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Nonlinear coupling of Langmuir and electron acoustic waves in a viscous plasma

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A nonlinear analysis of finite amplitude electron acoustic waves is considered in a viscous plasma. The two fluid two time scale model is used to describe the two temperature electron species in a fixed ion background. We have obtained two sets of modified Zakharov equations where the modification comes due to the presence of viscosity in the plasma system. We have shown that, for very low frequency, these viscosity modified Zakharov equations reduce to a modified nonlinear Schrödinger's equation where viscosity introduces a new term via collective effects. Perturbative analysis shows the formation of soliton structures with an oscillating tail. The relevance of the results is important in the context of astrophysical and laboratory plasma. *Published by AIP Publishing.*
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I. INTRODUCTION

The nonlinear wave theory has a vast application not specifically to plasma physics but to any physical medium where waves can propagate. The nonlinear Schrödinger's equation (NLSE) and Zakharov's equation are two significant equations encountered in the nonlinear wave dynamics and turbulence theory. In recent years, the investigation on NLSE has drawn huge attention due to its applicability in quantum physics, plasma physics, ocean wave analysis, nonlinear optics, turbulence theory, and many other different phases of physics. This equation successfully describes the envelope dynamics of a quasi-monochromatic plane wave (the carrying wave) propagating in a weakly nonlinear dispersive medium when dissipative processes are negligible. Whereas, the Zakharov equations model the non-linear interaction between high-frequency electromagnetic waves and low-frequency plasma waves on the two time scales two fluid theory.[1] The high frequency waves correspond to fast varying electromagnetic fields while the low frequency waves are related to slow varying density perturbations with slow varying amplitude of electromagnetic fields. Hence, the time-dependent processes in the plasma can be separated into processes on two separate time scales, one fast and one slow. Previously, this two fluid two time scale theory has been employed to study the nonlinear interaction of the electron acoustic wave with the electron plasma wave in unmagnetized plasma,[2] where solitary solutions are obtained for electric field concentration and density depletion regions. Nonlinear interaction of quantum electron plasma wave with quantum electron acoustic waves (EAWs) has also been studied[3] and a modified coupled Zakharov equation has been obtained. In both papers, a two time scale two fluid theory has been implemented to obtain a set of Zakharov's equations for plasma with two temperature electron components. Plasma with two temperature electron species can support the electron acoustic wave. Characteristically, these electrons are different from each other mainly because one of the electron components is hot and has energy of the order of KeV, while the other is cold and has energy not greater than 60 eV. It is obvious that the less energetic electron component or the cold electrons correspond to the slow time scale, whereas the hot electrons correspond to fast time scale. In this paper, we want to investigate nonlinear coupling of the electron acoustic wave with electron plasma in the presence of electron viscosity. It is shown that viscosity may incorporate important plasma phenomena through collective effects. However, before going to a detailed discussion of the work, we want to include a quick review of this electron acoustic mode.

The investigation of nonlinear propagation of the electron acoustic wave has drawn considerable attention because of its relevance in interpreting various satellite observations in different parts of the magnetosphere. It has been emphasized through several research studies that EAW plays significant role in the generation process of Broadband Electrostatic Noise (BEN) in the terrestrial cusp of the magnetosphere and hiss in polar cusp regions.[4–8] When less energetic cold plasma from the earth's ionosphere mixes with high energetic hot plasma of the earth's magnetosphere, an intense packet of the electron acoustic wave is formed in the flow boundary region.[7,9–12] The study of the electron acoustic wave is also crucial in laboratory plasma with two temperature electron species, beam plasma interaction, and stimulated electron acoustic scattering in the laser plasma interaction.[13–15]

The general mechanism of this particular mode is that the cold electrons behave like massive particles and oscillate in a background of Boltzmann distributed hot electrons which provide the oscillation energy.[16] The electron acoustic

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speed in such plasma is given by $c_{se} = \sqrt{T_h/m_{eff}}$, where m_{eff} is the modified cold electron mass enhanced by the equilibrium density ratio of hot and cold electron species as, $m_{eff} = m(n_{h0}/n_{c0})$,[6,17] where T_h is the hot electron temperature, m is the electron mass, and n_{c0} and n_{h0} represent the equilibrium cold and hot electron density, respectively. This mode is basically an analog to ion acoustic wave with cold electrons playing the role of ions providing the inertia.[2,18–20] In the fast time scale of electrons, ions are taken to be at rest forming a positively charged background and maintaining charge neutrality throughout the plasma. The phase velocity of the wave lies between the thermal velocity of cold and hot electron species as $v_c < \omega/k < v_h$, where $v_{c,h} = (T_{c,h}/m)^{1/2}$ and k is the wave vector. As only high frequency electrons are involved in whole dynamics, there is always a chance of wave damping. However, according to the study of Gary and Tokar,[21] there exist some parameter regime of density and temperature for which this wave can sustain electrostatic fluctuations while propagating in plasma. The first condition is the cold electron temperature (T_c) should be much less than the hot electron temperature (T_h), i.e., $T_c \ll T_h$ and the second is, the hot electron density (n_h) should be much larger than the cold electron density (n_c), i.e., $n_h \gg n_c$.

In this paper, we have considered a viscous plasma medium with two temperature electron components with stationary ions in the background. A two fluid two time scale theory has been employed to derive a set of modified Zakharov equations. The characteristic hot electron plasma frequency and the electron acoustic frequency give rise to two distinct time scales in the system that can be designated as fast and slow, respectively. Actually, fast motion is mainly governed by the hot electron fluid, whereas, the slow timescale movement is mainly due to the cold electron fluid. With the help of this two time scale theory, we have formulated a set of two nonlinear equations which couple slow density perturbation with the slowly varying amplitude of the rapidly varying electric field. These coupled equations are similar to Zakharov's equations with a correction term appearing due to presence of viscosity in the system. These equations are called viscosity modified Zakharov's equations. Later, we have shown that for low frequency consideration these equations can be reduced to the nonlinear Schrodinger equation where a new modification arises due to the presence of viscosity. A perturbative method to get small amplitude approximate solutions of the viscosity modified NLSE has been prescribed.

II. GOVERNING EQUATIONS FOR EAW IN VISCOUS PLASMA

We consider an un-magnetized, viscous plasma system comprising of ions and two electron species of different temperature and density, namely- hot electrons and cold electrons. The ions being heavier in mass do not participate in the high frequency dynamics of electrons except charge neutralization. The hot electrons are extremely energetic compared to the cold electrons and have a prevailing number density. On the other hand, viscous cold electrons have less energy and respond to a slower time scale compared to hot electrons. The set of fluid equations for both the hot and cold electron components is given as follows:

$$mn_h\left[\frac{\partial \vec{v}_h}{\partial t} + (\vec{v}_h.\nabla)\vec{v}_h\right] = -en_h\vec{E} - \nabla P_h, \tag{1}$$

$$mn_c\left[\frac{\partial \vec{v}_c}{\partial t} + (\vec{v}_c.\nabla)\vec{v}_c\right] = -en_c\vec{E} - \nabla P_c + \nu\nabla^2\vec{v}_c, \tag{2}$$

$$\frac{\partial n_h}{\partial t} + \nabla.(n_h\vec{v}_h) = 0, \tag{3}$$

$$\frac{\partial n_c}{\partial t} + \nabla.(n_c\vec{v}_c) = 0, \tag{4}$$

$$\nabla.\vec{E} = 4\pi e(n_0 - n_h - n_c). \tag{5}$$

Here, subscript "h" corresponds to the hot electrons and subscript "c" corresponds to the cold electrons. n, v, and P denote density, velocity, and pressure, respectively. e and m are the electronic mass and charge. $\vec{E}$ is the total electric field, and ν is the coefficient of viscosity.

The local density fluctuation due to the presence of the low frequency electron acoustic wave would alter the restoring electric field, thus affecting the high frequency Langmuir wave. To describe the nonlinear coupling between these two waves in a viscous plasma, we employ the two time scale theory. We will split the perturbation of the variables into two parts: the first part would correspond to slow variation with time and the second part to fast variation with time. As the cold electron does not take part in fast dynamics, quantities related with cold electrons have only slow variation with time

$$n_c = n_{c0} + \delta n_c; \quad n_h = n_{h0} + \delta n_h + \tilde{n}_h, \tag{6}$$

$$\vec{v}_c = \delta\vec{v}_c; \quad \vec{v}_h = \delta\vec{v}_h + \tilde{\vec{v}}_h, \tag{7}$$

$$\vec{E} = \vec{E}_s + \tilde{\vec{E}}, \tag{8}$$

n_{h0}, n_{c0} are the equilibrium densities of hot electrons and cold electrons. The tilde sign corresponds to the fast varying components, whereas, δ and subscript "s" are associated with slow varying components. The average of fast variables, when taken over many oscillations, is equal to zero. Thus, the average of hot electron density over many rapid oscillations is nothing but $\langle n_h\rangle = n_{h0} + \delta n_h$, which means that the hot electrons are moving so quickly that they manage to keep plasma quasi neutral in the slow time scale perturbation.

The fast time scale analysis of hot electron dynamics yields first the Zakharov equation which describes the evolution of the fast varying electric field $\tilde{\vec{E}}$.[2] While deriving the equation, we divide the rapidly varying electric field $\tilde{\vec{E}}$ into one slowly varying amplitude part and one fast oscillating part as follows:

$$\tilde{\vec{E}} = \frac{1}{2}\left[\vec{\bar{E}}(\vec{r},t)e^{-i\omega t} + c.c\right]. \tag{9}$$

Keeping in mind that the time derivative of slowly varying complex amplitude $\vec{\bar{E}}$ is defined to be very small, we obtain the first Zakharov equation

$$-2i\omega_{ph}\frac{\partial\vec{\bar{E}}}{\partial t}-v_{th}^2\nabla(\nabla.\vec{\bar{E}})+\omega_{ph}^2\frac{\delta n}{n_{h0}}\vec{\bar{E}}=0. \quad (10)$$

In order to obtain the second Zakharov equation, slow time scale components of the fluid equations have to be considered. Starting with the hot electron momentum equation, we take an average of the equation over many rapid cycles so that the terms linear in the fast time scale quantities approach zero and we obtain the following equation:

$$\frac{\partial\delta\vec{v}_h}{\partial t}+\langle(\vec{v}_h.\nabla)\vec{v}_h\rangle=-\frac{e}{m}\vec{E}_s-\frac{K_BT_h}{mn_{h0}}\nabla\delta n_e. \quad (11)$$

Now, the leading order terms of the fast varying component of the hot electron momentum equation will give $\frac{\partial\vec{v}_h}{\partial t}\approx-\frac{e}{m}\vec{\bar{E}}$ which yields the following equation:[2,3,22]

$$\langle(\vec{v}_h.\nabla)\vec{v}_h\rangle\approx\frac{e^2}{4m^2\omega_{ph}^2}\nabla|\bar{E}|^2. \quad (12)$$

Remembering time variation of slow quantities is defined to be negligible

$$\frac{e^2}{4m^2\omega_{ph}^2}\nabla|\bar{E}|^2=-\frac{e}{m}\vec{E}_s-\frac{K_BT_h}{mn_{h0}}\nabla\delta n_h. \quad (13)$$

Now, we consider the momentum equation of cold viscous electrons

$$\frac{\partial\delta\vec{v}_c}{\partial t}+(\delta\vec{v}_c.\nabla)\delta\vec{v}_c=-\frac{e}{m}\vec{E}_s-\frac{K_BT_c}{mn_{c0}}\nabla\delta n_c+\nu\nabla^2\delta v_c. \quad (14)$$

As the cold electron temperature is much lesser than the hot electron temperature, the cold electron pressure effect is negligible in the above equation. Equation (13) basically counts the effect of the fast varying electric field on the slow varying component of the electric field. So, we replace the value of the slow varying electric field component $\vec{E}_s$ from Eq. (13) in Eq. (14) to get

$$\frac{\partial\delta\vec{v}_c}{\partial t}=\frac{e^2}{4m^2\omega_{ph}^2}\nabla|\bar{E}|^2+\frac{K_BT_h}{mn_{h0}}\nabla\delta n_h+\nu\nabla^2\delta v_c. \quad (15)$$

Now, operating ∇ on both sides of the above equation, we replaced $\delta\vec{v}_c$ in terms of δn_c using the cold electron continuity equation $\frac{\partial\delta n_c}{\partial t}+n_{c0}\nabla.\vec{\delta v}_c=0$ and obtained[2,3,22]

$$\left[\frac{\partial^2}{\partial t^2}-c_{se}^2\nabla^2\right]\delta n=\frac{e^2n_{c0}}{4m^2\omega_{ph}^2}\nabla^2|\bar{E}|^2+\frac{\nu}{n_{c0}}\frac{\partial}{\partial t}\nabla^2(\delta n), \quad (16)$$

where $\delta n_h=-\delta n_c=\delta n$ and $c_{se}=\sqrt{T_h/m(n_{h0}/n_{c0})}$ is the electron acoustic velocity. The above equation is the modified second Zakharov equation in the presence of viscosity in three dimensional space. This equation describes the impact of rapidly varying electric field fluctuations on slow varying density fluctuations. Equations (10) and (16) thus describe two coupled Zakharov equations. Now, in order to find solutions and analyze the equations, we will consider one dimensional form of the two equations and introduce dimensionless variables for convenience as follows:

$$\chi=\frac{\bar{E}}{8\sqrt{\pi n_{c0}K_BT_h}};\quad n=\frac{\delta n}{4n_{c0}}, \quad (17)$$

$$\xi=\frac{x}{\lambda_D}\left(2\sqrt{\frac{n_{c0}}{n_{h0}}}\right);\quad \tau=t\omega_{ph}\left(\frac{2n_{c0}}{n_{h0}}\right). \quad (18)$$

Finally, Eqs. (10) and (16) reduce to a simplified form and we obtain a set of two normalized viscosity modified Zakharov equations in 1-D[2,3,22]

$$\left[\frac{\partial^2}{\partial\tau^2}-\frac{\partial^2}{\partial\xi^2}-\mu_v\frac{\partial}{\partial\tau}\left(\frac{\partial^2}{\partial\xi^2}\right)\right]n=\frac{\partial^2}{\partial\xi^2}|\chi|^2, \quad (19)$$

$$i\frac{\partial\chi}{\partial\tau}+\frac{\partial^2\chi}{\partial\xi^2}=n\chi, \quad (20)$$

where $\mu_v=2\nu/\omega_{ph}n_{c0}\lambda_D^2$ and $\lambda_D=\sqrt{K_BT_h/4\pi n_{h0}e^2}$ is the Debye length. These modified Zakharov's equations give a complete description of the interaction between the high frequency Langmuir wave and the slow frequency electron acoustic wave as they include the effect of viscosity. However, our equations actually describe the evolution of low frequency responses of density and electric field fluctuations. The coupling of the low frequency dynamics is caused due to the ponderomotive force derived from the high frequency electric field. It is to be noted that in the absence of viscosity, i.e., $\mu_v=0$, the Zakharov equations for EAW[2] can be exactly restored.

III. MODIFIED NONLINEAR SCHRÖDINGER EQUATION

Near very low frequency, second order time derivatives offer negligible significance and from Eq. (19) we can write

$$\left(1+\mu_v\frac{\partial}{\partial\tau}\right)n\approx-|\chi|^2. \quad (21)$$

Operating $\left(1-\mu_v\frac{\partial}{\partial\tau}\right)$ on both sides of the above equation

$$n\approx-|\chi|^2+\mu_v\frac{\partial}{\partial\tau}|\chi|^2. \quad (22)$$

Replacing the value of n in Eq. (20), we obtain the Schrödinger equation with a completely new modification of nonlinear dissipation arising due to viscosity

$$i\frac{\partial\chi}{\partial\tau}+\frac{1}{2}\frac{\partial^2\chi}{\partial\bar{\xi}^2}+|\chi|^2\chi=\mu_v\chi\frac{\partial}{\partial\tau}|\chi|^2, \quad (23)$$

where $\bar{\xi}=\sqrt{2}\xi$. This equation is novel in literature and offers a complete description of nonlinear coupling of two waves in a dissipative system. An exact solution of this above dissipative system is not available; however, we can treat this equation as a perturbed NLS equation where the small perturbation term arises due to viscosity μ_v. Now, the functional form of the solution of this perturbed NLS is assumed to be the same as the soliton solution but the four soliton parameters vary with time τ as the soliton propagates through plasma. Thus, the solution of χ from Eq. (23) can be given by the following equation,

$$\chi(\bar{\xi},\tau) = \eta(\tau)\,\mathrm{sech}\left[\eta(\tau)\left\{\bar{\xi} - q(\tau)\right\}\right]\exp\left[i\phi(\tau) - i\delta(\tau)\bar{\xi}\right]. \tag{24}$$

Using the variational method, the dependence of four soliton parameters η, δ, ϕ, q on time τ is found to have the following functional form:

$$\eta = \eta_0 = Const., \tag{25}$$

$$q = \frac{\delta_0}{\beta}(e^{-\beta\tau} - 1), \tag{26}$$

$$\phi = \frac{1}{2}\eta^2\tau + \frac{3}{4}\frac{\delta_0^2}{\beta}(e^{-2\beta\tau} - 1) - \frac{\delta_0^2}{\beta}(e^{-\beta\tau} - 1), \tag{27}$$

$$\delta = \delta_0 e^{-\beta\tau}, \tag{28}$$

where $\beta = 8/15\mu_v\eta_0^4$. Using the above relations, the final solution of the normalized electric field χ [Eq. (24)] has been plotted with respect to the normalized space variable and it exhibits soliton nature with an oscillating tail (Fig. 1). We have also plotted the same solution at different times with a lesser value of μ_v or the viscosity coefficient (Fig. 2). This plot clearly shows that the lesser value of μ_v accounts for the less oscillating tail. However, there is no decay or growth of the final electric field solution with time due to the presence of viscosity in the system.

IV. CALCULATION OF THE GROWTH RATE OF THE MODIFIED NONLINEAR SCHRÖDINGER EQUATION

In Sec. III, the analytical solution obtained through the perturbation method has shown that there exists no growth or decay of the solution. To verify the fact, in this section we will outline a detailed analytical study to find the growth rate of the solution of Eq. (23). We consider initially that there exists a stable solution of the following form:

$$\chi = \chi_0(\tau)\exp\left[-i\int_0^{\tau}\delta(\tau')d\tau'\right] \tag{29}$$

and this solution satisfies [Eq. (23)] so that

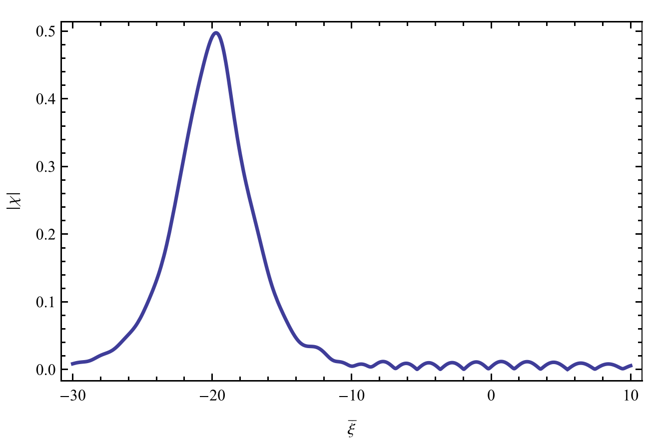

FIG. 1. Numerical solution of the modified Schrodinger equation at $t = 20$ with $\mu_v = 0.6$ initial amplitude 0.5. The normalized electric field (χ) is plotted against normalized space variable $\bar{\xi}$.

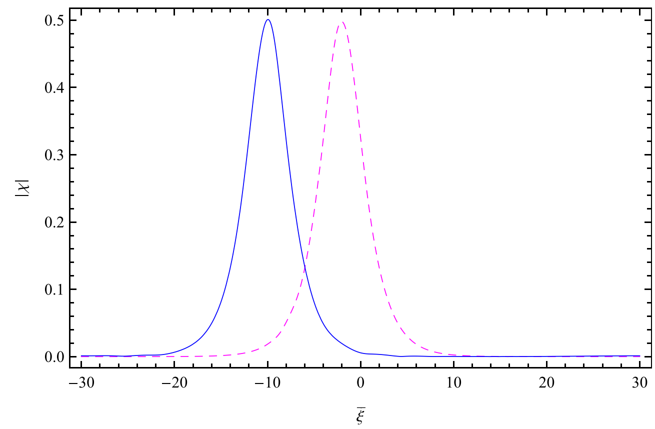

FIG. 2. Numerical solution of the modified Schrodinger equation at $t = 2, 10$ with $\mu_v = 0.1$ initial amplitude 0.5. The normalized electric field (χ) is plotted against normalized space variable $\bar{\xi}$.

$$i\frac{d\chi_0}{d\tau} + \delta(\tau)\chi_0 = -\chi_0|\chi_0|^2 + 2\mu_v\chi_0^2\frac{d\chi_0}{d\tau}. \tag{30}$$

Next, we consider a slightly different perturbative solution as

$$\chi = [\chi_0 + \tilde{\chi}]\exp[-i\delta\tau], \tag{31}$$

where $|\tilde{\chi}| \ll |\chi_0|$ and we obtain from our modified NLSE

$$i\frac{\partial\tilde{\chi}}{\partial\tau} + \frac{\partial^2\tilde{\chi}}{\partial\xi^2} + |\chi_0|^2(\tilde{\chi} + \tilde{\chi}^*) = \mu_v\chi_0^2\frac{\partial}{\partial\tau}(\tilde{\chi} + \tilde{\chi}^*). \tag{32}$$

If the real and imaginary part of $\tilde{\chi}$ and its complex conjugate is represented as $\tilde{\chi} = \rho + i\theta$ and $\tilde{\chi}^* = \rho - i\theta$, we can get two coupled equations for ρ and θ

$$-\frac{\partial\theta}{\partial\tau} + \frac{\partial^2\rho}{\partial\xi^2} + 2|\chi_0|^2\rho = 2\mu_v|\chi_0|^2\frac{\partial\rho}{\partial\tau}, \tag{33}$$

$$\frac{\partial\rho}{\partial\tau} + \frac{\partial^2\theta}{\partial\xi^2} = 0. \tag{34}$$

Operating $\frac{\partial^2}{\partial\xi^2}$ on both equations and eliminating θ, we get a fourth order equation for ρ

$$\frac{\partial^2\rho}{\partial\tau^2} + \frac{\partial^4\rho}{\partial\xi^2} + 2|\chi_0|^2\frac{\partial^2\rho}{\partial\xi^2} = 2\mu_v|\chi_0|^2\frac{\partial}{\partial\tau}\left(\frac{\partial^2\rho}{\partial\xi^2}\right). \tag{35}$$

To derive the dispersion relation, a Fourier analysis is performed considering $\rho \sim \exp i(\tilde{k}\xi - \tilde{\omega}\tau)$ which gives

$$\tilde{\omega}^2 \approx \tilde{k}^2\left[\tilde{k}^2 - |\chi_0|^2\left(2 + \mu_v^2|\chi_0|^2\tilde{k}^2\right)\right]. \tag{36}$$

It is evident from the dispersion relation that viscosity does not incorporate any kind of instability or decay or growth to the system. However, the presence of viscosity coefficient μ_v in the dispersion relation suggests that it accounts for the oscillation of the wave. This result is in accordance with the solution of soliton with the oscillating tail, obtained from the perturbative analysis.

V. DISCUSSION

The physics of nonlinear evolution of waves and instabilities has always played a significant role as it helps us to understand various plasma phenomena occurring in the astrophysical environment as well as in the laboratory. In this paper, we have considered plasma with ions and two electron components with different temperatures in a viscous plasma medium. A two fluid two time scale theory has been applied to explore the nonlinear interaction of the electron acoustic wave with the Langmuir wave. It is shown that a set of modified Zakharov's equations describes the cumulative nonlinear interactions which can modulate the wave amplitude on large spatial and temporal scales. At the low frequency regime, these two modified Zakharov's equations are reduced to a completely new equation which resembles the nonlinear Schrodinger's equation with a new term arising due to the presence of viscosity in the plasma system. As the exact solution of this equation is difficult to obtain, a perturbation method has been employed to get an approximate solution. This solution is found to have a soliton like nature with an oscillating tail. However, no decay or growth of the solution can be sighted from the analysis. To ensure this fact, we have further calculated the growth rate of our modified NLSE. The dispersion relation shows that the frequency of the wave is not a complex number but a real quantity, which implies that there is no decay or dissipation of the wave amplitude, however, the viscosity coefficient causes oscillation. In nonlinear fibre optics, stationary soliton solutions with oscillating tails have been reported in systems with small dispersion.[23,24] Similar kind of localized structures are found to have oscillatory decaying tails originating from diffraction in the optical pattern forming system.[25] However, this type of solution is novel for a plasma system and may have some implications in interpreting observational and experimental characteristics of nonlinear EAW.[26–28] Further studies may yield different characteristics of these newly developed viscosity modified Zakharov's and NLSE equations. This work can also be extended for different classes of plasma systems.

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