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Nonlinear electron acoustic waves in presence of shear magnetic field

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Nonlinear electron acoustic waves are studied in a quasineutral plasma in the presence of a variable magnetic field. The fluid model is used to describe the dynamics of two temperature electron species in a stationary positively charged ion background. Linear analysis of the governing equations manifests dispersion relation of electron magneto sonic wave. Whereas, nonlinear wave dynamics is being investigated by introducing Lagrangian variable method in long wavelength limit. It is shown from finite amplitude analysis that the nonlinear wave characteristics are well depicted by KdV equation. The wave dispersion arising in quasineutral plasma is induced by transverse magnetic field component. The results are discussed in the context of plasma of Earth's magnetosphere. © *2013 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4848717]

I. INTRODUCTION

Study of nonlinear wave propagation in a magnetized plasma is of considerable interest from both theoretical and experimental point of view. Inclusion of magnetic field in a plasma system can change entirely the dynamics of a wave introducing a different space and time scale as compared to un-magnetized plasma. With respect to the wave property, magnetic field introduces a cut-off in the linear dispersion relation.[1–6] Several past analyses have considered one or two dimensional forms to describe magnetized plasma where the directions of motion are perpendicular to the magnetic field. However, in actual plasma scenario, rapid motion of electrons in the direction parallel to the magnetic field can cause breakdown of the ideal one or two dimensional formulation. Hence, it will be more practical and challenging to impose a three dimensional magnetic field structure in the fluid theory to describe plasma. The present paper is devoted to enquire about the effects of ambient magnetic field on characteristics of nonlinear electron acoustic wave (EAW) propagation in the presence of two electron species of vastly different temperature by employing Lagrangian transformation technique. The analysis includes the effect of self generated magnetic field perturbations also.

Before entering into illustration of the work, let us look back on some of the basic facts and relevance of this particular EAW propagation in plasma. This is a high frequency electrostatic wave that can exist in plasma with two electron species—hot electrons and cold electrons. Less energetic cold plasma from earth's ionosphere when mixes with high energetic hot plasma of earth's magnetosphere, intense packet of electron acoustic wave is formed in the flow boundary region. Thus, this wave has huge applicability towards interpreting various satellite observations in different parts of earth's magnetosphere. High frequency part of broadband electrostatic noise (BEN) emissions observed by Fast Auroral Snapshot (FAST), POLAR satellite, Viking satellite near auroral region,[7–10] Hiss emissions observed by Dynamics Explorer-1 (DE-1) satellite across the polar cusp,[11,12] and several other high frequency emissions can be explained in terms of electron acoustic wave.[13,14] Study of electron acoustic wave is also crucial in laboratory plasma with two temperature electron species, beam plasma interaction and stimulated electron acoustic scattering in the laser plasma interaction.[15–17]

The electron acoustic speed in such a plasma is given by $c_{se} = \sqrt{T_h/m_{eff}}$, where m_{eff} is the modified cold electron mass enhanced by the equilibrium density ratio of hot and cold electron species as, $m_{eff} = m(n_{h0}/n_{c0})$.[18] The general mechanism of this particular mode is that the cold electrons behave like massive particles and oscillate in a background of Boltzmann distributed hot electrons which provide the oscillation energy.[19] This mode is basically an analog to ion acoustic wave with cold electrons playing the role of ions providing the inertia.[20–23] In the fast time scale of electrons, ions are taken to be at rest forming positively charged background and maintaining charge neutrality throughout the plasma. The phase velocity of the wave lies between the thermal velocity of cold and hot electron species as $v_c < \omega/k < v_h$, where $v_{c,h} = (T_{c,h}/m)^{1/2}$, k is the wave vector. As only high frequency electrons are involved in whole dynamics, there is always a chance of wave damping. However, according to the study of Gary and Tokar,[24] there exists some parameter regime of density and temperature for which this wave can sustain electrostatic fluctuations while propagating in plasma. First is, cold electron temperature (T_c) should be much less than hot electron temperature (T_h), i.e., $T_c \ll T_h$ and second is, hot electron density (n_h) should be much larger than cold electron density (n_c), i.e., $n_h \gg n_c$.

In both space plasma or laboratory plasma, where this wave can possibly exist, plasma is found to be magnetized. This instigates researchers to carry out numerous studies on propagation of EAW in the presence of magnetic field. Most of the approaches are accomplished by using either reductive perturbation technique (RPT) or Sagdeevs pseudo potential technique.[4,19,25–27]

On the other hand, Lagrangian transformation has been proven to be competent for years to find exact arbitrary amplitude solution of nonlinear systems.[28–31] In our previous study also, this powerful tool was successfully employed to obtain exact solution when nonlinear propagation of EAW in the presence of uniform magnetic field was considered and approximate solution when both magnetic field and dispersion were present.[32,33] In continuation to this types of analysis, this paper aims to pursue a detailed investigation employing Lagrangian transformation technique, on a full nonlinear three dimensional problem of EAW propagation in magnetized plasma medium where the magnetic field fluctuations due to plasma motion are incorporated. Though, due to the mathematical intricacies involved here, it is nearly impossible to get an exact analytical solution of this particular problem, a small but finite amplitude wave solution can be attained through perturbative analysis. The result indicates formation of a solitary wave due to interplay between nonlinear wave steepening and dispersion processes. It should be noted that the usual wave dispersion is absent here as the whole analysis is performed for a quasi-neutral plasma in a long wavelength limit (i.e., $k\lambda_D \ll 1$, λ_D is Debye length). It is the magnetic field fluctuation which through some collective process induces wave dispersion in plasma. This seems to be the most intriguing finding of our work.

The paper is organized as follows. In Sec. II, basic equations and Lagrangian method are described. The linear mode dispersion relation and small amplitude analysis are presented in Sec. III. Finally, the results are discussed in appropriate physical context in Sec. IV.

II. GOVERNING EQUATIONS AND LAGRANGIAN TRANSFORMATION TECHNIQUE

We consider a homogeneous, collisionless plasma medium composed of background ions, inertia-less hot electrons and inertial cold electrons immersed initially in a magnetic field of intensity $\mathbf{B}$ in the direction of $\hat{z}$ axis. Fluid theory is applied to investigate the low frequency wave propagation across the magnetic field, where small fluctuations of the magnetic field occur in transverse direction due to plasma current. These fluctuations, in turn, have an impulsive effect on plasma flow.

In comparison with highly mobile electrons, massive ions can be assumed to be stationary. They form a positively charged background and maintain the charge neutrality throughout the plasma. Inertialess hot electrons, on the other hand, follow Boltzmann distribution under weak field consideration. For them, electrostatic force is being balanced by the pressure gradient force, as follows:

$$\frac{eE}{m} = -\frac{T_h}{m}\frac{\partial \ln n_h}{\partial x}. \tag{1}$$

Hence, the set of governing equations of this particular plasma system encloses the complete dynamics of inertial cold electrons along with Maxwell's equations as given below

$$mn_c\frac{d\mathbf{v}_c}{dt} = -n_c e\left[\mathbf{E} + \frac{1}{c}\mathbf{v}_c \times \mathbf{B}\right], \tag{2}$$

$$\nabla \times \mathbf{B} = -\frac{4\pi e n_c v_c}{c}, \tag{3}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \tag{4}$$

$$\nabla . \mathbf{E} = 4\pi e(n_0 - n_c - n_h), \tag{5}$$

where, n_c, n_h, n_0 are densities of cold electron, hot electron, and ion fluid, respectively, $\mathbf{v}_c$ is the cold electron velocity, T_h is the hot electron temperature with $\mathbf{E}$ being the electric field. Equation (3) is written in the context that the current flow is solely carried by cold electrons and not by hot electrons. Because, the hot electrons being Boltzmann distributed have no directed velocity to contribute to the current flow. Again, the term due to displacement current is neglected in this equation due to the fact that we are considering very low frequency wave mode for which fluctuation of electric field is minimal. We proceed by combining the above equations

$$m\frac{d\mathbf{v}_c}{dt} = T_h\nabla \ln n_h + \frac{1}{4\pi n_c}(\nabla \times \mathbf{B}) \times \mathbf{B}, \tag{6}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[\mathbf{v}_c \times \mathbf{B} + \frac{mc}{e}\frac{d\mathbf{v}_c}{dt}\right]. \tag{7}$$

The above equations are now split up in component form assuming all the variables are functions of x and t only. The three components of momentum equation of cold electrons are

$$\left(\frac{\partial}{\partial t} + v_{cx}\frac{\partial}{\partial x}\right)v_{cx} = \frac{T_h}{m}\frac{\partial \ln n_h}{\partial x} - \frac{1}{4\pi m n_c}\frac{\partial}{\partial x}\left(\frac{B_z^2 + B_y^2}{2}\right), \tag{8}$$

$$\left(\frac{\partial}{\partial t} + v_{cx}\frac{\partial}{\partial x}\right)v_{cy} = \frac{1}{4\pi m n_c}B_x\frac{\partial B_y}{\partial x}, \tag{9}$$

$$\left(\frac{\partial}{\partial t} + v_{cx}\frac{\partial}{\partial x}\right)v_{cz} = \frac{1}{4\pi m n_c}B_x\frac{\partial B_z}{\partial x}. \tag{10}$$

The equation of continuity for cold electrons and Poisson's equation, respectively, are,

$$\frac{\partial n_c}{\partial t} + \frac{\partial}{\partial x}(n_c v_{cx}) = 0, \tag{11}$$

$$\frac{\partial E}{\partial x} = 4\pi e(n_0 - n_c - n_h). \tag{12}$$

Finally, the three components of magnetic field equations are

$$\frac{\partial B_x}{\partial t} = 0, \tag{13}$$

$$\frac{\partial B_y}{\partial t} + \frac{\partial}{\partial x}(B_y v_{cx} - B_x v_{cy}) = -\frac{c}{4\pi e}\frac{\partial}{\partial x}\left(\frac{B_x}{n_c}\frac{\partial B_z}{\partial x}\right), \tag{14}$$

$$\frac{\partial B_z}{\partial t} + \frac{\partial}{\partial x}(B_x v_{cz} - B_z v_{cx}) = \frac{c}{4\pi e}\frac{\partial}{\partial x}\left(\frac{B_x}{n_c}\frac{\partial B_y}{\partial x}\right). \tag{15}$$

where B_x is considered to be invariant in both time and space. Henceforth, it would be denoted as B_{x0}. Normalizing magnetic field components by initial magnetic field in $\hat{z}$ direction, i.e., B_{z0}, the above equations can be written as

$$\left(\frac{\partial}{\partial t}+v_{cx}\frac{\partial}{\partial x}\right)b_y+b_y\frac{\partial v_{cx}}{\partial x}-\alpha\frac{\partial v_{cy}}{\partial x}=-\frac{c\alpha B_{z0}}{4\pi e}\frac{\partial}{\partial x}\left(\frac{1}{n_c}\frac{\partial b_z}{\partial x}\right), \tag{16}$$

$$\left(\frac{\partial}{\partial t}+v_{cx}\frac{\partial}{\partial x}\right)b_z+b_z\frac{\partial v_{cx}}{\partial x}-\alpha\frac{\partial v_{cz}}{\partial x}=\frac{c\alpha B_{z0}}{4\pi e}\frac{\partial}{\partial x}\left(\frac{1}{n_c}\frac{\partial b_y}{\partial x}\right). \tag{17}$$

Here, b_y, b_z are normalized magnetic field components along $\hat{y}, \hat{z}$ directions, respectively. $\alpha = B_{x0}/B_{z0}$ is a dimensionless quantity.

The symmetry occurring in the governing equations, Eqs. (8)–(10) and (16) and (17), invokes us to apply Lagrangian fluid description where the Eulerian coordinates (x, t) are transformed to the new Lagrangian coordinates (ξ, τ) through the following relation:[34]

$$\xi = x-\int_0^\tau v_{cx}(\xi,\tau)d\tau; \quad \tau = t. \tag{18}$$

The most beneficial consequence of this method is that the nonlinear convective derivative in the governing equations is now simply transformed to time derivative in the new Lagrangian variable presentation reducing the governing equations to simpler forms. The relation between old and new space derivatives at a certain time is determined by the initial density profile and instantaneous density profile as

$$\frac{\partial}{\partial \tau}\equiv\frac{\partial}{\partial t}+v_{cx}\frac{\partial}{\partial x}, \quad \frac{\partial \xi}{\partial x}\equiv\frac{n_c(\xi,\tau)}{n_c(\xi,0)}. \tag{19}$$

However, before transforming the whole system to the new Lagrangian coordinates, let us normalize time, space and velocity in the following manner

$$\hat{\tau}=\tau v_A/\delta_e, \quad \hat{\xi}=\xi/\delta_e, \quad \hat{v}_c = v_c/v_A, \tag{20}$$

where hat signifies normalized variables, $v_A = B_0/\sqrt{4\pi n_{c0} m}$ is the Alfven velocity, and $\delta_e = c/\omega_p$, $\omega_p = \sqrt{4\pi n_{c0}e^2/m}$ being cold electron plasma frequency. Now, under Lagrangian transformation, our governing equations evolve into the following set of equations:

$$\frac{\partial}{\partial \tau}\left(\frac{1}{n_c}\right)=\frac{1}{n_c(\xi,0)}\frac{\partial v_{cx}}{\partial \xi}, \tag{21}$$

$$\frac{\partial}{\partial\hat{\tau}}\left(\frac{b_y}{\hat{n}_c}\right)-\frac{\alpha}{\hat{n}_c(\xi,0)}\frac{\partial \hat{v}_{cy}}{\partial\hat{\xi}}=-\frac{\alpha}{\hat{n}_c(\xi,0)}\frac{\partial}{\partial\hat{\xi}}\left(\frac{1}{\hat{n}_c(\xi,0)}\frac{\partial b_z}{\partial\hat{\xi}}\right), \tag{22}$$

$$\frac{\partial}{\partial\hat{\tau}}\left(\frac{b_z}{\hat{n}_c}\right)-\frac{\alpha}{\hat{n}_c(\xi,0)}\frac{\partial \hat{v}_{cz}}{\partial\hat{\xi}}=\frac{\alpha}{\hat{n}_c(\xi,0)}\frac{\partial}{\partial\hat{\xi}}\left(\frac{1}{\hat{n}_c(\xi,0)}\frac{\partial b_y}{\partial\hat{\xi}}\right), \tag{23}$$

$$\frac{\partial \hat{v}_{cx}}{\partial\hat{\tau}}=\frac{v_{th}^2}{v_A^2}\frac{\hat{n}_c}{\hat{n}_c(\xi,0)}\frac{\partial \ln n_h}{\partial\hat{\xi}}-\frac{1}{\hat{n}_c(\xi,0)}\frac{\partial}{\partial\hat{\xi}}\left(\frac{b_z^2+b_y^2}{2}\right), \tag{24}$$

$$\frac{\partial \hat{v}_{cy}}{\partial\hat{\tau}}=\alpha\frac{1}{\hat{n}_c(\xi,0)}\frac{\partial}{\partial\hat{\xi}}\left(\frac{b_z^2+b_y^2}{2}\right), \tag{25}$$

$$\frac{\partial \hat{v}_{cz}}{\partial\hat{\tau}}=\alpha\frac{1}{\hat{n}_c(\xi,0)}\frac{\partial b_z}{\partial\hat{\xi}}. \tag{26}$$

We proceed by eliminating $\hat{v}_{cx}$ from Eq. (24) by using Eq. (21) to produce the following equation:

$$\frac{\partial^2}{\partial\hat{\tau}^2}\left(\frac{1}{\hat{n}_c}\right)=\frac{v_{th}^2}{v_A^2}\frac{1}{\hat{n}_c(\xi,0)}\frac{\partial}{\partial\hat{\xi}}\left[\frac{\hat{n}_c}{\hat{n}_c(\xi,0)}\frac{\partial}{\partial\hat{\xi}}\ln(n_0-n_c)\right] -\frac{1}{\hat{n}_c(\xi,0)}\frac{\partial}{\partial\hat{\xi}}\left[\frac{1}{\hat{n}_c(\xi,0)}\frac{\partial}{\partial\hat{\xi}}\left(\frac{b_z^2+b_y^2}{2}\right)\right]. \tag{27}$$

In deriving this equation, we have used the fact that quasi-neutrality is sustained throughout the plasma for long wave length analysis so that $n_h = n_0 - n_c$. Further simplification is achieved if we introduce another variable, $\zeta = \int^{\xi}\hat{n}_{c0}(\xi',0)d\xi'$, which reduces the equations to the following final forms:

$$\frac{\partial^2}{\partial\hat{\tau}^2}\left(\frac{1}{\hat{n}_c}\right)=\frac{v_{th}^2}{v_A^2}\frac{\partial}{\partial\zeta}\left[\hat{n}_c\frac{\partial}{\partial\zeta}\ln(n_0-n_c)\right]-\frac{\partial^2}{\partial\zeta^2}\left(\frac{b_z^2+b_y^2}{2}\right), \tag{28}$$

$$\frac{\partial}{\partial\hat{\tau}}\left(\frac{b_y}{\hat{n}_c}\right)-\alpha\frac{\partial\hat{v}_{cy}}{\partial\zeta}=-\alpha\frac{\partial^2 b_z}{\partial\zeta^2}, \tag{29}$$

$$\frac{\partial}{\partial\hat{\tau}}\left(\frac{b_z}{\hat{n}_c}\right)-\alpha\frac{\partial\hat{v}_{cz}}{\partial\zeta}=\alpha\frac{\partial^2 b_y}{\partial\zeta^2}, \tag{30}$$

$$\frac{\partial\hat{v}_{cy}}{\partial\hat{\tau}}=\alpha\frac{\partial b_y}{\partial\zeta}, \tag{31}$$

$$\frac{\partial\hat{v}_{cz}}{\partial\hat{\tau}}=\alpha\frac{\partial b_z}{\partial\zeta}. \tag{32}$$

These final equations describe general evolution dynamics for an electron acoustic wave propagation across a variable magnetic field in a two electron fluid plasma. These equations are basically coupled differential equations of second order and due to the mathematical complications involved here, it is nearly impossible to obtain an exact solution from these set of equations. No matter, a perturbative analysis will do well to find finite amplitude solution.

III. LINEAR AND NONLINEAR ANALYSIS

Perturbative scheme is implemented first to identify the basic linear mode involved here and then to investigate the nonlinear propagation characteristics of the wave mode.[35] So, we expand the variables about their equilibrium values such that the perturbation entities have value much smaller than unity

$$n_c = 1+\tilde{n} \quad b_z = 1+b_{z1} \quad b_y = 0+b_{y1}$$
$$\hat{v}_{cy} = 0+v_{cy1} \quad \hat{v}_{cz} = 0+v_{cz1}. \tag{33}$$

For a linear analysis, the applied perturbations are presumed to be small enough, so that the terms up to first order only are retained. Hence, under this linear approximation, Eq. (28), and Eq. (30), Eq. (32) combined together, reduce to the following two equations:

$$\frac{\partial^2 \tilde{n}}{\partial \hat{\tau}^2} = \frac{c_{se}^2}{v_A^2}\frac{\partial^2 \tilde{n}}{\partial x^2} + \frac{\partial^2 b_{z1}}{\partial x^2}, \tag{34}$$

$$\frac{\partial}{\partial \hat{\tau}}(b_{z1} - \tilde{n}) = 0. \tag{35}$$

These relations hold when α^2 is taken to be small $\sim\tilde{n}$, which means that the magnetic field is not exactly perpendicular to the wave motion, but it has a very slight inclination angle. The above Eq. (35) implies $b_{z1} = \tilde{n}$ and consequently Eq. (34) takes the form of following wave equation:

$$\frac{\partial^2 \tilde{n}}{\partial \hat{\tau}^2} = \left(1 + \frac{c_{se}^2}{v_A^2}\right)\frac{\partial^2 \tilde{n}}{\partial x^2}. \tag{36}$$

Now, Fourier transform of this equation, assuming the density perturbation of the form of a sinusoidal wave ($\sim\exp i(\omega t - kx)$), yields

$$\omega^2 = \left(1 + \frac{c_{se}^2}{v_A^2}\right)k^2. \tag{37}$$

It is the linear dispersion relation for electron magneto sonic wave which is an acoustic type wave propagating in the direction perpendicular to magnetic field. The wave dynamics is regulated by both kinetic pressure and magnetic pressure. Actually, the kind of magnetic and electric field perturbations (implicit) employed here correspond to excitation of magneto sonic waves whose characteristic frequency is much less than the electron cyclotron frequency. The dispersion relation clearly shows that the wave is non-dispersive in nature. However, the later analysis will exhibit that for weakly nonlinear case, the wave is described by Kdv equation and the dispersion in quasineutral plasma arises due to magnetic field fluctuations.

Now, with complete perception of linear wave mode involved here, we continue our study to nonlinear regime where the terms up to second order are retained in the final Eqs. (28)–(32). Defining $M = c_{se}^2/v_A^2$, Eq. (36) is re-written in the following way:

$$\left(\frac{\partial}{\partial \hat{\tau}} - \sqrt{1+M^2}\frac{\partial}{\partial x}\right)\left(\frac{\partial}{\partial \hat{\tau}} + \sqrt{1+M^2}\frac{\partial}{\partial x}\right)\tilde{n} = 0. \tag{38}$$

This symbolizes two waves propagating in two opposite directions, i.e., positive and negative $\hat{x}$. Actually, for small amplitude nonlinearity, ξ, τ no longer remain Lagrangian variables but become equivalent to x, t. Now, if we consider waves moving in positive $\hat{x}$ direction only, then for these waves the following relation holds in a weakly nonlinear limit:

$$\frac{\partial}{\partial \hat{\tau}} = -\sqrt{1+M^2}\frac{\partial}{\partial x}. \tag{39}$$

According to the perturbation scheme given in Eq. (33), we substitute the values of density, magnetic field, and velocity in our final set of Eqs. (28)–(32) and retain terms up to second order. Next, we eliminate all other variables except density $\tilde{n}$, we get the following equation in terms of density perturbation:

$$\left(\frac{\partial}{\partial \hat{\tau}} + \sqrt{1+M^2}\frac{\partial}{\partial x}\right)\tilde{n} + \frac{3+4M^2}{4\sqrt{1+M^2}}\frac{\partial^2 \tilde{n}^2}{\partial x^2} + \frac{\alpha^2}{(1+M^2)^2}\frac{\partial^3 \tilde{n}}{\partial x^3} = 0, \tag{40}$$

where we have used Eq. (39) to impose that we are concerned with unidirectional waves. A further transformation of coordinates, $X = x - \sqrt{1+M^2}\hat{\tau}$, $\bar{\tau} = \hat{\tau}$, renders the usual form of KdV

$$\frac{\partial \tilde{n}}{\partial \bar{\tau}} + 2\left(\sqrt{1+M^2} - \frac{1}{4\sqrt{1+M^2}}\right)\tilde{n}\frac{\partial \tilde{n}}{\partial X} + \frac{\alpha^2}{(1+M^2)^2}\frac{\partial^3 \tilde{n}}{\partial X^3} = 0. \tag{41}$$

Hence, the small amplitude electron acoustic wave propagation across a shear magnetic field is depicted by Korteweg-de-Vries equation just like the case of EAW propagation in an unmagnetized plasma. The difference is that the coefficient of dispersive term is proportional to α^2, where, $\alpha = B_{x0}/B_{z0}$. This implies, the dispersion is basically developed due to transverse magnetic field, the physical mechanism being totally different from other cases. The dispersion arrests the nonlinear steepening of waves and permits propagation of stationary solitary waves (or periodic) perpendicular to the magnetic field. Thus, the solution of the above KdV equation is given by solitary traveling wave solution

$$\tilde{n} = U\mathrm{sech}^2\left[\sqrt{\frac{\phi U}{12\beta}}\left(x - \frac{\phi U}{3}\tau'\right)\right]. \tag{42}$$

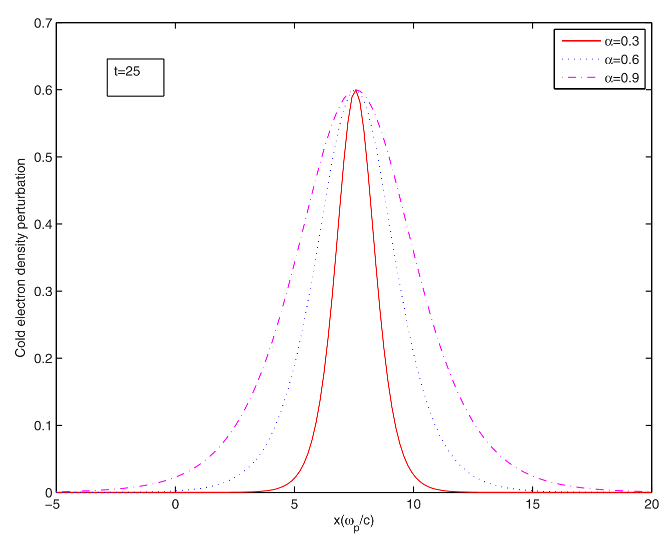

FIG. 1. Electron acoustic solitary wave solutions in the presence of variable magnetic field disperses as the value of α increases as $\alpha = 0.3, 0.6, 0.9$, at normalized time $t = 25$, for $M = 0.1$ and $U = 0.6$. Normalized cold electron density perturbation ($\tilde{n}$) is plotted against normalized space variable $x(\omega_p/c)$.

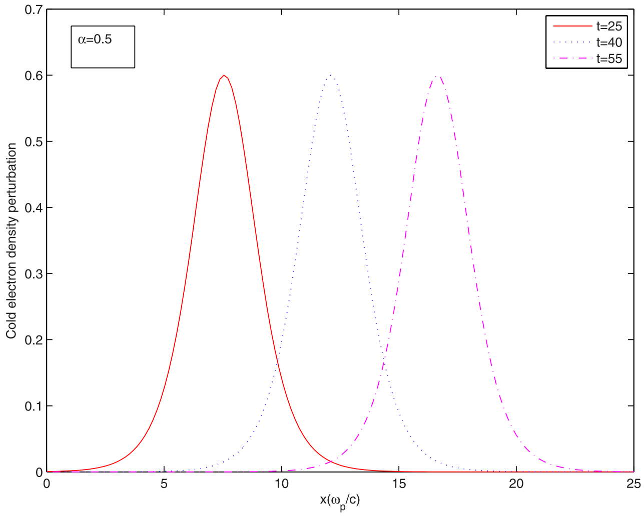

FIG. 2. Electron acoustic solitary wave propagation in the presence of variable magnetic field at normalized time $t = 25, 40, 55$ with $\alpha = 0.5$, $M = 0.1$, and $U = 0.6$. Normalized cold electron density perturbation ($\tilde{n}$) is plotted against normalized space variable $x(\omega_p/c)$.

Here, $\phi = 2(\sqrt{1+M^2} - \frac{1}{4\sqrt{1+M^2}})$ is the nonlinearity coefficient, $\beta = \alpha^2/(1+M^2)^2$ is the dispersion coefficient, U is the normalized speed of the solitary wave. A plot of the cold electron density solution is shown in Fig. 1 for different values of α. It is seen that the solitary waves disperse as the value of α increases for a certain given time. Fig. 2 shows propagation of solitary waves through plasma.

IV. CONCLUSION

In this paper, a detailed study is conducted on electron acoustic soliton propagation across a shear magnetic field in plasma with two different temperatures. A three dimensional magnetic field is included in the fluid theory to describe such a plasma system. Linear analysis indicates the propagation of electron magneto sonic waves where the wave is non-dispersive in nature. In the nonlinear analysis, Lagrangian variables are introduced to simplify the convective nonlinear terms and obtain a reduced form of governing equations. However, we find it nearly impossible to obtain an exact solution analytically from these coupled nonlinear partial differential equations. Nevertheless, essential physics can be extracted if small amplitude perturbation is employed to find quasi linear solutions. The analysis exhibits that the nonlinear wave dynamics is well described by KdV equation for weakly nonlinear case. The equation reflects that the dispersion is proportional to α^2 which is basically the normalized magnetic field intensity in transverse direction. For nonlinear regime, this α^2 accounts for wave dispersion and prevents steepening of wave allowing KdV soliton to propagation across the magnetic field. These results can be useful in interpreting electron acoustic solitary wave propagation, reported by satellites in different parts of magnetosphere. For quasi-neutral plasma of magnetosphere, usual wave dispersion is absent ruling out the possibility of solitary wave formation. Whereas, our results show that dispersion due to magnetic field may be another viable mechanism for inducing wave dispersion and generation of solitary waves.

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