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# Nonlinear electron acoustic cyclotron waves in presence of uniform magnetic field

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Nonlinear electron acoustic cyclotron waves (EACW) are studied in a quasineutral plasma in presence of uniform magnetic field. The fluid model is used to describe the dynamics of two temperature electron species in a stationary charge neutral inhomogeneous background. In long wavelength limit, it is shown that the linear electron acoustic wave is modified by the uniform magnetic field similar to that of electrostatic ion cyclotron wave. Nonlinear equations for these waves are solved by using Lagrangian variables. Results show that the spatial solitary wave-like structures are formed due to nonlinearities and dispersions. These structures transiently grow to larger amplitude unless dispersive effect is actively operative and able to arrest this growth. We have found that the wave dispersion originated from the equilibrium inhomogeneity through collective effect and is responsible for spatiotemporal structures. Weak dispersion is not able to stop the wave collapse and singular structures of EACW are formed. Relevance of the results in the context of laboratory and space plasmas is discussed.  [http://dx.doi.org/10.1063/1.4799776]

## I. INTRODUCTION

The study of large amplitude oscillations and waves has been highly appreciated among physicist due to their applications in many plasma based experiments and observations.[1] In laboratory plasma, laser assisted particle acceleration,[2] beam plasma interactions inertial fusion, and in space plasma heating are common examples where such large amplitude nonlinear waves play a significant role. The other motivation behind this work is that there is a class of nonlinear problem which can be solved analytically by Lagrangian transformation method.[3] In plasma physics, exact nonlinear solution are rare in literature. Here, we have demonstrated an example of such nonlinear wave, namely, Electron Acoustic Cyclotron Wave (EACW) and prescribed a method of exact solution. In absence of magnetic field the mode is simply electron acoustic wave (EAW) mode and the name suggests that electrons are involved in the wave dynamics, and therefore, it is a high frequency wave.[4] This wave arises in a plasma with highly dense energetic electrons which we often call as hot electrons and diluted cold electrons, respectively,[5,6] in the background of inhomogeneous static positively charged ions. Because of the high mobility of the hot electrons, this particular wave has a chance to be in the collisionless damping regime unless some criteria for density and temperature of two electron species are fulfilled.[7] First, the hot electron density must be higher than cold electron density and second, cold electron temperature should be much less than that of hot electrons.[8] The electron acoustic speed in such plasma is given by $c_{se} = \sqrt{T_h/m_{eff}}$, where $T_h$ is the hot electron temperature and $m_{eff}$ is the modified cold electron mass enhanced by the equilibrium density ratio of hot and cold electron species as, $m_{eff} = mn_{h0}/n_{c0}$. The general mechanism of this particular mode is that the cold electrons behave like massive particles and oscillate in a background of Boltzmann distributed hot electrons which provide the oscillation energy.[9] The phase velocity of the wave lies between the thermal velocity of cold and hot electron species as $v_c < \omega/k < v_h$, where $v_{c,h} = (T_{c,h}/m)^{1/2}$, $T_c$ is the temperature of cold electron, $k$ is the wave vector.

Because of two electron species and their rich varieties, several studies have been performed on the analysis of nonlinear behavior of EAW.[10–13] Some of the analyses describe modulation of EAW, nonlinear Schrödinger equation (NLSE) for this mode, or formation of electrostatic solitary waves for different conditions in unmagnetized plasma.[14,15] However, the usual occurrence of EAW in earth's magnetosphere is inspiring for a detailed investigation of large amplitude analysis of this mode in a magnetized plasma. Few past studies are devoted to investigate the nonlinear behavior of EAW and coupling with cyclotron waves in a magnetized plasma.[16,17] But the essence of the convective nonlinearity with its full glory was missing. It has been argued for years that EAW has potential importance in interpreting Broadband Electrostatic Noise (BEN) in geomagnetic tail, hiss in polar cusp region, and other emissions in parts of magnetosphere.[15,18,19] Fast Auroral Snapshot observations in Auroral Kilometric Radiation source region have detected intense packet of large amplitude nonlinear electron acoustic waves;[20] whereas POLAR and Geotail observations also signify nonlinear solitary waves associated with this mode. Also in recent past, this wave has been found to exist in laboratory plasma with two temperature electron population.[21] It has also vast applications in stimulated laser back scattered experiments[22] and also in simulation.[23]

In this paper, we have presented a detailed study of fully nonlinear (EACW) in presence of a uniform magnetic field using Lagrangian transformation. In general, governing equations have $x, t$ dependence, therefore, solutions of equations which are functions of $x - ut$ (where $u$ is the velocity of moving frame) can be constructed. However, these solutions are unable to predict any information about the dynamics of the system. Lagrangian technique is basically a transformation of Eulerian coordinates$(x, t)$ to a new coordinate system where the convective nonlinearity reduces to first order temporal variation[24] and enables us to obtain more simple equations which can be analyzed easily. This formulation is more interesting than the usual stationary state $[f(x-ut)]$ solutions as it confers more enriched solution with information about temporal evolution of plasma state. Here, we have applied this technique in a inhomogeneous plasma system consisting of stationary ions, cold electrons, and hot electrons in presence of a uniform magnetic field to obtain evolution equation for electron density. Our analysis indicates that there is an interplay between nonlinearity and dispersion to form a solitary wave like structure. It should be mentioned that we have considered here the long wavelength electron acoustic cyclotron mode (i.e., $k\lambda_D \ll 1$, where $\lambda_D$ is the Debye length); therefore, the dispersion due to electric field gradient is absent. However, inhomogeneous density distribution associated with hot electron pressure through collective effect serves the similar role of dispersion. It is shown that weak dispersion will not be able to stop density from collapsing, whereas dispersion being balanced with nonlinearity results in a nonlinear oscillations in time.

The paper is organized as follows. In Sec. II, basic equations and linear mode dispersion are written down. The Lagrangian method describing the nonlinear EACW is presented in Sec. III. Finally, the results are discussed in Sec. IV.

## II. BASIC EQUATIONS AND LINEAR WAVE

We consider a three component collisionless plasma system, consisting of background ions, inertial cold electrons, and inertia-less hot electrons immersed in a static magnetic field $B_0$ along the $\hat{z}$ axis. The ions, having a larger mass than that of electrons, are, therefore, unable to respond in electrons timescale and thus form a static background of positive charge and thereby maintaining charge neutrality throughout the plasma. We are considering waves propagating perpendicular to the applied magnetic field. However, to describe the actual scenario in plasma, magnetic field $\vec{B}_0$ and propagation vector $\vec{k}$ is not exactly perpendicular, but the angle between them deviates a little from $\pi/2$ similar to what happens in ion cyclotron wave.[25] So the hot electrons can have a velocity component along $\vec{B}_0$ and move in the direction of magnetic field to maintain charge neutrality in that direction also. These electron fluid the electric field is being balanced by the pressure gradient and form Boltzmann distribution. Neglecting the inertial term from hot electron momentum equation, we have

$$eE = -T_h \frac{\partial \ln n_h}{\partial x}. \tag{1}$$

In above equation, the thermal pressure of hot electrons is taken as $(p_h = n_h T_h)$. Assuming electric field $E = -\partial\varphi/\partial x$, integrating both side we have $n_h = n_{h0}\exp(e\varphi/T_h)$, where $n_{h0}$ is equilibrium hot electron density and independent of $x$, $\varphi$ is the electrostatic potential and $e$ is the magnitude of electronic charge. For cold electrons, it is their high effective mass $m_{eff}(= m n_{h0}/n_{c0})$ that prevents them responding in same time scale of hot electrons. Therefore, we consider the full dynamics of this cold electron fluid. Now the governing equations of the cold electron fluid are given by the following system of equations:

$$\frac{\partial n_c}{\partial t} + \frac{\partial}{\partial x}(n_c v_{cx}) = 0, \tag{2}$$

$$\left(\frac{\partial}{\partial t} + v_{cx}\frac{\partial}{\partial x}\right) v_{cx} = -\frac{e}{m}E - \frac{eB_0}{mc} v_{cy}, \tag{3}$$

$$\left(\frac{\partial}{\partial t} + v_{cx}\frac{\partial}{\partial x}\right) v_{cy} = \frac{eB_0}{mc} v_{cx}. \tag{4}$$

Here, Eq. (2) is the continuity equation for cold electron fluid, Eqs. (3) and (4) are $\hat{x}$ and $\hat{y}$ component of cold electron momentum equation and $n_c$ and $v_{cx}, v_{cy}$ are density and velocities of cold electron species in $x$, $y$ directions. These equations are closed by Poisson equation for electric field

$$\frac{\partial E}{\partial x} = 4\pi e(n_0 - n_c - n_h), \tag{5}$$

where $n_0$ is equilibrium ion density. Here, we have considered all the dynamical variables are function of coordinate $x$ and time $t$. It may be noted that for simplicity, we will consider long wavelength $(k\lambda_D \ll 1)$ EAW; therefore, in Eq. (5), we ignore electric field gradient. With these equations, first, we want to investigate the linear dispersion relation for this particular wave mode for neutral plasma. Linearizing these equations in a homogeneous background, we find the dispersion relation for this mode

$$\omega^2 = \Omega_c^2 + k^2 c_{se}^2, \tag{6}$$

where $c_{se}$ is the electron acoustic speed. This equation clearly indicates that the electron acoustic mode is modified by constant electron cyclotron frequency and can be termed as the dispersion relation for electron acoustic cyclotron wave mode. This shows that the cold electrons suffer an acoustic type oscillation; whereas $\Omega_c^2$ term is the contribution of the Lorentz's force exerted on the electrons. This wave is dispersive in presence of magnetic field as the group velocity, calculated from the above dispersion relation, comes out to be function of $k$. In absence of magnetic field, the dispersive character of the wave vanishes.

## III. NONLINEAR ANALYSIS

In this section, we shall study the nonlinear behavior of **EACW** in presence of uniform magnetic field. Aiming at obtaining full nonlinear solution, let us introduce the Lagrangian Coordinates $(\xi,\tau)$, such that $\xi$ is function of $x$ and $t$. The transformation relation between Lagrangian $(\xi,\tau)$ and Eulerian $(x,t)$ coordinates is provided as below

$$\xi = x - \int_0^\tau v_{cx}(\xi,\tau)d\tau; \quad \tau = t. \tag{7}$$

The advantage of this Lagrangian formalism is that, the nonlinear convective derivatives in governing equations become merely time derivatives in our new coordinates

$$\frac{\partial}{\partial t} + v_{cx}\frac{\partial}{\partial x} = \frac{\partial}{\partial \tau}. \tag{8}$$

Whereas at a certain time, the space derivative operators of both coordinates $\xi$ and $x$ are related by initial density profile and density at that time

$$\frac{\partial}{\partial x} = \frac{n_c(\xi,\tau)}{n_c(\xi,0)}\frac{\partial}{\partial \xi}. \tag{9}$$

We proceed for implementing Lagrangian transformation to the system of governing equations and the new set of equations namely the continuity and momentum equations for cold electrons are as follows:

$$\frac{\partial}{\partial \tau}\left(\frac{1}{n_c}\right) = \frac{1}{n_c(\xi,0)}\frac{\partial v_{cx}}{\partial \xi}, \tag{10}$$

$$\frac{\partial v_{cx}}{\partial \tau} = \frac{T_h}{m}\frac{n_c}{n_c(\xi,0)}\frac{\partial \ln n_h}{\partial \xi} - \Omega_c v_{cy}, \tag{11}$$

$$\frac{\partial v_{cy}}{\partial \tau} = \Omega_c v_{cx}. \tag{12}$$

Note that in Eq. (11), we have replaced electric field from Eq. (1). Combining Eqs. (10) and (12) and integrating it with respect to $\tau$, we get the following equation for $v_{cy}$:

$$\frac{1}{n_c(\xi,0)}\frac{\partial v_{cy}}{\partial \xi} = \Omega_c\left(\frac{1}{n_c} - \frac{1}{n_c(\xi,0)}\right). \tag{13}$$

To obtain the above equation, we have taken at $\tau = 0$, $n_c = n_c(\xi,0)$ and $\partial v_c/\partial \xi = 0$. Next, we eliminate all other variables except hot and cold electron density from Eqs. (10) and (13), and finally obtain

$$\left[\frac{\partial^2}{\partial \tau^2} + \Omega_c^2\right]\left(\frac{1}{n_c}\right) = \frac{\Omega_c^2}{n_c(\xi,0)} + \frac{T_h}{m}\frac{1}{n_c(\xi,0)}\frac{\partial}{\partial \xi}\left[\frac{n_c}{n_c(\xi,0)}\frac{\partial \ln n_h}{\partial \xi}\right]. \tag{14}$$

From the quasineutrality condition (5) neglecting electric field gradient consistent with long wavelength EAW mode, we have

$$n_0 = n_c + n_h, \tag{15}$$

where $n_0$ is the equilibrium ion density. At $\tau = 0$, $n_0 = n_{h0} + n_c(\xi,0)$. Now, $n_{h0}$ is equilibrium hot electron density, which is independent of $x$. So, to satisfy the quasineutrality condition at $\tau = 0$, ion density should be taken as function of space since initially cold electron density is inhomogeneous. This relaxes the assumption of uniform ion density. Moreover, the inhomogeneous density is a common feature in plasma and thus, closely related to more realistic situation. Inserting this neutrality condition Eq. (15), we obtain

$$\left[\frac{\partial^2}{\partial \tau^2} + \Omega_c^2\right]\left(\frac{1}{n_c}\right) = \frac{\Omega_c^2}{n_c(\xi,0)} + \frac{T_h}{m}\frac{1}{n_c(\xi,0)}\frac{\partial}{\partial \xi} \times \left[\frac{n_c}{n_c(\xi,0)}\frac{\partial}{\partial \xi}\ln\left\{n_0(\xi)\left(1 - \frac{n_c}{n_0(\xi)}\right)\right\}\right]. \tag{16}$$

Here, we will use the inequality $n_c/n_{h0} < 1$ (therefore $n_c/n_0 \ll 1$), for existence of EAW in a plasma. Using this fact in the above equation together with equilibrium ion density distribution $n_0(\xi) = n_0\exp(-\alpha^2\xi^2)$ in Eq. (16), we obtain

$$\left[\frac{\partial^2}{\partial \hat{\tau}^2} + 1\right]\left(\frac{1}{\hat{n}_c}\right) = \frac{1}{\hat{n}_c(\xi,0)} - a^2\frac{1}{\hat{n}_c(\xi,0)}\frac{\partial}{\partial \hat{\xi}}\left[\frac{\hat{n}_c\hat{\xi}}{\hat{n}_c(\xi,0)}\right] - \frac{1}{\hat{n}_c(\xi,0)}\frac{\partial}{\partial \hat{\xi}}\left[\frac{\hat{n}_c}{\hat{n}_c(\xi,0)}\frac{\partial}{\partial \hat{\xi}}\left(\hat{n}_c e^{\hat{\alpha}^2\hat{\xi}^2}\right)\right], \tag{17}$$

where $\alpha$ is identified as inverse of equilibrium ion inhomogeneity scale length. The hat variables in above equation represent the normalized variables. The cold electron density $n_c$ is normalized to its equilibrium value, i.e., $n_{c0}$, space is normalized as $\hat{\xi} = \xi\Omega_c/c_{se}$ time is normalized to inverse of electron gyro-frequency like $\hat{\tau} = \Omega_c\tau$, and $\hat{\alpha} = \alpha c_{se}/\Omega_c \equiv \rho_{se}/L_n$, where $\rho_{se}$ is cold electron larmor radius and $L_n$ is inhomogeneous density scale length and denoted $a^2 = 2\hat{\alpha}^2(n_0/n_{c0})$, a dimensionless parameter. Note that the term proportional to $a^2$ explicitly contains $n_c\xi$, which is essentially nonlinear since $\xi$ involves cold electron velocity $v_c$. This clearly indicates the manifestation of convective nonlinearity through collective effect and frequently we designate the term $a^2$ as nonlinearity parameter in our analysis. Hereafter, for simplicity of notation, we will remove all the hats and work with these normalized variables. Now, because of certain symmetry of the Eq. (17), it can be solved by using a separation of variables. Heading for obtaining full nonlinear solution, we first write the cold electron density as

$$n_c(\xi,\tau) = \psi(\tau)N(\xi). \tag{18}$$

At $\tau = 0$, we can represent $n_c(\xi,0) = \psi(0)N(\xi)$ and $\psi(0)N(0)$ can be taken as unity since we are working with normalized variables. Without any loss of generality, we can use $\psi(0) = N(0) = 1$. Substituting $n_c(\xi,\tau)$ in Eq. (17) and separating for time and space variables, we get

$$\frac{1}{\psi^2}\left[\left(\frac{\partial^2}{\partial \tau^2} + 1\right)\frac{1}{\psi} - 1 + a^2\psi\right] = -\frac{\partial^2}{\partial \xi^2}\left(Ne^{\hat{\alpha}^2\xi^2}\right) = \beta, \tag{19}$$

where $\beta$ is an arbitrary separation constant to be determined later. The two evolution equations for space and time profile of cold electron density are given by

$$\frac{d^2}{d\xi^2}(Ne^{\hat{\alpha}^2\xi^2}) = -\beta, \tag{20}$$

$$\left(\frac{d^2}{d\tau^2}+1\right)\frac{1}{\psi} - 1 + a^2\psi = \beta\psi^2. \tag{21}$$

The spatial part of density can be easily integrated out from Eq. (20) with the condition, at $\xi \to 0$, $N=1$, and $dN/d\xi \to 0$, and we have

$$N(\xi) = \left(1 - \frac{1}{2}\beta\xi^2\right)e^{-\hat{\alpha}^2\xi^2}. \tag{22}$$

The arbitrary constant $\beta$ can be determined from the following density conservation condition over the whole space, e.g., $\int_{-\infty}^{+\infty} N(\xi,0)d\xi = 1$. The straightforward integration gives us

$$\beta = 4\hat{\alpha}^2\left(1 - \frac{\hat{\alpha}}{\sqrt{\pi}}\right). \tag{23}$$

The parameter $\hat{\alpha}$ is a normalized equilibrium density scale length which is positive. The above relation tells us that $\beta$ must be positive. The condition $\beta > 0$ implies that $\hat{\alpha} < \sqrt{\pi}$. From this we may write the inequality as $L_n > \rho_{se}/\sqrt{\pi}$. The opposite limit, i.e., $L_n < \rho_{se}/\sqrt{\pi}$, which is in contradiction with fluid theory hence negative value of $\beta$ is ruled out. We have solved Eq. (21) using MATLAB to obtain the time dependent part of the density. In this solution, we have taken the initial conditions at $\tau = 0$, $d\psi/d\tau = 0$, and $\psi = 1$. The solution is given in Fig. 1 for a typical value of $\hat{\alpha} = 0.27$ and $n_0/n_{c0} = 5.5, 9.0$. The time variation of the density amplitude shows nonlinear oscillations as expected. To write the complete solution, we also need to convert the Lagrangian variables to Eulerian and is done by means of the Eqs. (9) and (7) which is given by $\xi = x\psi$. Therefore, the cold electron density in terms of original $(x,t)$ variables becomes

$$n_c = \psi(t)\left(1 - \frac{1}{2}\beta x^2\psi(t)^2\right)e^{-\hat{\alpha}^2x^2\psi(t)^2}, \tag{24}$$

where $\psi(t)$ has to be determined from Eq. (21). We emphasize that the obtained expression of cold electron density given in Eq. (24) is a complete solution of full nonlinear system of our governing equations. We have pictorially represented the density solution in space and time for different values of parameters as shown in Fig. 2.

## IV. DISCUSSIONS AND SUMMARY

In this paper, we have explored the space and time evolution of cold electron density in a plasma with inhomogeneous ions, cold electrons, and hot electrons in presence of externally applied uniform magnetic field. The Lagrangian transformation is employed in a fluid model which fruitfully simplifies the complicated nonlinear problem into a solvable form. Small amplitude (linear) analysis shows electron acoustic wave is modified due to cyclotron motion and produce similar result as ion cyclotron waves. The nonlinear equation is then solved exploiting the symmetry of the equation using separation of variable method. The full nonlinear analysis shows nonlinear oscillation of the density amplitude in time, whereas there is a localized density structures in space. These results are manifestation of an interplay between nonlinearity and dispersion. However, it should be mentioned that the dispersion present here is not usual dispersion due to electric field gradient (since we have taken neutral plasma throughout), but it is a kind of wave dispersion associated with equilibrium inhomogeneity. As it is mentioned before, the convective nonlinearity which was hidden in $\xi$ is reflected in the term which is proportional to $a^2$ in Eq. (17). Depending on the strength of this nonlinearity density solution goes to large amplitude and eventually shows singular behavior. In such a situation, the wave dispersion cannot stop the density singularity which is shown in the right (red) Fig. 2. The spatial structures that we have found show qualitative similarity with satellite observations.[20]

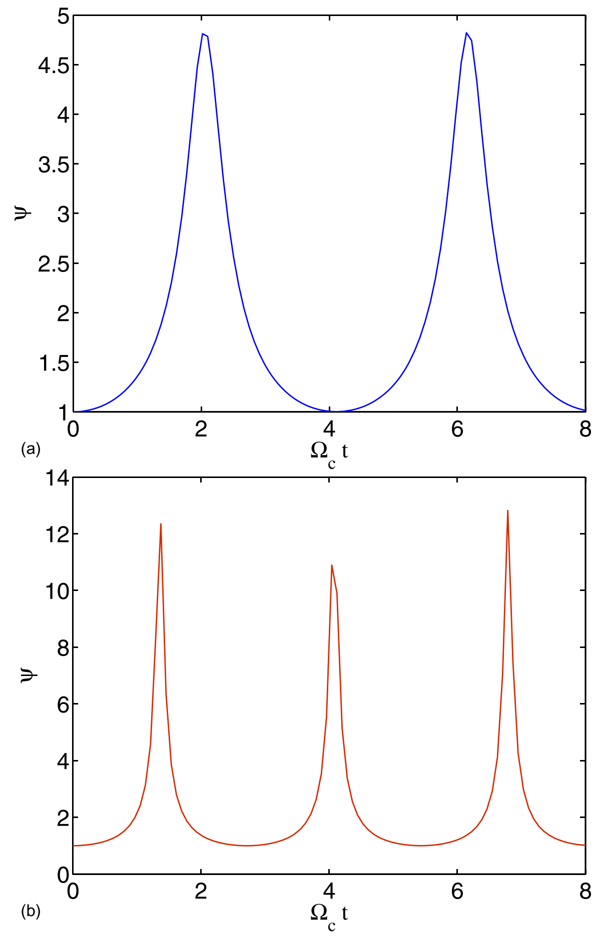


FIG. 1. Normalized time dependent density amplitude $\psi$ for cold electron species are shown. For the left panel $\hat{\alpha} = 0.27$, $n_0/n_{c0} = 5.5$ and for the right side figure $\hat{\alpha} = 0.27$, $n_0/n_{c0} = 9.0$. The left-hand side figure shows that the density amplitude exhibits nonlinear oscillation, whereas on the right figure for large values for the nonlinearity parameter, density goes towards singularity.

In this work, it has been demonstrated that the Lagrangian fluid description is a powerful tool for studying the complex dynamics of an arbitrary large amplitude electron acoustic cyclotron waves. The nonlinear equation presented here has a similarity with unmagnetized electron warm plasma oscillation.[26] However, the physical situation

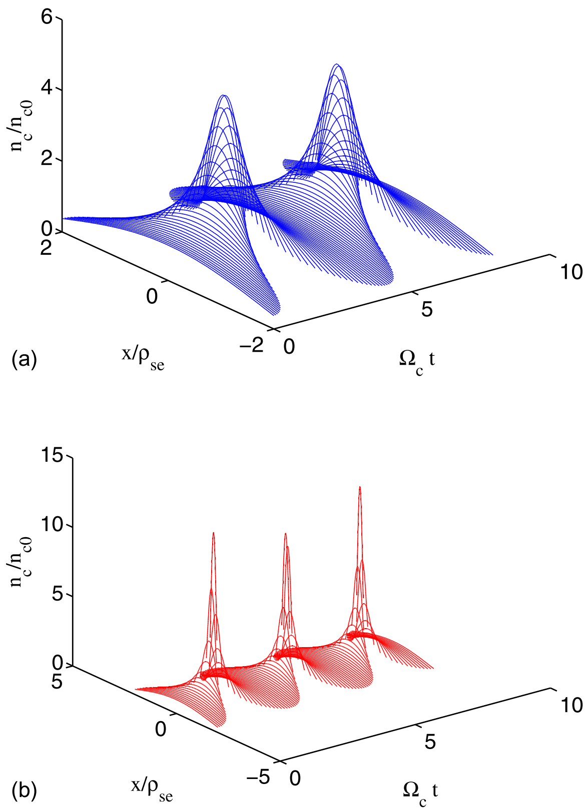


FIG. 2. The solution for normalized cold density $n(x,t)$ is shown. For the left panel $\hat{\alpha} = 0.27$, $n_0/n_{c0} = 5.5$ and for the right side figure $\hat{\alpha} = 0.27$, $n_0/n_{c0} = 9.0$. The left-hand side figure shows that the density exhibits nonlinear oscillation in time but localized in space, whereas on the right figure for large values for the nonlinearity parameter, density goes towards singular value.

here is very different and moreover magnetic field changes the problem to a different scenario. We feel that such solution will be useful in a manifold similar physical situations.[27,28] The nonlinear, time-dependent solutions for EACW oscillations involving dispersive and nonlinear effects are self contained and hope to improve our understanding to study nonlinear phenomena in the frontier of modern physics and technology.

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