\omega_{pd}^2\ll\omega_{pi}^2$, i.e., $\mu_d(1-\delta)\ll 1$, we recover the normalized phase velocity of the dust ion acoustic wave obtained by Shukla and Silin[7] as follows:

$$\lambda^2=\frac{1}{\delta}. \tag{34}$$

In un-normalized form, which becomes as follows:

$$\omega^2=z_i^2\frac{n_{i0}}{n_{e0}}k^2\frac{T_e}{m_i}. \tag{35}$$

Also for the two component electron ion plasma, $\delta=1$ gives the phase velocity of the usual ion acoustic wave as

$$\lambda^2=1+\sigma(\text{Since }\ \sigma\ll 1)\Rightarrow\lambda^2\approx 1\Rightarrow\omega^2=z_ik^2\frac{T_e}{m_i}. \tag{36}$$

Equating the terms next higher order in ε, we obtain the following relations:

$$N_{d\tau}^{(1)}+N_d^{(1)}V_{d\xi}^{(1)}+N_{d\xi}^{(1)}V_d^{(1)}=\lambda N_{d\xi}^{(2)}-V_{d\xi}^{(2)}, \tag{37}$$

$$V_{d\tau}^{(1)}+V_d^{(1)}V_{d\xi}^{(1)}+\mu_d\delta Q^{(1)}\Phi_\xi^{(1)}=\mu_d\Phi_\xi^{(2)}+\lambda V_{d\xi}^{(2)}, \tag{38}$$

$$N_{i\tau}^{(1)}+N_i^{(1)}V_{i\xi}^{(1)}+N_{i\xi}^{(1)}V_i^{(1)}=\lambda N_{i\xi}^{(2)}-V_{i\xi}^{(2)}, \tag{39}$$

$$V_{i\tau}^{(1)}+V_i^{(1)}V_{i\xi}^{(1)}-\lambda N_i^{(1)}V_{i\xi}^{(1)}+N_i^{(1)}\Phi_\xi^{(1)}=\lambda V_{i\xi}^{(2)}-\Phi_\xi^{(2)}-\sigma N_{i\xi}^{(2)}-\bar{\nu}V_i^{(1)}, \tag{40}$$

$$\Phi_{\xi\xi}^{(1)}=(1-\delta)N_d^{(2)}-N_i^{(2)}-(1-\delta)\,\delta Q^{(2)}-(1-\delta)N_d^{(1)}\delta Q^{(1)}+\delta\left(\Phi^{(2)}+\frac{{\Phi^{(1)}}^2}{2}\right), \tag{41}$$

$$\delta Q^{(2)}=-z\beta_d\delta Q^{(1)}\Phi^{(1)}-\frac{\beta_d}{2}{\Phi^{(1)}}^2-\frac{z^2}{2}\beta_d{\delta Q^{(1)}}^2-\frac{z}{(z+\sigma)}\beta_d N_i^{(1)}\delta Q^{(1)}+\beta_d N_i^{(2)}-\beta_d\Phi^{(2)}, \tag{42}$$

where the subscripts ξ and τ denote differentiation.

Eliminating $V_{d\xi}^{(2)}$ from Eqs. (37) and (38) and using (29) we obtain

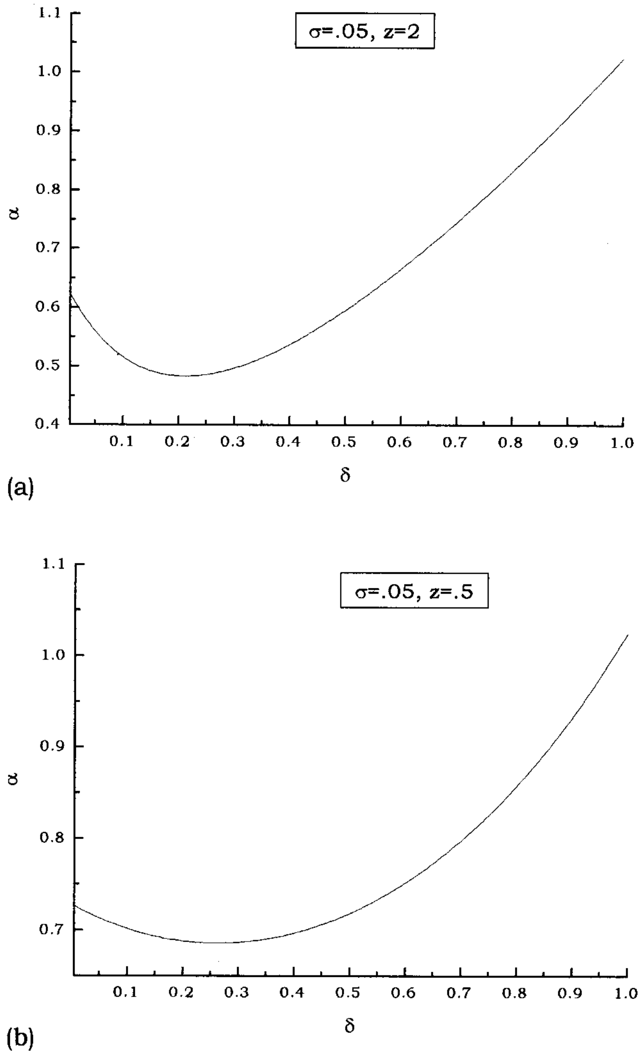


FIG. 1. Variation of coefficient of nonlinear term α with δ for (a) $\sigma=0.05$, $z=2$ and (b) $\sigma=0.05$, $z=0.5$.

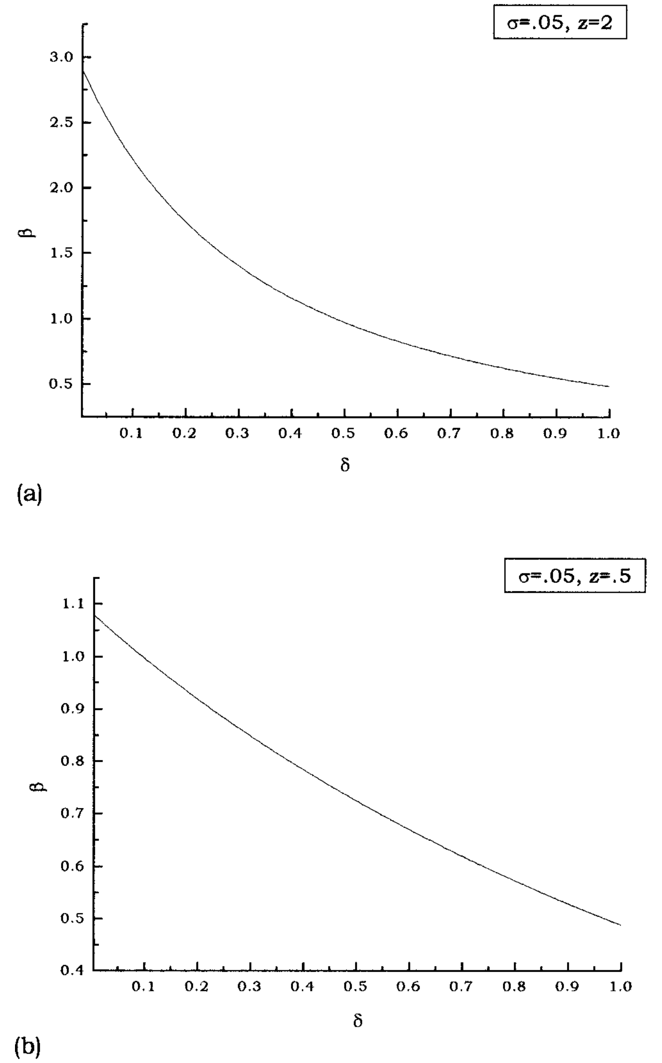


FIG. 2. Variation of coefficient of dispersion term β with δ for (a) $\sigma=0.05$, $z=2$ and (b) $\sigma=0.05$, $z=0.5$.

$$N_{d\xi}^{(2)}=-2\mu_d\frac{\Phi_\tau^{(1)}}{\lambda^3}+\frac{1}{\lambda^2}\left[\frac{3\mu_d^2}{\lambda^2}+\frac{\mu_d\beta_d(1+\sigma-\lambda^2)}{\lambda^2-\sigma}\right]\times\Phi^{(1)}\Phi_\xi^{(1)}-\frac{\mu_d}{\lambda^2}\Phi_\xi^{(2)}. \quad (43)$$

Again eliminating $V_{i\xi}^{(2)}$ from Eqs. (39) and (40) and using (30) we obtain

$$N_{i\xi}^{(2)}=\frac{2\lambda}{(\lambda^2-\sigma)^2}\Phi_\tau^{(1)}+\frac{(3\lambda^2-\sigma)}{(\lambda^2-\sigma)^3}\Phi^{(1)}\Phi_\xi^{(1)}+\frac{1}{\lambda^2-\sigma}\Phi_\xi^{(2)}+\bar{\nu}\frac{\lambda}{(\lambda^2-\sigma)}\Phi^{(1)}. \quad (44)$$

Using (29)–(31) into (42) we obtain

$$\delta Q^{(2)}=\beta_d N_i^{(2)}-\beta_d\Phi^{(2)}-\frac{1}{2(\lambda^2-\sigma)^2}\frac{z^2\beta_d^3}{(\sigma+z)^2}\times[(1+\sigma-\lambda^2)^2+(1+\sigma+z)^2]\Phi^{(1)^2}. \quad (45)$$

Finally differentiating equation (41) with respect to ξ and using the relations (29)–(31), (43)–(45), we obtain the following modified KdV equation of the dust ion acoustic wave:

$$\Phi_\tau^{(1)}+\alpha\Phi^{(1)}\Phi_\xi^{(1)}+\beta\Phi_{\xi\xi\xi}^{(1)}+\gamma\Phi^{(1)}=0, \quad (46)$$

where

$$\alpha=\beta\left[\begin{array}{l}\dfrac{\{1+\beta_d(1-\delta)\}(3\lambda^2-\sigma)}{(\lambda^2-\sigma)^3}-\delta-\dfrac{3\mu_d^2(1-\delta)}{\lambda^4}-\dfrac{3\mu_d\beta_d(1-\delta)(1+\sigma-\lambda^2)}{\lambda^2(\lambda^2-\sigma)}\\ -\dfrac{z^2\beta_d^3(1-\delta)}{(\sigma+z)^2(\lambda^2-\sigma)^2}\{(1+\sigma-\lambda^2)^2+(1+\sigma+z)^2\}\end{array}\right], \quad (47)$$

$$\gamma=\bar{\nu}\beta\frac{\lambda}{(\lambda^2-\sigma)^2}[1+\beta_d(1-\delta)], \tag{48}$$

$$\beta=\frac{1}{2}\left[\frac{\mu_d(1-\delta)}{\lambda^3}+\frac{\lambda\{1+\beta_d(1-\delta)\}}{(\lambda^2-\sigma)^2}\right]^{-1}. \tag{49}$$

For $\sigma\ll 1$ and for two component electron ion plasma $\delta=1$ and $\bar{\nu}=0$ gives $\alpha\approx 1$ and $\beta\approx\frac{1}{2}$ and the KdV equation (46) becomes the following:

$$\Phi_{\tau}^{(1)}+\Phi^{(1)}\Phi_{\xi}^{(1)}+\tfrac{1}{2}\Phi_{\xi\xi\xi}^{(1)}=0, \tag{50}$$

which is the KdV equation of the usual ion acoustic wave.[37] Integrating (46) and using the boundary conditions (21), we obtain Sech^2 time evolution solitary wave form approximate solution as

$$\Phi^{(1)}=\Phi_m^{(1)}(\tau)\mathrm{Sech}^2\sqrt{\frac{\alpha\Phi_m^{(1)}(\tau)}{12\beta}}(\xi-U\tau), \tag{51}$$

where $\Phi_m^{(1)}=\Phi_0^{(1)}e^{-\gamma\tau}$ and $U=\frac{\alpha\Phi_0^{(1)}}{6}e^{-\gamma\tau}$. (52)

$\Phi_m^{(1)}$ is the soliton amplitude. From (52) it is seen that the soliton amplitude is exponentially decaying with time. From (47)–(49) it is seen that the coefficients of the nonlinear term, dispersive term and the damping factor depend on δ, the dusty plasma parameter z and on the term β_d arising from the dust charging mechanisms. It is also seen that for fixed dusty plasma parameters, the damping factor is proportional to the collision frequency.

IV. NUMERICAL RESULTS

Figures 1(a) and 1(b) shows the variation of the coefficient of the nonlinear term α with the number density ratio δ for $\sigma=0.05$[17] and $z=2, 0.05$, respectively. Figure 1(a) ($\sigma=0.05$, $z=2$) shows that α is positive and decreases as δ increases, i.e., as $(z_d n_{d0}/n_{i0})(=1-\delta)$ decreases up to some critical value of $\delta(\delta_{\mathrm{cr}}\approx 0.22)$, at which $\alpha_{\min}\approx 0.483$ and after that α increases according as δ. Similarly Fig. 1(b) ($\sigma=0.05$, $z=0.5$) shows the same nature but in this case $\delta_{\mathrm{cr}}\approx 0.26$ and the corresponding value of α is $\alpha_{\min}\approx 0.685$. The comparative studies of Figs. 1(a) and 1(b) show that for $\delta=1$, α becomes independent of the dusty plasma parameter z and in both the graphs $\alpha\approx 1$. This happened because for $\delta=1$, the wave becomes ion acoustic wave.

Figures 2(a) and 2(b) show the variation of the coefficient of dispersive term β with δ for the same parameters as used in Fig. 1(a, b). Both figures show that β decreases as δ increases, i.e., $(z_d n_{d0}/n_{i0})(=1-\delta)$ decreases and, finally, $\beta\approx 0.5$ for $\delta=1$, i.e., for the pure ion acoustic mode.

Figures 3(a) and 3(b) show the variation of the wave amplitude $\Phi_m^{(1)}$ with τ for $\sigma=0.05$, $z=2$, 0.5, $\bar{\upsilon}=1$, 5 and for $\delta=0.01$. Both figures show that $\Phi_m^{(1)}$ is positive and decreases as τ increases. This shows that the dust ion acoustic wave admits only positive potential.

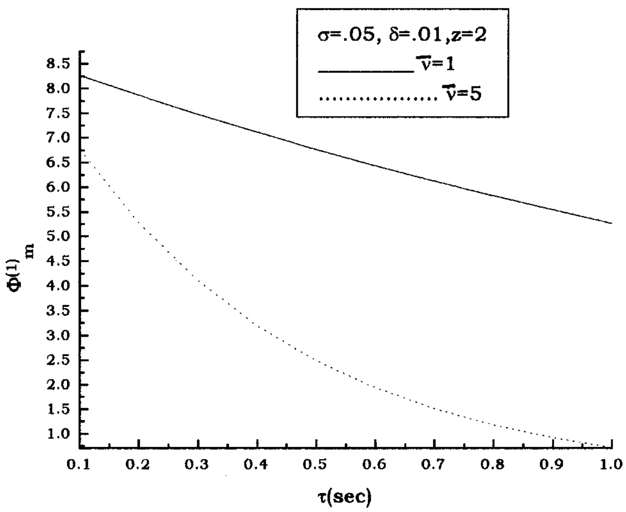
V. DISCUSSION

The properties of a nonlinear dust ion acoustic wave considering charge variation of the dust grains have been studied. The dust ion acoustic wave is the ion acoustic wave in a dusty plasma media with phase velocity of the order of $(1/\delta)^{1/2}c_i$. This phase velocity of the dust ion acoustic wave is modified due to the variation of the charge on the dust grain.

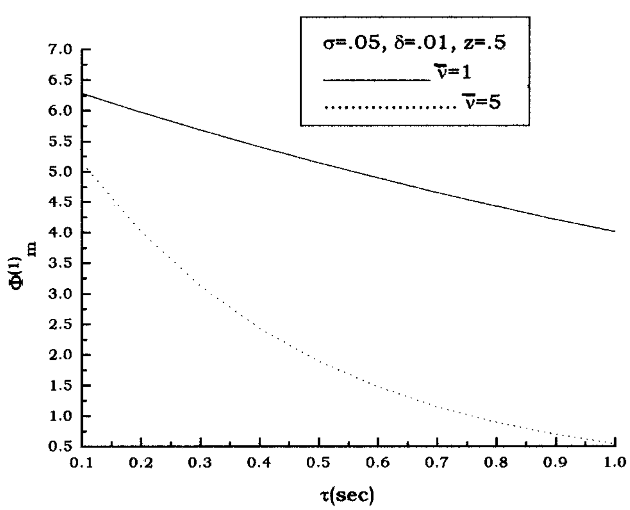
Moreover, from the present study of the damped KdV equation of the dust ion acoustic waves, the following new interesting features, different from ion acoustic waves, are seen:

(a) The coefficient of the nonlinear term α in the KdV equation is positive and decreases to its minimum at some critical value of δ; it again increases up to 1 (ion acoustic case; for $\delta=1$).

(b) The coefficient of the dispersive term β in the KdV equation decreases up to 0.5 (ion acoustic case; for $\delta=1$) as δ increases, i.e., $(z_d n_{d0}/n_{i0})(=1-\delta)$ decreases.



(a)



(b)

FIG. 3. Variation of wave amplitude $\Phi_m^{(1)}$ with τ for (a) $\sigma=0.05$, $\delta=0.01$, and $z=2$ and (b) $\sigma=0.05$, $\delta=0.01$, and $z=5$. Solid lines for $\bar{\nu}=1$ and dashed lines for $\bar{\nu}=5$.

(c) The wave amplitude $\Phi_m^{(1)}$ is exponentially decaying with time due to the ion dust collision and it admits only a positive potential, i.e., the dust ion acoustic wave admits only a compressive soliton. The damping arises due to the ion dust collision under the assumption that the ion hydrodynamical time is much smaller than the ion dust collision time.

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