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Nonlinear evolution of Alfvén waves in a finite beta plasma

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A general form of the derivative nonlinear Schrödinger (DNLS) equation, describing the nonlinear evolution of Alfvén waves propagating parallel to the magnetic field, is derived by using two-fluid equations with electron and ion pressure tensors obtained from Braginskii [in *Reviews of Plasma Physics* (Consultants Bureau, New York, 1965), Vol. 1, p. 218]. This equation is a mixed version of the nonlinear Schrödinger (NLS) equation and the DNLS, as it contains an additional cubic nonlinear term that is of the same order as the derivative of the nonlinear terms, a term containing the product of a quadratic term, and a first-order derivative. It incorporates the effects of finite beta, which is an important characteristic of space and laboratory plasmas.

I. INTRODUCTION

Recently, there has been interest in the asymptotic evolution of Alfvén waves propagating parallel to the ambient magnetic field and its modulation stability. Understanding the nonlinear behavior of Alfvén waves is an essential prerequisite for developing a perception of the magnetic fluctuations and Alfvénic turbulence that commonly occur in plasmas.

Theoretical studies of the nonlinear evolution of Alfvén waves were carried out by Taniuti and Washimi,[1] Hasegawa,[2] Furutani,[3] Patel and Dasgupta,[4] and others who have modeled the evolution equation in terms of a nonlinear Schrödinger (NLS) equation. Rogister[5] first obtained a derivative nonlinear Schrödinger (DNLS) equation for the asymptotic evolution of the finite amplitude Alfvén wave starting from the kinetic theory. The same kind of equation was obtained from the two-fluid equations for cold plasma by Mio *et al.*,[6] Mjølhus,[7] and Ichikawa *et al.*,[8] who also examined the problem of modulational stability and the solitary wave solution. Spangler and Sheerin,[9] Khanna and Rajaram,[10] Sakai and Sonnerup,[11] and Mjølhus and Wyller[12] considered the effect of finite β and showed that the modulational stability critically depends on β. It is indicated that the derivative of the nonlinear term becomes important when the wavelength of the carrier wave is comparable to the scale length of variation of the envelope. Numerical studies of the onset of Alfvénic turbulence from the DNLS equation were done by Spangler, Sheerin, and Payne[13] and Ghosh and Papadopoulos.[14]

Patel and Dasgupta[4] have demonstrated the possibility of the existence of Alfvénic solitons in the Earth's magnetosphere. Plasmas with a finite value of β commonly occur in planetary magnetospheres of Earth, Jupiter, and Saturn.[15,16] In such situations a more detailed search for finite β effects on nonlinearity and dispersion is desirable as a shifted balance between the two may recast the entire scenario of the nonlinear evolution of Alfvén waves and the structure of Alfvénic turbulence.

With this end in view, we have derived a modified form of the DNLS as an evolution equation of the complex wave amplitude of the Alfvén wave. The important point in this derivation is that we have assumed that the (nondimensionalized) wavenumber k of the carrier wave is comparable to ϵ, the slowness parameter for the spatial and temporal scale. It is to be noted that $k \gg \epsilon$ for the usual form of the NLS[4] with cubical nonlinearity of the type $q|\hat{B}|^2\hat{B}$ ($\hat{B}$ is wave amplitude, q is a coefficient), while the derivative nonlinear term of the type $q_1(\partial/\partial\xi)(|\hat{B}|^2\hat{B})$ predominates in the opposite limit, namely, $k \to 0$.

We have not worked under any of these two extreme approximations, with the consequence that both the cubic nonlinear term (i.e., $q|\hat{B}|^2\hat{B}$) and the derivative of the cubic nonlinear term [i.e., $iq_1(\partial/\partial\xi)|\hat{B}|^2\hat{B}$] occur in our derivation. The dependence of the coefficients of nonlinear terms, q and q_1 and β, k, and other parameters of the system is, however, more involved, but they approach their standard values: $q \to 0$, while $q_1 \to -\frac{1}{4}$ as $k \to 0$. Another nonlinear term occurs that contains the product of a quadratic term and a first-order derivative [of the form $iq_2|\hat{B}|^2(\partial\hat{B}/\partial\xi)$]. This additional contribution is a consequence of our assumption on k and using Braginskii's pressure tensor model.[17] The expressions of pressure tensor as given by Braginskii, though appropriate for a collisional plasma, also involve the effects caused by stresses perpendicular to the velocity. Such stresses do not result in dissipation of energy; on the contrary, the effects are dispersive even in the linear approximation and remain finite in the weak collision limit $\Omega_i\tau \gg 1$ (Ω_i is the ion cyclotron frequency, τ is the ion–electron collision time).

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The interpretation of such viscous stresses arising from the longitudinal gradient of transverse velocity components was given earlier by Kaufman.[18] For the reasons explained above, the Braginskii pressure tensor yields in our case important contributions that depend on beta even in the limit of negligible collision. The DNLS equation thus obtained is more general and describes the nonlinear evolution of Alfvén waves adequately. It is also applicable to low-frequency waves propagating through plasma with high values of beta.

II. DERIVATION

For a uniform plasma consisting of ions and electrons and placed in a constant magnetic field, the system of equations is

$$\frac{\partial n_e}{\partial t} + \nabla\cdot(n_e \mathbf{v}_e) = 0, \tag{1a}$$

$$\frac{\partial n_i}{\partial t} + \nabla\cdot(n_i \mathbf{v}_i) = 0, \tag{1b}$$

$$m_e n_e \frac{d\mathbf{v}_e}{dt} = -n_e e\left(\mathbf{E} + \frac{1}{c}\mathbf{v}_e\times\mathbf{B}\right) - \nabla\cdot\mathsf{P}_e, \tag{1c}$$

$$m_i n_i \frac{d\mathbf{v}_i}{dt} = n_i e\left(\mathbf{E} + \frac{1}{c}\mathbf{v}_i\times\mathbf{B}\right) - \nabla\cdot\mathsf{P}_i, \tag{1d}$$

$$\nabla\times\mathbf{B} = \frac{4\pi e}{c}(n_i\mathbf{v}_i - n_e\mathbf{v}_e) + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}, \tag{1e}$$

$$\nabla\cdot\mathbf{B} = 0, \tag{1f}$$

$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \tag{1g}$$

$$\nabla\cdot\mathbf{E} = 4\pi e(n_i - n_e), \tag{1h}$$

where the symbols have their usual meaning, with subscripts e, i for electrons and ions. Here P_e and P_i are the pressure tensors.

We shall express Eqs. (1a)–(1h) in dimensionless form following the procedure of Kakutani *et al.*[19] We shall use the quasineutrality condition, $n_e \approx n_i = n$, and eliminate electron velocity by the relation

$$\mathbf{v}_e = \mathbf{v}_i - (c/4\pi e n_0)\nabla\times\mathbf{B}. \tag{2}$$

The basic two-fluid equations, after eliminating the electron variables, can now be written in dimensionless form as

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}) = 0, \tag{3a}$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla\times(\mathbf{v}\times\mathbf{B}) = -\frac{1}{R_i}\nabla\times\frac{d\mathbf{v}}{dt} - \frac{\beta_i}{R_i}\nabla\times\left(\frac{\nabla\cdot\mathsf{P}_i}{n}\right), \tag{3b}$$

$$\frac{d\mathbf{v}}{dt} = \frac{1}{n}(\nabla\times\mathbf{B})\times\mathbf{B} + \frac{1}{R_e}\frac{d}{dt}\left(\frac{\nabla\times\mathbf{B}}{n}\right) - \frac{1}{1+\mu_1}\frac{1}{n}(\beta_i\nabla\cdot\mathsf{P}_i + \mu_1\beta_e\nabla\cdot\mathsf{P}_e), \tag{3c}$$

where $\mathbf{v}$ is the ion velocity (with suffix i omitted) and the variables n, $\mathbf{v}$, and $\mathbf{B}$ are made dimensionless, respectively, by n_0, the equilibrium density, V_0, the characteristic (Alfvén) velocity, and B_0, the steady magnetic field in the z direction; t and z are normalized by T, a characteristic time (Alfvén time), and L, a characteristic length associated with the system, so that $V_0 = L/T$. The quantities R_i and R_e are the ion and electron cyclotron frequencies expressed in terms of the characteristic frequency T^{-1} and $\beta_{i,e} = v^2_{\text{th}\,i,e}/V_0^2$, where $v_{\text{th}\,i,e}$ is the ion (electron) thermal velocity, $\beta = (\beta_i + \mu_1\beta_e)/1 + \mu_1$, with μ_1 the electron–ion mass ratio.

In Eqs. (3b) and (3c), P_i and P_e are the ion and electron pressure tensors. We use the expressions given by Braginskii.[17] For each species, the elements of pressure tensor are written as

$$P_{zz} = -\eta_0 w_{zz}, \tag{4a}$$

$$P_{xx} = -(\eta_0/2)(w_{xx} + w_{yy}) - (\eta_1/2)(w_{xx} - w_{yy}) - \eta_3 w_{xy}, \tag{4b}$$

$$P_{yy} = -(\eta_0/2)(w_{xx} + w_{yy}) + (\eta_1/2)(w_{xx} - w_{yy}) + \eta_3 w_{xy}, \tag{4c}$$

$$P_{xy} = P_{yx} = -\eta_1 w_{xy} + (\eta_3/2)(w_{xx} - w_{yy}), \tag{4d}$$

$$P_{xz} = P_{zx} = -\eta_2 w_{xz} - \eta_4 w_{yz}, \tag{4e}$$

$$P_{yz} = P_{zy} = -\eta_2 w_{yz} + \eta_4 w_{xz}, \tag{4f}$$

with

$$w_{\alpha\beta} = \frac{\partial v_\alpha}{\partial x_\beta} + \frac{\partial v_\beta}{\partial x_\alpha} - \frac{2}{3}\nabla\cdot\mathbf{v}\delta_{\alpha\beta},$$

$\delta_{\alpha\beta}$ being the Kronecker delta. For ions, the coefficients $\eta_0^{(i)}$, $\eta_1^{(i)}$, etc., are given as

$$\eta_0^{(i)} \approx n_i T_i \tau_i,$$

$$\eta_1^{(i)} \approx n_i T_i/\Omega_i^2\tau_i, \quad \eta_2^{(i)} = 4\eta_1^{(i)},$$

$$\eta_3^{(i)} \approx n_i T_i/2\Omega_i, \quad \eta_4^{(i)} = 2\eta_3^{(i)}.$$

Here, n_i, T_i, and τ_i are, respectively, the ion density, temperature (in electron-volts), and collision time. For electrons, the expressions for $\eta_0^{(e)}, \eta_1^{(e)}, \ldots$, are similar to those for ions given above, but contain the electron density, temperature, and collision time.

We normalize the collision time in terms of Alfvén time and assume

$$\Omega_{i,e}\tau_{i,e} \gg 1. \tag{5}$$

We rewrite Eqs. (1)–(3) in terms of $B_{(\pm)}$, $v_{(\pm)}$ defined in the usual way

$$B_\pm = B_x \pm iB_y, \quad v_\pm = v_x \pm iv_y, \tag{6}$$

where the subscript $\pm$ denotes the left (right) circularly polarized wave. The field variables are perturbed as follows:

$$\begin{bmatrix} n \\ v_z \\ v_\pm \\ B_\pm \end{bmatrix} = \begin{vmatrix} 1 \\ 0 \\ 0 \\ 0 \end{vmatrix} + \begin{vmatrix} \tilde{n} \\ \tilde{v}_z \\ \tilde{v}_\pm \\ \widetilde{B}_\pm \end{vmatrix}. \tag{7}$$

We now substitute the expressions for the pressure tensor and use (5) and (6). For propagation along the longitudinal direction, to which we address ourselves, only the off-diagonal stress components P_{xz} and P_{yz}, which depend on the longitudinal gradient of the transverse components of velocity (i.e., $\partial v_x/\partial z$ and $\partial v_y/\partial z$), make important contributions provided $\Omega_{i,e}\tau_{i,e} \gg 1$. A contribution of $\sim \eta_0(\partial^2 v_z/\partial z^2)$ also appears from P_{zz} in the z component of the equation. However, in view of our scaling [see Eq. (14) later], appropriate for nonlinear propagation of Alfvén waves, it affects neither the dispersion nor the nonlinear terms. It is to be noted that $\eta(\partial^2 v_z/\partial z^2) \sim O(\mu^2\epsilon^2) = O(\epsilon^3)$, while the DNLS is obtained by retaining nonlinearities only to $O(\mu^3\epsilon) = O(\epsilon^{5/2})$. We thus obtain the equations for the left circularly polarized wave as (tildes are omitted)

$$\frac{\partial n}{\partial t} + \frac{\partial v_z}{\partial z} = -\frac{\partial}{\partial z}(n v_z), \tag{8}$$

$$\frac{\partial v_z}{\partial t} + \beta\frac{\partial n}{\partial z} = -\frac{1}{2}\frac{\partial}{\partial z}|B_+|^2 - v_z\frac{\partial}{\partial z}v_z + \beta n\frac{\partial n}{\partial z} - \beta n^2\frac{\partial n}{\partial z}, \tag{9}$$

$$\begin{aligned}\frac{\partial^2 B_+}{\partial t^2} - \frac{\partial^2 B_+}{\partial z^2} + i(\Gamma+\alpha)\frac{\partial^3 B_+}{\partial t\,\partial z^2} - \frac{\Gamma}{R_i}\frac{\partial^4 B_+}{\partial z^4} &= -\frac{\partial^2}{\partial t\,\partial z}(v_z B_z) - \frac{\partial}{\partial z}\left(n\frac{\partial B_+}{\partial z} + v_z\frac{\partial v_+}{\partial z}\right)\\ &\quad - i\Gamma\frac{\partial}{\partial z}\left(\frac{\partial n}{\partial z}\cdot\frac{\partial v_+}{\partial z}\right) + \frac{\Gamma}{R_i}\frac{\partial^2}{\partial t\,\partial z}\left(\frac{\partial n}{\partial z}\cdot\frac{\partial v_+}{\partial z}\right)\\ &\quad - \frac{\Gamma}{R_i}\frac{\partial^3}{\partial z^3}\left(v_z\frac{\partial v_+}{\partial z} + n\frac{\partial B_+}{\partial z}\right) - i\Gamma\frac{\partial^3}{\partial z^3}(v_2 B_+),\end{aligned} \tag{10}$$

with $\Gamma \simeq \beta/R_i$, $2\alpha = (1/R_i - 1/R_e)$, $1/R_e \to 0$.

We now employ a modified form of the reductive perturbation method[20] to obtain the DNLS. We expand the variables as

$$v_+ = [\mu\hat{v}_+(\epsilon z,\epsilon t,\ldots) + \mu^3 v_+^{(3)}(\epsilon z,\epsilon t,\ldots)]e^{i\psi} + \cdots + \text{c.c.}, \tag{11}$$

$$B_+ = \mu\hat{B}_+(\epsilon z,\epsilon t,\ldots)e^{i\psi} + \cdots + \text{c.c.}, \tag{12}$$

$$n = \mu^2 n^{(0)}(\epsilon z,\epsilon t,\ldots) + \cdots, \tag{13}$$

$$v_z = \mu^2 v_z^{(0)}(\epsilon z,\epsilon t,\ldots) + \cdots, \tag{14}$$

while

$$\frac{\partial}{\partial t} = -im\omega + \epsilon\frac{\partial}{\partial t_1} + \epsilon^2\frac{\partial}{\partial t_2} + \cdots, \quad \frac{\partial}{\partial z} = imk + \epsilon\frac{\partial}{\partial z_1} + \epsilon^2\frac{\partial}{\partial z_2} + \cdots. \tag{15}$$

Substituting the above expansions in Eqs. (8)–(10) and setting $\mu = \epsilon^{1/2}$, $k \ll \epsilon$, so that the nonlinearity of order $\mu^3\epsilon$ is balanced by the dispersive term of order $\mu\epsilon^2$, we obtain the following equation for the slow space-time evolution of the amplitude, $\hat{B}_+$:

$$i\frac{\partial\hat{B}_+}{\partial\tau} + p\frac{\partial^2\tilde{B}_+}{\partial\xi^2} = iq_1\frac{\partial}{\partial\xi}(|\hat{B}_+|^2\hat{B}_+) + iq_2|\hat{B}_+|^2\frac{\partial}{\partial\xi}\hat{B}_+ + q|\hat{B}_+|^2\hat{B}_+, \tag{16}$$

where

$$q_1 = -\frac{k}{2D_\omega(v_g^2-\beta)}\left[1 - v_g\left(\frac{\omega}{k}+1\right)\right],$$

$$q_2 = -\frac{k}{2D_\omega(v_g^2-\beta)}\left[1 - v_g\frac{\omega^2}{k^2}\left(\frac{2\omega}{k}-1\right)\right],$$

$$q = -\frac{k^2[1-\Gamma(k^2/R_i)]}{2D_\omega(v_g^2-\beta)} \times\left[1 - v_g\left(\frac{k}{\omega} + \frac{k}{\omega+\Gamma k^2}\right)\right],$$

and

$$D = D(k,\omega) = -\omega^2 + k^2\left(1 - \Gamma\frac{k^2}{R_i}\right) \mp (\Gamma+\alpha)\omega k^2,$$

$$D_\omega = \frac{\partial D}{\partial\omega},\quad D_k = \frac{\partial D}{\partial k},\quad v_g = -\frac{D_k}{D_\omega},$$

$$p = \frac{1}{2}\frac{dv_g}{dk},\quad \xi = \epsilon(z - v_g t),\quad \tau = \epsilon^2 t.$$

Now, we note that Eq. (16) is the slow space-time evolution equation for the Alfvén wave when both ϵ (spectral width) and the central wavenumber k are of the same order. It is to be noted that the term containing the cubic nonlinearity is of the same order as the other two terms of the rhs of Eq. (16) when the powers of k in the expressions for q_1, q_2, and q are taken into account. In the limit of $k\to 0$, $\omega\to 0$, but as $k/\omega \to 1$ we note that

$$\begin{aligned}&q \to 0\\ &q_1 \to -\tfrac{1}{4}\\ &q_2 \sim \tfrac{3}{4}\{(\Gamma+\alpha)k/[1-\beta-2(\Gamma+\alpha)k]\} \to 0,\\ &v_g \to 1.\end{aligned} \tag{17}$$

Thus the results of earlier work can be recovered from Eq. (16). It is to be noted that p, the dispersive term in the DNLS, reduces to

$$-(\Gamma+\alpha) \sim (\beta + \tfrac{1}{2})/R_i, \quad \text{for } k\to 0.$$

Thus for higher values of beta, the dispersion is significantly changed and together with the emergence of new nonlinear terms, the conditions for modulational instability and soliton formations are altered.

III. MODULATIONAL INSTABILITY: LOCALIZED SOLUTIONS

We investigated the linearized stability of Eq. (16) in a straightforward manner by assuming

$$\hat{B}_+ = (b_0 + \delta b)\exp[i(K_0\xi - \Omega\tau + \delta\varphi)], \tag{18}$$

where $\delta b(\xi,\tau)$ and $\delta\varphi(\xi,\tau)$ are the perturbations in amplitude and phase. Substituting (18) in Eq. (16) we linearize equations for δb and $\delta\varphi$ (after separating the real and imaginary parts) and assume a space-time dependence of δb and $\delta\varphi$ as $\exp[i(K_1\xi - \Omega_1\tau)]$. We obtain for Ω, the frequency of the carrier wave,

$$\Omega = pK_0^2 + b_0^2[q - K_0(q_1 + q_2)]. \tag{19}$$

The condition for modulational instability is obtained as

$$(q_1 b_0^2)^2 + p^2K_1^4 - 2b_0^2K_1^2p[K_0(q_1+q_2) - q] < 0. \tag{20}$$

The modulational instability criterion of Mjølhus and Wyller[12] can be recovered from (20) in the long wavelength limit $k_1 \to 0$ and putting $q_2 = q = 0$. In Eq. (20) q introduces a more destabilizing effect and q_2 has a stabilizing effect.

The growth rate of modulational instability can be obtained from Eq. (20). The growth rate γ is obtained from the expression

$$\gamma^2 = -q_1^2 b_0^4 - p^2K_1^4 + [2pK_0(q_1+q_2) - 2pq]b_0^2K_1^2. \tag{21a}$$

In our case, p and q_1 are negative, while q and q_2 are positive. The growth rate depends on both b_0^2 and K_1^2 and an absolute maximum is found to exist at

$$b_0^2 = (|p|/q_1^2)[(|q_1| + q_2)K_0 - q],$$
$$K_1^2 = \frac{1}{2}\left[K_0\left(1 + \frac{q_2}{q_1}\right) - \frac{q}{q_1}\right]. \tag{21b}$$

The maximum growth rate is given by

$$\gamma_{\max} = \frac{1}{2}|p|\left[K_0\left(1+\frac{q_2}{q_1}\right) - \frac{q}{q_1}\right]^2. \tag{21c}$$

With $p = (\beta + \alpha)/R_i$, the growth rate may be significantly increased for high values of β. Furthermore, we note that q_2, which arises mainly because of the Braginskii model, induces a stabilizing influence. On the other hand, q, which occurs as a coefficient of cubic nonlinearity, tends to increase the growth rate.

The DNLS equation obtained by previous workers (i.e., with $q_2 = q = 0$) possesses a solitary wave solution in the form of a solitary modulation envelope. An exact solution has been worked out by Kaup and Newell.[21] We have been able to find the solitary wave solution of Eq. (16) in a way similar to that followed by Ichikawa *et al.*[8] We briefly outline the procedure and restrict ourselves to the case of a "hyperbolic soliton," although the "algebraic soliton" solution can also be obtained.

We assume

$$\hat{B}_+ = b(\xi,\tau)\exp[i\chi(\xi,\tau)] \tag{22}$$

and also assume a traveling wave solution for $b(\xi,\tau)$ and $\chi(\xi,\tau)$. With $y = \xi - \lambda\tau$, λ being the propagation velocity,

$$b(\xi,\tau) = b(y), \tag{23}$$

$$\chi(\xi,\tau) = K\xi - \Omega\tau + \theta(y). \tag{24}$$

We substitute Eqs. (22)–(24) in Eq. (16) and obtain the following two equations for b and θ:

$$p\frac{d^2b}{dy^2} + (\Omega - pK^2)b + (\lambda - 2pK)b\frac{d\theta}{dy} - pb\left(\frac{d\theta}{dy}\right)^2 = (q - QK)b^3 - Qb^3\frac{d\theta}{dy}, \tag{25}$$

$$p\frac{d\theta}{dy} = \frac{A}{b^2} + \frac{\lambda - 2pK}{2} + \frac{R}{4}b^2, \tag{26}$$

with $Q = q_1 + q_2$, $R = 3q_1 + q_2$, and where A is a constant of integration, to be determined from the boundary condition $d\theta/dy \to 0$ as $|y| \to \infty$. Combining Eqs. (25) and (26) we obtain the equation for $b^2(\Phi = b^2)$:

$$\begin{aligned} P^2\left(\frac{d\Phi}{dy}\right)^2 &= \frac{1}{3}\left(\frac{R^2}{4} - RQ\right)\Phi^4 \\ &+ [2(q - QK)p - (\lambda - 2pK)Q]\Phi^3 \\ &+ \left[4A\left(\frac{R}{2} - Q\right) - (\lambda - 2pK)^2 \right. \\ &\left. - 4(q + QK)p\Phi_0\right]\Phi^2 \\ &+ 4B\Phi - 4A^2 = F(\Phi), \end{aligned} \tag{27}$$

where

$$\lim_{|y|\to\infty}\Phi = \Phi_0$$

and

$$A = -\frac{R}{4}\Phi_0^2 - \frac{\lambda - 2pK}{2}\Phi_0,$$

$$4B = \tfrac{2}{3}R(R - Q)\Phi_0^3 + 2\{(\lambda - 2pK)[(R/2) - Q] + (q - QK)p\}\Phi_0^2 + 2(\lambda - 2pK)^2\Phi_0.$$

It is verified that the rhs of Eq. (27), $F(\Phi)$, has a multiple root at $\Phi = \Phi_0$ and $F'(\Phi_0) = 0$. Thus the solution for Φ can be obtained from Eq. (27). After a straightforward, but lengthy, calculation we obtain

$$\Phi = b^2 = \Phi_0 + \frac{2F_2}{-F_3 + \delta\sqrt{F_3^2 - 4F_2F_4}\cosh[\sqrt{F_2}(y - y_0)]},$$

where

$$F_2 = \tfrac{1}{2}F''(\Phi_0), \quad F_3 = \tfrac{1}{6}F'''(\Phi_0), \quad F_4 = \tfrac{1}{24}F'''',$$

and $\delta = +1$ for a bright soliton; -1 for a dark soliton.

The condition for the existence of a solitary wave and envelope depends on whether $F''(\Phi_0)$ and $F_3^2 - 4F_2F_4$ are positive. Moreover, if $|F_3| \geqslant \sqrt{F_3^2 - 4F_2F_4}$ the existence of a solitary envelope with two spikes is possible.

IV. DISCUSSIONS

A more general form of the DNLS equation describing the nonlinear evolution of the complex amplitude of the Alfvén wave is derived. It contains a term with a cubic nonlinearity, a term containing a product of a quadratic term with a derivative term, in addition to the usual derivative nonlinear term. It can be easily verified that this form of the DNLS [Eq. (16)] reduces to the standard form obtained by other researchers in the limit $k \to 0$, $\omega \to 0$, $k/\omega \to 1$. The dispersion relation incorporates the effects of finite beta because the Braginskii model of the pressure tensor has been used.

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