Nonlinear low frequency wave propagation in electronegative dusty plasma: Effects of *adiabatic* and *nonadiabatic* charge variations

Subrata Sarkar,[1,a)] Samiran Ghosh,[2,b)] Manoranjan Khan,[1,c)] and M. R. Gupta[1]
[1]*Department of Instrumentation Science, Jadavpur University, Kolkata-700 032, India*
[2]*Department of Applied Mathematics, University of Calcutta 92, Acharya Prafulla Chandra Road, Kolkata-700 009, India*

(Received 26 July 2011; accepted 12 August 2011; published online 16 September 2011)

The effects of both *adiabatic* and *nonadiabatic* charge variations on small but finite amplitude nonlinear dust acoustic wave (DAW) have been investigated in an electronegative dusty plasma in presence of a static magnetic field. It is found that in case of *adiabatic* charge variations, the nonlinear wave is governed by the Zakharov-Kuznetsov (ZK) equation which yields the usual solitary wave solution. On the other hand, in case of *nonadiabatic* charge variations, the dynamics is governed by the Zakharov-Kuznetsov-Burgers' (ZKB) equation which exhibits shock like structures. The results are discussed in the context of cometary plasma.
[doi:10.1063/1.3632979]

I. INTRODUCTION

Electronegative dusty plasmas are very ubiquitous in modern technological applications such as plasma coating and plasma processing of materials[1–3] as well as in astrophysics.[4–8] Electrons and ions frequently strike on the surface of the dust grain immersed in the plasma due to their thermal energy. Because of the higher thermal velocity, electrons strike more rapidly on the dust grain than ions and thus make the dust grain negatively charged. However, presence of magnetic field changes the charging characteristics of dust grains in the plasma and also makes the dusty plasma anisotropic. In presence of weak magnetic field, the gyro-radii of both electrons and ions become higher than the dust grain radius, and in this case, the effect of magnetic field on the charging process can be neglected.[9] However, in presence of strong magnetic field, electrons become magnetized in the charging process which results in the reduction of charge on the dust grains. With a further increase of magnetic field, ions also become magnetized resulting in the increase of charge on the dust grains.[10] The presence of negative ions also reduce the charge on dust grains (because of their lower thermal velocity than electrons) by reducing the floating potential acquired by the dust grains[11] and for very low electron concentration (i.e., for very high negative ion concentration) which is very common in low-temperature reactive plasmas,[12] dust grains become positively charged.[13,14] The linear wave propagation characteristics of both dust acoustic wave (DAW)[15] and dust ion acoustic wave (DIAW)[16] have been investigated in electronegative dusty plasma in absence of magnetic field.[13,17] Recently, the DAW instabilities are investigated in electronegative magnetized dusty plasma.[18]

Moreover, the charge on the dust grains is not fixed, it varies with each space time coordinate and thus becomes an extra dynamical variable. The variation of dust charge is associated with the change of floating potential of the plasma. For very high charging frequency compared to dust oscillation frequency, the dust charge instantaneously reaches its equilibrium value at each space-time point determined by the local electrostatic potential and this phenomena is known as *adiabatic* dust charge variation.[19–21] On the other hand, when the charging frequency is low and comparable to the dust oscillation frequency, the dust charge does not instantaneously reach its equilibrium value, instead, produces an anomalous dissipation in the dusty plasma. This phenomena is known as *nonadiabatic* dust charge variation.[22,23] Due to this *nonadiabatic* dust charge variation, the linear modes become damped,[24,25] whereas in the nonlinear regime, this dissipation leads to the formation of shock wave in a dusty plasma.[22,23,26–29] In an un-magnetized electronegative one dimensional (1D) dusty plasma, the effects of both *adiabatic* and *nonadiabatic* dust charge variations on nonlinear DAWs are investigated.[30,31] Recently, the dust acoustic shock wave due to *nonadiabatic* charge variations in a magnetized electronegative 1D dusty plasma is also investigated.[32]

In this paper, we have investigated the effects of both *adiabatic* and *nonadiabatic* dust charge variations on nonlinear DAWs in a magnetized electronegative three-dimensional (3D) dusty plasma using well known reductive perturbation technique (RPT). In case of adiabatic dust charge variations, we have obtained the Zakharov-Kuznetsov (ZK) equation which have usual solitary wave solutions. On the other hand, for *nonadiabatic* dust charge variations, we get Zakharov-Kuznetsov-Burgers' (ZKB) equation. An analytical as well as numerical solution has been derived for both the cases. It has been shown that the *adiabatic* assumption is valid only for large dust grain, whereas for smaller dust grain *nonadiabatic* assumption is to be used. The presence of magnetic field only gives a correction to the dispersion whereas presence of negative ion reduces charge variation induced dissipation and the shock

[a)]Electronic mail: ssarkar08ju@gmail.com.
[b)]Electronic mail: sran_g@yahoo.com.
[c)]Electronic mail: mkhan_ju@yahoo.com.

strength. The formation of shock initiates the density localization which makes the problem relevant to astrophysical phenomena like star formation.

In the manuscript, Sec. II contains the physical assumptions and basic equations describing the model. The nonlinear evolution equations are derived in Sec. III. The analytical and numerical solutions are given in Secs. IV and V, respectively. Finally, the conclusion is presented in Sec. VI.

II. PHYSICAL ASSUMPTIONS AND BASIC EQUATIONS

(1) An unbounded, magnetized, collisionless electronegative dusty plasma constituting of electrons, positive ions, negative ions, and negatively charged dust particles is considered. The charge on the dust grains is not fixed, it varies continuously with time. The external constant magnetic field $\vec{B}$ is acting along z direction i.e., $\vec{B} = B_0\hat{z}$, B_0 is the magnitude of the magnetic field, and $\hat{z}$ is the unit vector in the z direction. The plasma is overall quasi-neutral and the quasi-neutrality condition is $n_{e0} + z_d n_{d0} + n_{-0} - n_{+0} = 0$, where z_d is the number of electrons residing on the dust grain and n_{j0} is the equilibrium number density of jth (e for electron, $+$ for positive ion, and $-$ for negative ion) species plasma.

(2) The gyro-radii of electrons and both ions (positive and negative) are also assumed large compared with the wavelength of the perturbation as well as dust size. Thus, for very low frequency motion in electronegative magneto dusty plasma, the effects of magnetic field on electrons and both ions (positive and negative) are insignificant and their number density distributions can be taken as Boltzmanian

$$n_e = n_{e0}\exp\left(\frac{e\varphi}{T_e}\right), \quad n_+ = n_{+0}\exp\left(-\frac{e\varphi}{T_+}\right),$$
$$n_- = n_{-0}\exp\left(\frac{e\varphi}{T_-}\right), \tag{1}$$

where $T_\pm$ and T_e are the temperatures of positive (negative) ion and electron, respectively.

(3) The gyro-radii of both electrons and ions (positive and negative) are large compared to the dust size. So the normalized expressions for electron and ion (positive and negative) currents for spherical dust grain of radius r_d can be approximated as[11,30–32]

$$I_e = -J_e\exp\left(\frac{Q_d e}{4\pi\epsilon_0 r_d T_e}\right),$$
$$I_- = -J_-\exp\left(\frac{Q_d e}{4\pi\epsilon_0 r_d T_-}\right),$$
$$I_+ = J_+\left(1 - \frac{Q_d e}{4\pi\epsilon_0 r_d T_+}\right), \tag{2}$$

where $J_s = \pi r_d^2 e n_s\sqrt{8T_s/m_s}$; m_s and T_s are the mass and temperature of $s(= e, +, -)$ th species.

It is always convenient to write equations in the dimensionless form; for this purpose, the time and space scales are made dimensionless in units of dust plasma frequency $\omega_{pd}\left(=\sqrt{z_d^2 e^2 n_{d0}/\epsilon_0 m_d}\right)$ and plasma Debye length $\lambda_D\left(=\sqrt{\lambda_{De}^{-2} + \lambda_{D+}^{-2} + \lambda_{D-}^{-2}}\right)$; where, $\lambda_{Ds}\left(=\sqrt{\epsilon_0 T_s/n_{s0}e^2}\right)$ is the sth species particle Debye length. The velocity and plasma potential are made dimensionless in units of dust acoustic speed $c_d\left(=\sqrt{z_d T_e \alpha_d/m_d}\right)$ and T_e/e, respectively. The dust charge Q_d is made dimensionless in units of its equilibrium value $z_d e$ so that charge on dust grains becomes $Q - 1$ (dust grains are negatively charged), where Q is the fluctuating (dimensionless) dust charge. The other dimensionless parameters are: $\alpha_d = z_d n_{d0}/\delta n_{+0}$, $\delta_+ = n_{e0}/n_{+0}$, $\delta_- = n_{-0}/n_{+0}$, $\delta = \delta_+ + 1/\sigma_+ + \delta_-/\sigma_-$, and $\sigma_\pm = T_\pm/T_e$.

Thus, in dimensionless form, the basic equations for the dynamics of low frequency DAW in electronegative magnetized dusty plasmas are as follows:

$$\frac{\partial N_d}{\partial t} + \nabla\cdot(N_d\mathbf{V_d}) = 0, \tag{3}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{V_d}\cdot\nabla\right)\mathbf{V_d} = -\frac{(Q-1)}{\alpha_d}\nabla\Phi + \omega_{cd}(Q-1)\mathbf{V_d}\times\hat{z} - \frac{\gamma_d\sigma_d}{\alpha_d}N_d^{\gamma_d - 2}\nabla N_d, \tag{4}$$

$$\delta\nabla^2\Phi = \delta_+\exp(\Phi) + \delta_-\exp\left(\frac{\Phi}{\sigma_-}\right) - \exp\left(-\frac{\Phi}{\sigma_+}\right) - (Q-1)(1-\delta_+-\delta_-)N_d, \tag{5}$$

where γ_d is adiabatic index, $\omega_{cd} = \Omega_{cd}/\omega_{pd}$ and Ω_{cd} & (= $z_d e B_0/m_d$) is the dust cyclotron frequency. The normalized dust charging equation becomes

$$\left(\frac{\omega_{pd}}{\nu_{ch}}\right)\frac{dQ}{dt} = \Lambda\left[\left(1 - \frac{z_0 Q}{z_0 + \sigma_+}\right)e^{-\frac{\Phi}{\sigma_+}} - A_+ e^{(\Phi + z_0 Q)} - A_- e^{\left(\frac{\Phi + z_0 Q}{\sigma_-}\right)}\right], \tag{6}$$

where $z_0 = z_d e^2/4\pi\epsilon_0 r_d T_e$ is a dusty plasma parameter, ν_{ch} is the charging frequency given by[32]

$$\nu_{ch} = \frac{r_d}{\sqrt{2\pi}}\frac{\omega_{p+}^2}{V_{t+}}\frac{(z_0 + \sigma_+)}{z_0\Lambda}, \quad \Lambda = \frac{\sigma_+\beta_{ch}}{(1 + \sigma_+ + \mu_2^-)}. \tag{7}$$

The expressions for other coefficients A_+, A_-, β_{ch}, and μ_2^- are given in Appendix A.

III. NONLINEAR EVOLUTION EQUATIONS

In order to study the small but finite amplitude nonlinear DAW using RPT, the independent variables are stretched as

$$\xi = \sqrt{\epsilon}(z - Ut), \quad \tau = \epsilon^{3/2}t, \ \eta = \sqrt{\epsilon}x, \quad \zeta = \sqrt{\epsilon}y, \tag{8}$$

where U is the normalized phase velocity of the linear DAW and ϵ is a small parameter that characterizes the strength of nonlinearity. The dependent variables are expanded in powers of ϵ in the following way:

$$\begin{pmatrix} N_d \\ \Phi \\ Q \\ V_{dx} \\ V_{dy} \\ V_{dz} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} N_d^{(1)} \\ \Phi^{(1)} \\ Q^{(1)} \\ 0 \\ 0 \\ V_{dz}^{(1)} \end{pmatrix} + \epsilon \begin{pmatrix} 0 \\ 0 \\ 0 \\ V_{dx}^{(1)} \\ V_{dy}^{(1)} \\ 0 \end{pmatrix} + \epsilon^2 \begin{pmatrix} N_d^{(2)} \\ \Phi^{(2)} \\ Q^{(2)} \\ V_{dx}^{(2)} \\ V_{dy}^{(2)} \\ V_{dz}^{(2)} \end{pmatrix} + \cdots. \tag{9}$$

Now using Eqs. (8) and (9) in the basic Eqs. (3)–(6), one can develop equations in different powers of ϵ (details are given in Appendix B).

A. Adiabatic charge variation: Zakharov-Kuznetsov equation

For *adiabatic* dust charge variations, it is assumed that the dust charging time scale $\left(\sim \nu_{ch}^{-1}\right)$ is much smaller than the dust oscillation time scale $\left(\sim \omega_{pd}^{-1}\right)$ so that for consistent perturbation

$$\frac{\omega_{pd}}{\nu_{ch}} \approx 0. \tag{10}$$

Thus, from Eq. (B7), one obtains

$$Q^{(1)} = -\beta_{ch}\Phi^{(1)}, \tag{11}$$

in the lowest order of ϵ and

$$Q^{(2)} + \beta_{ch}\Phi^{(2)} + \Lambda\left[\frac{z^2(1+B_-)Q^{(1)^2}}{2} - \frac{1}{2}\frac{(1-\sigma_+^2(1+B_-))}{\sigma_+^2}\Phi^{(1)^2} - \frac{z(1-\sigma_+(z+\sigma_+)(1+B_-))}{\sigma_+(z+\sigma_+)}Q^{(1)}\Phi^{(1)}\right] = 0, \tag{12}$$

in the next higher order of ϵ.

The Eqs. (B1) and (11) self-consistently determine the normalized phase velocity U in the following way:

$$U = \sqrt{\frac{\gamma_d\sigma_d}{\alpha_d} + \frac{1}{(1+\alpha_d\beta_{ch})}}. \tag{13}$$

Also x and y components of $\vec{E} \times \vec{B}$ and diamagnetic drift velocities are given by

$$V_{dy}^{(1)} = U^2\frac{(1+\alpha_d\beta_{ch})}{\alpha_d\omega_{cd}}\frac{\partial\Phi^{(1)}}{\partial\eta}, \quad V_{dz}^{(1)} = -U^2\frac{(1+\alpha_d\beta_{ch})}{\alpha_d\omega_{cd}}\frac{\partial\Phi^{(1)}}{\partial\zeta}. \tag{14}$$

Finally, elimination of second order variables from Eqs. (B2)–(B6) and (12) yield the following ZK equation:

$$\frac{\partial\Phi^{(1)}}{\partial\tau} - \alpha\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial\xi} + \beta\frac{\partial^3\Phi^{(1)}}{\partial\xi^3} + \gamma\frac{\partial}{\partial\xi}\left(\frac{\partial^2}{\partial\eta^2} + \frac{\partial^2}{\partial\zeta^2}\right)\Phi^{(1)} = 0, \tag{15}$$

where

$$\alpha = \frac{(1+\alpha_d\beta_{ch})}{2\alpha_d U}\left[3U^2 + \frac{\gamma_d\sigma_d(\gamma_d-2)}{\alpha_d} + \frac{\alpha_d(\delta_+ + \frac{\delta_-}{\sigma_-^2} - \frac{1}{\sigma_+^2})}{\delta(1+\alpha_d\beta_{ch})^3} + \frac{\alpha_d^2\Lambda C}{(1+\alpha_d\beta_{ch})^3} - \frac{3\alpha_d\beta_{ch}}{(1+\alpha_d\beta_{ch})^2}\right], \tag{16}$$

$$\beta = \frac{1}{2U(1+\alpha_d\beta_{ch})^2}; \quad \gamma = \beta\left(1 + \frac{U^4(1+\alpha_d\beta_{ch})^2}{\omega_{cd}^2}\right), \tag{17}$$

and

$$C = (1+B_-)(z\beta_{ch}-1)^2 + \frac{2z\beta_{ch}}{\sigma_+(z+\sigma_+)} - \frac{1}{\sigma_+^2}. \tag{18}$$

B. Nonadiabatic charge variation: Zakharov-Kuznetsov Burgers' equation

For *nonadiabatic* dust charge variations, it is assumed that the dust charging time scale is small but finite compared to the dust oscillation time scale[22,31] so that

$$\frac{\omega_{pd}}{\nu_{ch}} = \nu\sqrt{\epsilon}, \tag{19}$$

where $\nu = O(1)$. Thus, in this case, in the second order of ϵ, instead of Eq. (12), the perturbed dust charging Eq. (B7) becomes

$$\nu U\frac{\partial Q^{(1)}}{\partial\xi} = Q^{(2)} + \beta_{ch}\Phi^{(2)} + \Lambda\left[\frac{z^2(1+B_-)Q^{(1)^2}}{2} - \frac{1}{2}\frac{(1-\sigma_+^2(1+B_-))}{\sigma_+^2}\Phi^{(1)^2} - \frac{z(1-\sigma_+(z+\sigma_+)(1+B_-))}{\sigma_+(z+\sigma_+)}Q^{(1)}\Phi^{(1)}\right]. \tag{20}$$

Proceeding as before, elimination of second order variables lead to the following ZKB equation

$$\frac{\partial\Phi^{(1)}}{\partial\tau} - \alpha\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial\xi} + \beta\frac{\partial^3\Phi^{(1)}}{\partial\xi^3} + \gamma\frac{\partial}{\partial\xi}\left(\frac{\partial^2}{\partial\eta^2} + \frac{\partial^2}{\partial\zeta^2}\right)\Phi^{(1)} = \mu\frac{\partial^2\Phi^{(1)}}{\partial\xi^2}, \tag{21}$$

where α, β, and γ are given by Eqs. (16) and (17) and μ is given by the following equation:

$$\mu = \nu\frac{\alpha_d\beta_{ch}}{2(1+\alpha_d\beta_{ch})^2}. \tag{22}$$

This relation shows that the Burgers' term μ vanishes in absence of dust charge variations $\beta_{ch} = 0$ and also for *adiabatic* dust charge variations ($\nu \approx 0$). Also, μ is independent of the strength of the magnetic field. Thus, the Burgers' term μ is present here only due to *nonadiabatic* dust charge fluctuations. Hence, it is clear that the *nonadiabatic* charge fluctuation plays a dissipative role and this anomalous dissipation occurs due to delay in the charging of dust grains.

IV. STATIONARY SOLUTION

To find the stationary solution of both the Eqs. (15) and (21), let us introduce the following wave frame:

$$\chi = V_f\tau - (k\xi + l\zeta + m\eta) = \sqrt{\epsilon}\left[\frac{c_d(\epsilon V_f + kU)t - (kz + ly + mx)}{\lambda_D}\right], \tag{23}$$

moving with velocity V_f (where k, l, and m are the direction cosines of the wave vector along z, y, and x axes, respectively, so that $k^2 + l^2 + m^2 = 1$).

A. Soliton solution

The ZK equation (15) is an exactly integrable system. So, to obtain the exact analytical solution of this equation, we transform the equation in the above wave frame and then integrating the transformed equation once with respect to the variable χ subject to the boundary conditions both $\Phi^{(1)}$ and $d\Phi^{(1)}/d\chi \to 0$ as $|\chi| \to \infty$, the following equation is obtained:

$$\frac{d^2\varphi}{d\chi^2} = \frac{1}{D}\left[V_f\varphi - \left(\frac{\alpha k}{2}\right)\varphi^2\right]; \; D = \beta k^3 + k\gamma(l^2 + m^2). \tag{24}$$

Here, $\Phi^{(1)} = -\varphi(\varphi > 0)$ since the coefficient of nonlinearity in Eq. (15) implies potential $\Phi^{(1)} < 0$. The energy integral of this equation can be written as

$$\frac{1}{2}\left(\frac{d\varphi}{d\chi}\right)^2 + V(\varphi) = 0. \tag{25}$$

This equation can be interpreted as an equation of motion of a pseudo-particle of unit mass in a force field with potential energy $V(\varphi)$ (Sagdeev Potential) given by

$$V(\varphi) = \left(\frac{\alpha k}{6D}\right)\varphi^3 - \left(\frac{V_f}{2D}\right)\varphi^2, \tag{26}$$

where χ plays the role of time and φ plays a role of generalized coordinate. The necessary condition for the existence of soliton is

$$\frac{d^2V(\varphi)}{d\varphi^2}\Big|_{\Phi^{(1)}=0=\varphi} = -\frac{V_f}{2D} < 0. \tag{27}$$

This shows that the soliton solution exist only if $D > 0$ as $V_f > 0$, otherwise no stable soliton exists. It is clear that all k, l, m, β [Eq. (17)], and γ [Eq. (17)] are always positive. Finally, integrating (25) with respect to χ and using the boundary conditions as above, one can obtain the following single soliton solution:

$$\Phi^{(1)} = -\left(\frac{3V_f}{k\alpha}\right)\cosh^{-2}\left(\frac{\chi}{W}\right), \tag{28}$$

where $3V_f/k\alpha$ is the soliton amplitude and $W\left(=\sqrt{4D/V_f}\right)$ is the spatial width of the soliton.

B. Shock solution

The Burgers' term in ZKB equation (21) implies the possibility of existence of shock structure. The ZKB equation is not exactly integrable system and hence, exact analytical solution of ZKB is not possible. A particular type of solution of ZKB is possible, which exhibits only the monotonic shock structure.

To study the nature of the solution[33] of Eq. (21), transforming the Eq. (21) to the wave frame χ [Eq. (23)] and then integrating the transformed equation with respect to χ subject to the boundary conditions $\Phi^{(1)}$ and $d\Phi^{(1)}/d\chi \to 0$ as $|\chi| \to \infty$, the following equation $\left(\Phi^{(1)} = -\varphi\right)$ is obtained:

$$\frac{d^2\varphi}{d\chi^2} = \frac{1}{D}\left[V_f\varphi - \left(\frac{\alpha k}{2}\right)\varphi^2 - \mu k^2\frac{d\varphi}{d\chi}\right]. \tag{29}$$

This equation has a well known mechanical analogy: it describes a damped anharmonic oscillator whereas before χ plays the role of time and φ plays a role of generalized coordinate. In the $(\varphi, d\varphi/d\chi)$ plane, Eq. (29) has two singular points (0, 0) and $(2V_f/k\alpha, 0)$. Among these two, the former (with $\Phi^{(1)} = -\varphi = 0$) corresponds to the upstream state and latter (with $\Phi^{(1)} = -\varphi = -2V_f/k\alpha$) corresponds to the equilibrium downstream state. The singular point (0, 0) is always a saddle point.

To find the nature of the other singular point i.e., the nature of the shock structure, one can investigate the asymptotic behavior of the solution of Eq. (29). For this, one can substitute $\varphi = 2V_f/k\alpha + \varphi_1$, where $2V_f/k\alpha \gg \varphi_1$ in Eq. (29) and then linearizing it with respect to φ_1, one can obtain

$$\frac{d^2\varphi_1}{d\chi^2} + \left(\frac{\mu k^2}{D}\right)\frac{d\varphi_1}{d\chi} + \left(\frac{V_f}{D}\right)\varphi_1 = 0. \tag{30}$$

The solutions of this equation are proportional to exp $(p\chi)$,[33] where

$$p = -\frac{\mu k^2}{2D} \pm \left(\frac{\mu^2k^4}{4D^2} - \frac{V_f}{D}\right)^{\frac{1}{2}}. \tag{31}$$

It follows from this equation that the singular point $(2V_f/k\alpha, 0)$ [$\equiv (-2V_f/k\alpha, 0)$] is a stable focus for $\mu^2k^4 < 4V_fD$, which always corresponds to the oscillatory shock structures (dispersion dominates over dissipation). On the other hand, the singular point is a stable node for $\mu^2k^4 > 4V_fD$, which always corresponds to the monotonic shock structure (dissipation dominates over dispersion). For monotonic shock structure, the solution of Eq. (29) is given by

$$\Phi^{(1)} = -\varphi = -\frac{V_f}{k\alpha}\left[1 - \tanh\left(\frac{\chi}{2\mu k^2}\right)\right], \tag{32}$$

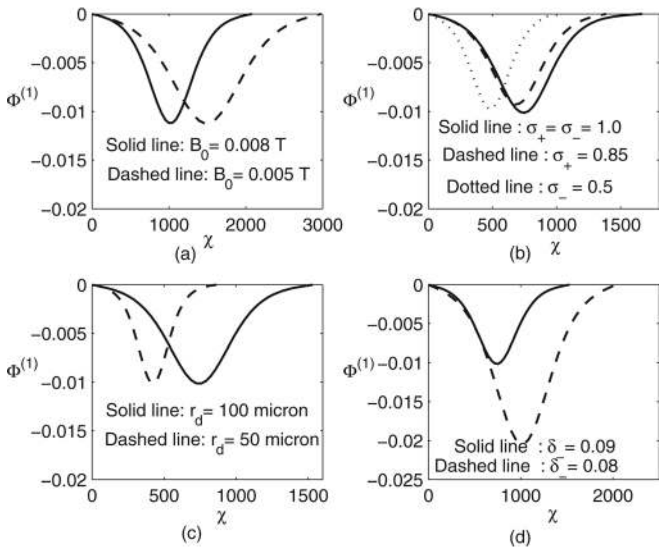

FIG. 1. Solitary wave profiles for different plasma parameters.

where $V_f/k\alpha$ and $2\,\mu k^2/V_f$ are the amplitude and width of the shock, respectively.

V. NUMERICAL SOLUTION

The plasma model considered in this paper is applicable to astrophysical bodies like Comets. To find the numerical solution of the Eqs. (15) and (21), the following representative plasma parameters of Comet Halley ($\sim 10^4$ km from the nucleus) are used:[4,7] $T_e = T_+ \approx 100$ ev; $m_+ = m_- = 1.6726 \times 10^{-27}\,kg$; dust mass density (for ice dust grains) $\rho_d = 9 \times 10^2\,kgm^{-3}$; $\gamma_d = 5/3$, and $n_{+0} \approx 10^9\,m^{-3}$. The density ratios δ_+ and δ_- are determined from the following equilibrium current balance equation:

$$I_{e0} + I_{+0} + I_{-0} = 0 \Rightarrow \frac{\delta_+ e^{-z_0}}{(z_0+\sigma_+)}\sqrt{\frac{m_+\sigma_+}{m_e}} + \frac{\delta_- e^{-z_0/\sigma_-}}{(z_0+\sigma_+)}\sqrt{\frac{\sigma_-\sigma_+ m_+}{m_-}} = 1, \tag{33}$$

for $z_0 = 2.4$. For the values of other physical parameters, we have taken dust grain radius $r_d = 10\ \mu$m, 100 μm.[34,35] Thus, $r_d = 100\ \mu$m yields $\omega_{pd}/\nu_{ch} = 4 \times 10^{-3}$, which justifies the *adiabatic* assumption (10)$[\omega_{pd}/\nu_{ch} \approx 0]$. On the other hand, for smaller dust grains of radius $r_d = 10\ \mu$m, the ratio becomes 0.4 and consequently, this justifies the *nonadiabatic* assumption (19) $[\omega_{pd}/\nu_{ch} = \nu\sqrt{\epsilon}]$. Also, for $r_d = 10\ \mu$m (100 μm), the ratios are $r_d/\rho_e \sim 10^{-3}(10^{-2}) < 1$ and $r_d/\rho_i \sim 10^{-5}(10^{-4}) < 1$ where, $\rho_s (= m_s V_{ts}/B_0 q_s)$ is the gyroradius of the $s(= e, i)$ species. Thus, the assumption of Boltzmann distribution of electrons and ions is justified.

Finally, for the above mentioned plasma parameters, the Eqs. (15) and (21) are solved numerically using fifth order Runge-Kutta-Felhberg scheme with the initial conditions $\Phi^{(1)} = 0$ and $d\Phi^{(1)}/d\chi = 10^{-5}$ at $\chi = -\infty$. For *adiabatic* charge variations, we obtain the solitary wave profiles as shown in Fig. 1, whereas for *nonadiabatic* charge variations, we get the shock like structures as shown in Figs 2 and 3. Fig. 1 shows that with the increase of magnetic field strength (B_0), the spatial width ($W \propto \sqrt{D}$) of the solitons decrease. This happens because of the fact that the dispersive term $D \propto \omega_{cd}^{-2}$ (other plasma parameters remain constant [Eqs. (17) and (24)]. Due to the same reason for lower values of B_0, the shocks are more dispersive than that of higher value of B_0 as shown in Fig. 2. Thus, the magnetic field has dispersive effect on the nonlinear wave.

In case of dust grain radius r_d, the qualitatively similar behavior is shown in Figs. 1(c) and 3. These two figures

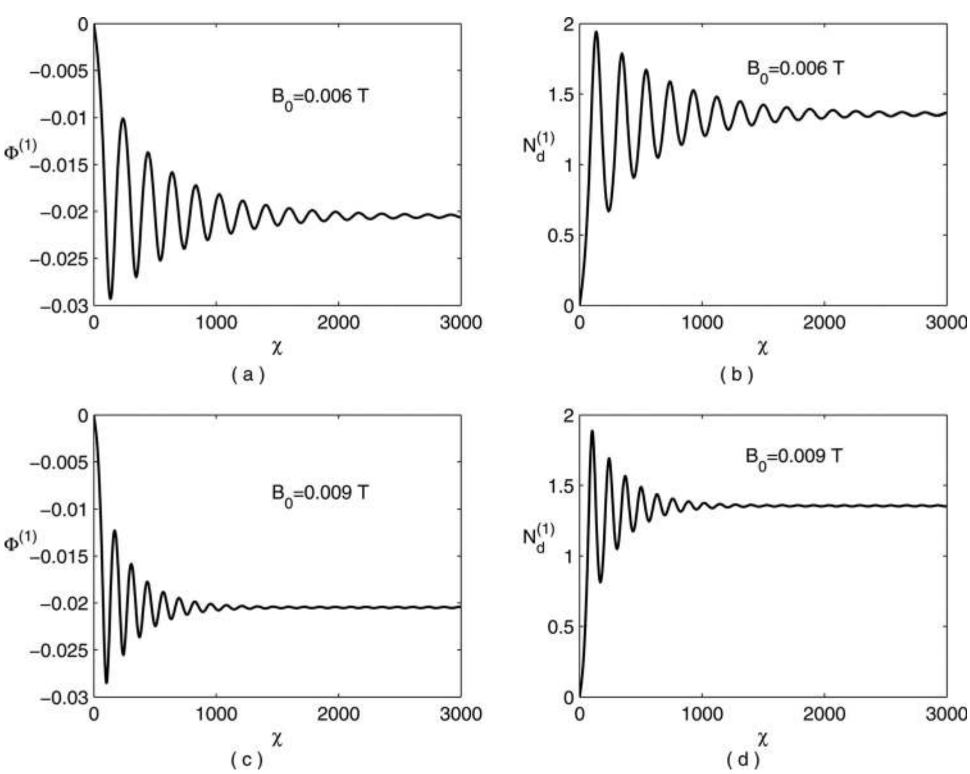

FIG. 2. Shock structures: Variations of potential $\Phi^{(1)}$ and dust density $N_d^{(1)}$ for different magnetic field strength.

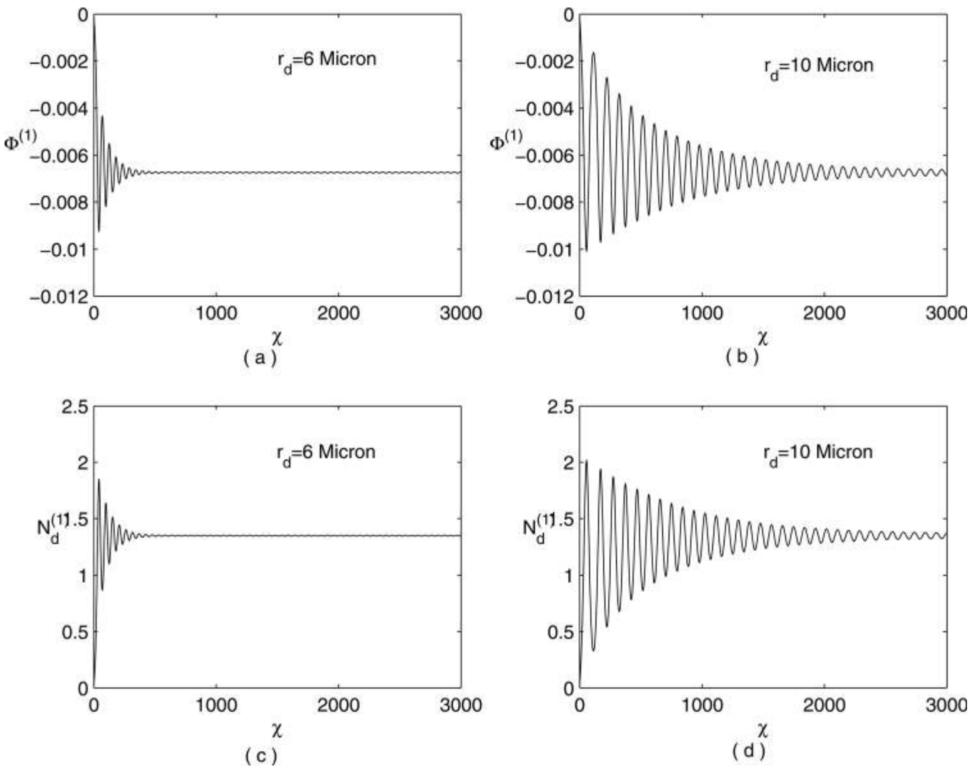

FIG. 3. Variation of potential $\Phi^{(1)}$ and dust density $N_d^{(1)}$ for different dust radius.

show that larger value of r_d, the solitons, and shocks are more dispersive than that of lower value of r_d. This is happened because of the fact that the Burgers term $\mu \propto r_d^{-2}$ (other plasma parameters remain constant). These two figures also show that both soliton and shock are compressive in nature.

The variations of shock strength (height) $2V_f/k\alpha$ with temperature ratios of both negative and positive ions are depicted in Fig. 4. This figure shows that the shock strength increases with the increase of both temperature ratio(σ_+, σ_-), whereas it decreases with the increase of negative ion density ratio(δ_-). These happen due to the fact that the coefficient of nonlinearity α is affected by the concentration and temperature of ions. If concentration of negative ion is increased, α increases and the shock strength (proportional to $1/\alpha$) decreases. In case of soliton, similar phenomena occurs and as a result of the fact, the soliton amplitude increases with temperature ratio, whereas it decreases with the increase of density ratio [Figs. 1(b) and 1(d)].

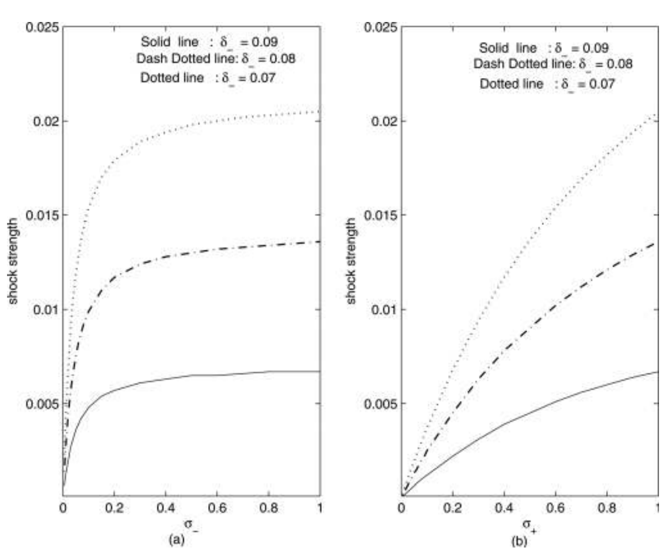

FIG. 4. Variations of shock strength with positive ion temperature ratio (σ_+) and negative ion temperature ratio (σ_-) for different negative ion concentration ratio (δ_-).

VI. CONCLUSIONS

In this paper, the nonlinear propagation characteristics of low frequency wave in electronegative magnetized dusty plasmas incorporating both *adiabatic* and *nonadiabatic* dust charge variations are investigated. The *adiabatic* assumption is valid only for large dust grains and the nonlinear wave is governed by the Zakharov-Kuznetsov equation which yields soliton solution. The *nonadiabatic* assumption is valid only for small dust grains and the dynamics of nonlinear wave is governed by the Zakharov-Kuznetsov Burgers equation which results shock like structures. Both magnetic field and dust grain radius introduce only the corrections in the dispersion of the nonlinear wave. On the other hand, negative ion concentration and temperature affect both the dispersion and amplitude of the wave. Also, presence of negative ion reduces the charge variation induced dissipation and shock strength. The solitons and shocks are compressive in nature. Therefore, the density of the dust particles is increased at the shock front and this increment of dust density due to passage of compressional shock wave followed by enhancement of gravitational attraction sometimes initiates the process of star formation.[36,37] Thus, the present investigation may be relevant to understand the density localization, particle acceleration in cometary dusty plasmas, and also the physics of the formation of stars in astrophysical plasmas.[36,37]

ACKNOWLEDGMENTS

The authors acknowledge the Council of Scientific and Industrial Research (CSIR), Government of India for providing financial support.

APPENDIX A: EXPRESSIONS OF DIFFERENT COEFFICIENTS

$$A_+ = \frac{\delta_+\sigma_+\exp(-z_0)}{(z_0+\sigma_+)}\sqrt{\frac{m_+}{m_e}\frac{T_e}{T_+}},\quad A_- = \frac{\delta_-\exp(-z_0/\sigma_-)}{(z_0+\sigma_+)}\sqrt{\frac{\sigma_+\sigma_- m_+}{m_-}}, \tag{A1}$$

$$\beta_{ch} = \frac{(z_0+\sigma_+)(1+\sigma_++\mu_2^-)}{z_0\sigma_+(1+z_0+\sigma_++\mu_1^-)}, \tag{A2}$$

$$\mu_1^- = \frac{(z_0+\sigma_+)(1-\sigma_-)A_-}{\sigma_-},\ \mu_2^- = \frac{\sigma_+(1-\sigma_-)A_-}{\sigma_-}. \tag{A3}$$

Note that the equilibrium current balance equation is given by $I_{e0}+I_{+0}+I_{-0}=0 \Rightarrow A_+ + A_- = 1$.

APPENDIX B: DETAILS OF THE CALCULATIONS

To the lowest order in ϵ, one can obtain the following first order continuity equation, x, y, and z components of the momentum equation and Poisson's equation:

$$N_d^{(1)} = \frac{1}{U}V_{dz}^{(1)} = -\frac{1}{\alpha_d U^2}\left(\Phi^{(1)} - \gamma_d\sigma_d N_d^{(1)}\right),\quad \Phi^{(1)} + \alpha_d(N_d^{(1)} - Q^{(1)}) = 0, \tag{B1}$$

$$V_{dx}^{(1)} = -\frac{1}{\alpha_d\omega_{cd}}\frac{\partial}{\partial\zeta}\left(\Phi^{(1)} - \gamma_d\sigma_d N_d^{(1)}\right),\quad V_{dy}^{(1)} = \frac{1}{\alpha_d\omega_{cd}}\frac{\partial}{\partial\eta}\left(\Phi^{(1)} - \gamma_d\sigma_d N_d^{(1)}\right). \tag{B2}$$

Similarly, to the next higher order of ϵ, one can obtain the following second order x and y components of momentum equation and Poisson's equation:

$$V_{dx}^{(2)} = -\frac{U}{\omega_{cd}}\frac{\partial V_{dy}^{(1)}}{\partial\xi},\ V_{dy}^{(2)} = \frac{U}{\omega_{cd}}\frac{\partial V_{dx}^{(1)}}{\partial\xi}, \tag{B3}$$

$$\left(\frac{\partial^2}{\partial\eta^2}+\frac{\partial^2}{\partial\zeta^2}+\frac{\partial^2}{\partial\xi^2}\right)\Phi^{(1)} = \Phi^{(2)} + \alpha_d\left(N_d^{(2)} - Q^{(1)}N_d^{(1)} - Q^{(2)}\right) + \frac{1}{2\delta}\left(\delta_+ + \frac{\delta_-}{\sigma_-^2} - \frac{1}{\sigma_+^2}\right)\Phi^{(1)^2} + \alpha_d(N_d^{(2)} - Q^{(2)} - N_d^{(1)}Q^{(1)}). \tag{B4}$$

Again following the same procedure, one can obtain the following equations in the next higher order continuity and z component of the momentum equations:

$$\frac{\partial N_d^{(1)}}{\partial\tau} + \frac{\partial(N_d^{(1)}V_{dz}^{(1)})}{\partial\xi} = U\frac{\partial N_d^{(2)}}{\partial\xi} - \frac{\partial V_{dz}^{(2)}}{\partial\xi} - \frac{\partial V_{dy}^{(2)}}{\partial\zeta} - \frac{\partial V_{dx}^{(2)}}{\partial\eta}, \tag{B5}$$

$$\frac{\partial V_{dz}^{(1)}}{\partial\tau} + \frac{1}{2}\frac{\partial}{\partial\xi}V_{dz}^{(1)^2} + \frac{1}{\alpha_d}\left[Q^{(1)}\frac{\partial\Phi^{(1)}}{\partial\xi} + \frac{\gamma_d\sigma_d(\gamma_d-2)}{2}\frac{\partial N_d^{(1)^2}}{\partial\xi}\right] = \frac{1}{\alpha_d}\frac{\partial}{\partial\xi}\left(\Phi^{(2)} - \gamma_d\sigma_d N_d^{(2)} - \alpha_d U V_{dz}^{(2)}\right). \tag{B6}$$

Now substituting Eqs. (8) and (9) in dust charging Eq. (6), one can obtain the following equation:

$$\begin{aligned}\left(\frac{\omega_{pd}}{\nu_{ch}}\right)&\left[-U\epsilon^{3/2}\frac{\partial Q^{(1)}}{\partial\xi} + \epsilon^{5/2}\left(\frac{\partial Q^{(1)}}{\partial\tau} - U\frac{\partial Q^{(2)}}{\partial\xi} + V_{dz}^{(1)}\frac{\partial Q^{(1)}}{\partial\xi}\right)\right]\\ &= -\epsilon\Lambda\left(Q^{(1)} + \beta_{ch}\Phi^{(1)}\right)\\ &\quad - \epsilon^2\left[Q^{(2)} + \beta_{ch}\Phi^{(2)} + \Lambda\left(\frac{z^2(1+B_-)Q^{(1)^2}}{2} - \frac{1}{2}\frac{(1-\sigma_+^2(1+B_-))}{\sigma_+^2}\Phi^{(1)^2} - \frac{z(1-\sigma_+(z+\sigma_+)(1+B_-))}{\sigma_+(z+\sigma_+)}Q^{(1)}\Phi^{(1)}\right)\right],\end{aligned} \tag{B7}$$

where $B_- = \frac{(1-\sigma_-^2)A_-}{\sigma_-^2}$.

[1]K. Ostrikov, Rev. Mod. Phys. **77**, 489 (2005).
[2]S. V. Vladimirov and K. Ostrikov, Phys. Rep. **393**, 175 (2004).
[3]M. A. Liberman and A. J. Lichtennberg, *Principles of Plasma Discharge and Materials Processing* Wiley, New York, 1994 Chap. 6.
[4]D. A. Mendis and M. Rosenberg, Ann. Rev. Astron. Astrophys. **32**, 419 (1994).
[5]A. Wekhof, Moon Planets **24**, 157 (1981).
[6]I. M. Podgorny, E. M. Dubinin, and P. L. Israelevich, Moon Planets **27**, 135 (1982).
[7]N. Divine, H. Fechtig, T. I. Gombosi, M. S. Hanner, H. U. Keller, S. M. Larson, D. A. Mendis, R. L. Newburn, R. Reinhard, Z. Sekanina, and D. K. Yeomans, Space Sci. Rev. **43**, 1 (1986).
[8]K. A. Anderson, C. W. Carlson, D. W. Curtis, R. P. Lin, H. Reme, J. A. Sauvaud, C. d'Uston, A. Korth, A. K. Richter, and D. A. Mendis, in *Proceedings of the 20th ESLAB Symposium on the Exploration of Halley's Comet, Heidelberg*, 27–31 October 1986, EA SP-250, p. 259.
[9]J. S. Chang and K. Spariosu, J. Phys. Soc. Jpn. **62**, 97 (1993).
[10]V. N. Tsytovich, N. Sato, and G. E. Morfill, New J. Phys. **5**, 43 (2003).
[11]A. A. Mamun and P. K. Shukla, Phys. Plasmas **10**, 1518 (2003); B. M. Annaratone and J. E. Allen, J. Phys. D **38**, 26 (2005).
[12]I. B. Denysenko, K. Ostrikov, S. Xu, M. Y. Yu, and C. H. Diong, J. App. Phys. **94**, 6097 (2003).
[13]N. D'Angelo, J. Phys. D **37**, 860 (2004).
[14]S. H. Kim and R. L. Merlino, Phys. Plasmas **13**, 052118 (2006).
[15]N. N. Rao, P. K. Shukla, and M. Y. Yu, Planet. Space Sci. **38**, 543 (1990); A. Barkan, R. L. Merlino, and N. D'Angelo, Phys. Plasmas **2**, 3563 (1995).
[16]P. K. Shukla and V. P. Silin, Phys. Scr. **45**, 508 (1992); R. L. Merlino, A. Barkan, C. Thompson, and N. D'Angelo, Phys. Plasmas **5**, 1607 (1998); Y. Nakamura, H. Bailung, and P. K. Shukla, Phys. Rev. Lett. **83**, 1602 (1999).
[17]S. V. Vladimirov, K. Ostrikov, M. Y. Yu, and G. E. Morfill, Phys. Rev. E **67**, 036406 (2003).
[18]M. Rosenberg, Phys. Scr. **79**, 015008 (2009); *ibid*, **81**, 015503 (2010).
[19]J. X. Ma and J. Liu, Phys. Plasmas **4**, 253 (1997); B. S. Xie, K. He, and Z. Huang, Phys. Lett. A **247**, 403 (1998).
[20]S. Ghosh, S. Sarkar, M. Khan, M. R. Gupta, and K. Avinash, Phys. Lett. A **298**, 49 (2002).
[21]S. K. El-Labany, W. M. Moslem, W. F. El-Taibany, and M. Mahmoud, Phys. Plasmas **11**, 3303 (2004); W. F. El-Taibany and R. Sabry, *ibid.* **12**, 082302 (2005).

[22]M. R. Gupta, S. Sarkar, S. Ghosh, M. Debnath, and M. Khan, Phys. Rev. E **63**, 046406 (2001).
[23]S. Ghosh, S. Sarkar, M. Khan, K. Avinash, and M. R Gupta, Phys. Plasmas **10**, 4 (2003).
[24]F. Melandso, T. Aslaksen, and O. Havnes, Planet. Space Sci. **41**, 321 (1993); R. K. Varma, P. K. Shukla, and V. Krishan, Phys. Rev. E **47**, 3612 (1993).
[25]M. R. Jana, A. Sen, and P. K. Kaw, Phys. Rev. E **48**, 3930 (1993).
[26]S. I. Popel, M. Y. Yu, and V. N. Tsytovich, Phys. Plasmas **3**, 12 (1996); S. I. Popel, V. N. Tsytovich, and M. Y. Yu, Astrophys. Space Sci. **256**, 107 (1998).
[27]S. I. Popel, A. A. Gisko, A. P. Golub, and T. V Losseva, Phys. Plasmas **7**, 6 (2000).
[28]S. Ghosh, S. Sarkar, M. Khan, and M. R. Gupta, IEEE Trans. Plasma Sci. **29**, 26 (2001).
[29]J. Heinrich, S. H. Kim, and R. L. Merlino, Phys. Rev. Lett. **103**, 115002 (2009).
[30]S. Ghosh, Phys. Plasmas **12**, 094504 (2005).
[31]S. Ghosh, M. Khan, M. R. Gupta, and R. Bharuthram, Phys. Plasmas **13**, 112305 (2006).
[32]S. Ghosh, Z. Ehsan, and G. Murtaza, Phys. Plasmas **15**, 023701 (2008).
[33]V. I. Karpman, *Nonlinear Waves in Dispersive Media* (Pergamon, London, 1975).
[34]J. A. M. McDonnell, P. L. Lamy, and G. S. Pankiewicz, “Physical properties of cometary dust,” in *Comets in the Post-Halley Era, Bamberg Meeting Proceedings*, edited by R. L. Newburn, M. Neugebauer, and J. Rahe (Kluwer Academic, Dordecht, 1991), p. 1043.
[35]E. Grun, D. P. Hamilton, M. Baguhl, R. Riemann, M. Horanyi, and C. Polanskey, Geophys. Res. Lett. **21**, 1035, doi: 10.1029/94GL00701 (1994).
[36]S. A. Kaplan and S. B. Pikelner, *Interstellar Medium* (Harvard University Press, Cambridge, MA, 1982).
[37]L. J. Spitzer, Jr., *Physical Process in Interstellar Medium* (Wiley, New York, 1978).