Modulational instability of ion acoustic waves in a multi-species collisionless unmagnetized plasma consisting of nonthermal and isothermal electrons

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Modulational instability of ion acoustic waves in a multi-species collisionless unmagnetized plasma consisting of nonthermal and isothermal electrons

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A nonlinear Schrödinger equation is derived to study the modulational instability of finite amplitude ion acoustic waves in a collisionless unmagnetized plasma consisting of warm adiabatic ions and two distinct populations of electrons, one is due to distributed energetic electrons described by Cairns *et al.* [Geophys. Res. Lett. **22**, 2709 (1995)] which generates the energetic electrons, and the other is the isothermal electrons. The instability condition and the maximum growth rate of instability have been investigated numerically. We have studied the effect of each parameter of the present plasma system on the maximum growth rate of instability. In particular, it is found that the maximum growth rate of instability increases with the increasing values of the wave number for any given set of values of the parameters associated with the present plasma system. It has also been shown that for any fixed value of the wave number, the maximum growth rate of instability increases with increasing values of the nonthermal parameter associated with the Cairns distributed energetic electrons. *Published by AIP Publishing.*
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I. INTRODUCTION

Nonlinear modulated ion acoustic (IA) waves in a plasma system can be described by a nonlinear Schrödinger equation (NLSE). The NLSE can be derived by using either the Krylov-Bogoliubov-Mitropolsky (KBM) method[1] or the Reductive Perturbation Method (RPM).[2,3] In a steady state, under different conditions, soliton solutions of the NLSE are possible, and these soliton solutions are known as bright envelop solitons[4–6] and dark (black and grey) envelop solitons.[4–6]

Several authors investigated the modulational instability (MI) of IA waves in multi-species unmagnetized or magnetized plasmas theoretically[7–14] and experimentally.[15,16] Kourakis and Shukla[17] considered the oblique MI of IA waves in a collisionless unmagnetized electron-ion plasma consisting of cold ions and two distinct populations of isothermal electrons at different temperatures. Esfandyari-Kalejahi and Asgari[18] investigated the oblique MI of IA waves in a collisionless unmagnetized electron-ion plasma system consisting of warm ions and two distinct populations of isothermal electrons. Esfandyari-Kalejahi *et al.*[19] investigated the effect of adiabatic ion temperature on the MI of IA waves in a collisionless unmagnetized electron-ion plasma system consisting of two distinct populations of isothermal electrons. Recently, some authors[20–22] have studied the MI of IA waves in a plasma consisting of two temperature electrons having the same non-Maxwellian distribution for the cold and hot electron species. In the present paper, we have considered MI of IA waves in a collisionless unmagnetized electron-ion plasma consisting of two distinct populations of electrons at different temperatures, one of which is in thermodynamic equilibrium and this species of electrons obeys the Maxwellian velocity distribution and the other group of electron species contains fast energetic electrons which can be modeled by nonthermal distribution as prescribed by Cairns *et al.*[23]

The observations of solitary structures with density depletion made by the Freja Satellite,[24] influenced Cairns *et al.*[23] to investigate how the presence of fast energetic electrons changes the properties of ion acoustic waves for both positive and negative density perturbations. For this purpose, they considered a model for the distribution function of electrons, which has the property that the number of particles in the neighbourhood of the point $v = 0$ is much smaller than the number of particles in the neighbourhood of the point $v = 0$ for the case of a Boltzmann distribution, where v is the velocity of the particle in phase space. Actually, in the distribution of electrons as prescribed by Cairns *et al.*,[23] the number of electrons in the neighbourhood of $v = 0$ decreases with increasing β_e for $0 \leq \beta_e < \frac{4}{3}$, and the number of electrons in the neighbourhood of $v = 0$ is almost zero as $\beta_e \to \frac{4}{3}$, where β_e is the parameter associated with the Cairns distributed electrons. Again, it is simple to check that the distribution function develops wings for increasing β_e, which become stronger as β_e increases, and at the same time, the center density in phase space drops, the latter as a result of the normalization of the area under the integral.[25] According to Verheest and Pillay,[25] we should not take values of $\beta_e > \frac{4}{7}$ as that stage might stretch the credibility of the Cairns model too far and consequently, the physically admissible bounds of β_e is given by $0 \leq \beta_e \leq \frac{4}{7} \approx 0.6$. In the literature, this distribution of electrons is known as the nonthermal distribution. On the other hand, the Cairns[23] distributed nonthermal electrons reduce to the isothermal (Boltzmann-Maxwellian) electrons when $\beta_e = 0$. Although the isothermal velocity distribution of electrons is a special case of Cairns[23] distributed electrons for

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$\beta_e = 0$, but from the physical point of view, when $\beta_e = 0$, then these electrons are in thermodynamic equilibrium, whereas for increasing $\beta_e\,(>0)$, the relaxation time of the energetic electrons is not so small to reach the thermal equilibrium.[26]

Two different populations of electrons at different temperatures (e.g., cool and hot electrons) are common in laboratories[27–30] and also in space plasmas.[31,32] On the basis of these observations, several authors[33–42] theoretically studied different linear and nonlinear properties of ion acoustic waves in a plasma consisting of two different populations of electron species at different temperatures along with one or two ion species. In these investigations, the distribution functions of both the electrons (cool and hot electrons) were taken to be isothermal, i.e., both cooler and hotter electron species are in thermodynamic equilibrium and obey the Boltzmann-Maxwellian velocity distribution. Plasmas consisting of two different species of electrons at different temperatures have been observed by various spacecraft missions, viz., FAST at the auroral region,[43–47] Viking Satellite,[48,49] S3-3 Satellite,[32] GEOTAIL,[50] and POLAR[47,51,52] missions in the magnetosphere. Studies of two-electron-temperature plasmas by these satellite observations of moving localized potential variations regions not necessarily indicate that the distribution functions of both the electron species are isothermal. Specifically, the observations of electric field structures by the FAST[43–47] satellite and Viking Satellite[48,49] in the auroral zone together with the Freja Satellite[24] observations of electric field structures in the auroral zone of the upper ionosphere indicate the existence of energetic electrons along with isothermal electrons. So, there should always exists a certain portion of electrons which are in thermodynamic equilibrium. Thus, there may exist two different species of electrons at different temperatures in the auroral zone of the upper ionosphere of Earth. Among these two species of electrons, one species of electrons is in thermodynamic equilibrium, and therefore, this species follows Boltzmann-Maxwellian distribution. Another population of fast energetic electron species follows the nonthermal velocity distribution of Cairns *et al.*[23] Although the actual mechanism for the formation of energetic electrons in the space plasma is yet to be an open problem, different non-Maxwellian distributions have been modeled in phase space to describe the behaviour of the energetic particles. In fact, the electrostatic wave structures observed by the Freja Satellite[24] can be best described by considering Cairns[23] distributed nonthermal electrons. Recently, Rufai *et al.*[53] considered a similar electron-ion magnetized collisionless plasma system consisting of cold ions and two electron species at different temperatures, a cooler one with a Boltzmann distribution and a hotter one with a nonthermal Cairns distribution, whereas Singh and Lakhina[54] considered the same plasma system of Rufai *et al.*[53] without the effect of the uniform static magnetic field. Rufai *et al.*[53] and Singh and Lakhina[54] investigated the arbitrary amplitude supersoliton structures in the above mentioned plasma systems.

Regarding the justification of describing the electrons as two different species of particles of the plasma with respect to the ion acoustic time scale, we want to mention the paper of Jones *et al.*[27] According to them, the validity of describing the electron components as two fluids is justified on the grounds that processes which produce such electron distributions have time scales much shorter than the relevant ion time scale. But for the present plasma system, to make a clear explanation of describing the electrons as two different species of particles (of same mass) of the plasma, which are following two different types of distribution functions (a cooler one with a Boltzmann distribution and a hotter one with a nonthermal Cairns distribution) with respect to the ion acoustic time scale, we consider the following linearised dispersion relation of the IA waves.

Starting from the equation of continuity of ions, the equation of motion of ions, and the Poisson equation along with the expression of adiabatic pressure term and the expressions of number densities of isothermal electrons and Cairns[23] distributed nonthermal electrons, the linearised dispersion relation of the IA waves can be written as

$$\frac{\omega^2}{k^2} = C_{s1}^2 \frac{1 + \dfrac{\gamma\sigma_{ie}}{1+\gamma\sigma_{ie}} k^2\lambda_D^2}{1 + k^2\lambda_D^2}, \tag{1}$$

where ω is the wave frequency, k is the wave number, and C_{s1} and λ_D are given by the following equations:

$$C_{s1}^2 = (1+\gamma\sigma_{ie})\frac{K_B T_{eff}}{m}, \tag{2}$$

$$\frac{n_{co}+n_{s0}}{T_{eff}} = (1-\beta_e)\frac{n_{co}}{T_{ce}} + \frac{n_{s0}}{T_{se}}, \tag{3}$$

$$\frac{1}{\lambda_D^2} = \frac{1-\beta_e}{\lambda_{Dce}^2} + \frac{1}{\lambda_{Dse}^2}, \tag{4}$$

$$\lambda_{Dce}^2 = \frac{K_B T_{ce}}{4\pi e^2 n_{c0}}, \quad \lambda_{Dse}^2 = \frac{K_B T_{se}}{4\pi e^2 n_{s0}}. \tag{5}$$

Here, n_{c0} (n_{s0}), T_{ce} (T_{se}), m, $-e$, and β_e are unperturbed nonthermal (isothermal) electron number density, average temperature of nonthermal (isothermal) electrons, mass of an ion, electronic charge of an electron, and the nonthermal parameter associated with the Cairns distributed nonthermal electrons, respectively. Again, $\sigma_{ie} = T_i/T_{eff}$, and $\gamma(=3)$ is the adiabatic index, K_B is the Boltzmann constant, and T_i is the average ion temperature. The dispersion relation (1) shows that the linearized velocity of the IA wave is C_{s1} with λ_D as the Debye length of the plasma system.

Now, for the plasma system consisting of warm adiabatic ions and only one species of isothermal electrons, the dispersion relation (1) assumes the following form:

$$\frac{\omega^2}{k^2} = C_{s2}^2 \frac{1 + \dfrac{\gamma\sigma_{ie}}{1+\gamma\sigma_{ie}} k^2\lambda_{Dse}^2}{1 + k^2\lambda_{Dse}^2}, \tag{6}$$

where

$$C_{s2}^2 = (1+\gamma\sigma_{ie})\frac{K_B T_{se}}{m}. \tag{7}$$

From the definition of C_{s1}^2, C_{s2}^2, λ_D^2, λ_{Dse}^2, and T_{eff} as given by (2), (7), (4), the second equation of (5), and (3), respectively, we get the following two inequalities:

$$C_{s2}^2 \le C_{s1}^2 \text{ if } (1-\beta_e)\frac{T_{se}}{T_{ce}} \le 1, \tag{8}$$

$$\lambda_{Dse}^2 > \lambda_D^2. \tag{9}$$

As $0 \le \beta_e < 1 \iff 0 < 1-\beta_e \le 1$ and $0 < \frac{T_{se}}{T_{ce}} \le 1$, the inequality $(1-\beta_e)\frac{T_{se}}{T_{ce}} \le 1$ is always true, and the inequalities (8) and (9) hold good for any values of the parameters. From the inequalities (8) and (9), we get

$$\frac{\lambda_D^2}{C_{s1}^2} < \frac{\lambda_{Dse}^2}{C_{s2}^2} < (\omega_{pi}^2)^{-1}, \tag{10}$$

where

$$\omega_{pi}^2 = \frac{4\pi e^2 n_0}{m} \tag{11}$$

with n_0 is the unperturbed ion number density.

The inequality (10) can also be written as

$$(\omega_p)^{-1} < (\omega_{pi})^{-1}, \tag{12}$$

where

$$\left(\omega_p^2\right)^{-1} = \frac{\lambda_D^2}{C_{s1}^2} \iff \lambda_D^2\omega_p^2 = C_{s1}^2. \tag{13}$$

The inequality (12) efficiently shows the validity of describing the electrons as two different species of particles (of same mass) of the plasma, which are following two different types of distribution functions (a cooler one with a Boltzmann distribution and a hotter one with a nonthermal Cairns distribution) with respect to the ion acoustic time scale.

For the present problem, we have considered the same plasma system of Islam *et al.*[55] without the effect of the magnetized field, i.e., here, we consider a collisionless unmagnetized electron-ion plasma consisting of two distinct populations of electrons at different temperatures, one of which obeys the Maxwellian velocity distribution and the other electron species follows the nonthermal distribution of Cairns *et al.*[23] Again, it is assumed that the average thermal temperature of isothermal electrons is less than that of the nonthermal electrons. Therefore, the consideration of such a plasma system gives us an opportunity to investigate the MI of IA waves in a collisionless unmagnetized plasma.

II. BASIC EQUATIONS

We consider a fully ionized collisionless unmagnetized plasma composed of warm adiabatic ions and two distinct populations of electrons, one nonthermally distributed electrons due to Cairns *et al.*[23] and the other one isothermally distributed electrons. The nonlinear behaviour of IA waves propagating along the x–axis can be described by the following set of equations:

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0, \tag{14}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{\sigma}{n}\frac{\partial p}{\partial x} = -\frac{\partial \phi}{\partial x}, \tag{15}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_{ce} + n_{se} - n, \tag{16}$$

$$p = n^\gamma. \tag{17}$$

Here, n, n_{ce}, n_{se}, u, p, ϕ, x and t are the ion number density, the nonthermal electron number density, the isothermal electron number density, the ion fluid velocity, the ion pressure, the electrostatic potential, the spatial variable and time, respectively, and these quantities have been normalized by n_0, n_0, n_0, $c_s (=\sqrt{K_BT_{ef}/m})$, $n_0K_BT_i$, K_BT_{ef}/e, $\lambda_{Df} (=\sqrt{K_BT_{ef}/4\pi n_0 e^2})$, and ω_{pi}^{-1}, where $\sigma = T_i/T_{ef}$ and T_{ef} is given by the following equation:

$$\frac{n_{c0}+n_{s0}}{T_{ef}} = \frac{n_{c0}}{T_{ce}} + \frac{n_{s0}}{T_{se}}. \tag{18}$$

It is important to note that $T_{ef} = T_{eff}$ when $\beta_e = 0$. Although the effective temperature of the present plasma system is T_{eff}, here we use T_{ef} as a modified effective temperature.

Based on the above-mentioned normalization of the independent and dependent variables, the normalized number density of Cairns[23] distributed nonthermal electrons can be written as

$$n_{ce} = \bar{n}_{c0}(1 - \beta_e\sigma_c\phi + \beta_e\sigma_c^2\phi^2)\exp[\sigma_c\phi], \tag{19}$$

and the normalized number density of isothermal electrons can be written as

$$n_{se} = \bar{n}_{s0}\exp[\sigma_s\phi], \tag{20}$$

where

$$\beta_e = \frac{4\alpha_e}{1+3\alpha_e} \quad \text{with } \alpha_e \ge 0, \tag{21}$$

$\bar{n}_{c0}$, $\bar{n}_{s0}$, σ_c, and σ_s are given by

$$\bar{n}_{c0} = \frac{n_{c0}}{n_0}, \quad \bar{n}_{s0} = \frac{n_{s0}}{n_0}, \quad \sigma_c = \frac{T_{ef}}{T_{ce}}, \quad \sigma_s = \frac{T_{ef}}{T_{se}}. \tag{22}$$

The unperturbed charge neutrality condition is given by

$$n_{c0} + n_{s0} = n_0, \tag{23}$$

using the first two equations of (22) and Eq. (23), it can be written as

$$\bar{n}_{c0} + \bar{n}_{s0} = 1. \tag{24}$$

Equation (18) can be written as

$$\bar{n}_{c0}\sigma_c + \bar{n}_{s0}\sigma_s = 1. \tag{25}$$

Again, introducing the new parameters σ_{sc} and n_{sc} as

$$\sigma_{sc} = \frac{T_{se}}{T_{ce}}, \quad n_{sc} = \frac{n_{s0}}{n_{c0}}, \tag{26}$$

and using Equations (24) and (25), the expressions of $\bar{n}_{c0}$, $\bar{n}_{s0}$, σ_c, and σ_s can be simplified as follows:

$$\bar{n}_{s0} = \frac{n_{sc}}{1+n_{sc}}, \quad \bar{n}_{c0} = \frac{1}{1+n_{sc}}, \tag{27}$$

$$\sigma_s = \frac{1+n_{sc}}{\sigma_{sc}+n_{sc}}, \quad \sigma_c = \sigma_{sc}\frac{1+n_{sc}}{\sigma_{sc}+n_{sc}}. \tag{28}$$

Therefore, γ, β_e, σ, σ_{sc}, and n_{sc} are the only parameters of the present plasma system. According to the discussions as given in Section I, we have considered the following case: $0 \le n_{sc} \le 1$ and $0 < \sigma_{sc} < 1$.

Expanding both n_{ce} and n_{se} as given by (19) and (20), respectively, using the charge neutrality condition and keeping the terms up to ϕ^3, we can write the Poisson equation (16) as

$$\frac{\partial^2\phi}{\partial x^2} = h_0 + h_1\phi + h_2\phi^2 + h_3\phi^3 - n, \tag{29}$$

where h_0, h_1, h_2, and h_3 are given by

$$h_0 = 1, \quad h_1 = (1-\beta_e\bar{n}_{c0}\sigma_c), \tag{30}$$

$$h_2 = \frac{1}{2}\left[\bar{n}_{s0}\sigma_s^2 + \bar{n}_{c0}\sigma_c^2\right], \tag{31}$$

$$h_3 = \frac{1}{6}\left[\bar{n}_{s0}\sigma_s^3 + \bar{n}_{c0}(1+3\beta_e)\sigma_c^3\right]. \tag{32}$$

III. DERIVATION OF THE NLSE

In order to study the MI of the IA waves in a collisionless unmagnetized plasma, we have used the RPM[2,3] to obtain the NLSE. We have also considered the following stretching of the coordinate and time:

$$\xi = \epsilon(x - V_g t), \quad \tau = \epsilon^2 t, \tag{33}$$

where V_g is a constant and ϵ is a small parameter measuring the weakness of the dispersion. We take the following perturbation of the field quantities to make a balance between the nonlinear and dispersive terms:

$$f = f^{(0)} + \sum_{l=1}^{\infty}\epsilon^l \sum_{a=-\infty}^{+\infty} f_a^{(l)}(\xi,\tau)\exp[ia\psi], \tag{34}$$

where

$$\psi = kx - \omega t, \tag{35}$$

k is the wave number, ω is the wave frequency, and $f = n$, u, and ϕ with $n^{(0)} = 1$, $u^{(0)} = \phi^{(0)} = 0$. Here, we have also used the following terminology: $f_{-a}^{(l)} = \bar{f}_a^{(l)}$, where "bar" denotes the complex conjugate.

To make a one-one correspondence between the multiple scale perturbation method[56,57] and the RPM,[2,3] we have the following consistency conditions:

$$(i)\ n_0^{(1)} = 0, \quad u_0^{(1)} = 0, \quad \phi_0^{(1)} = 0, \tag{36}$$

$$(ii)\ n_a^{(s)} = 0, \quad u_a^{(s)} = 0, \quad \phi_a^{(s)} = 0 \quad \text{for } s < |a|. \tag{37}$$

Again, in the perturbation expansion (34), there is only one term corresponding to $a = 0$ and also for $-a = 0$, and consequently, without loss of generality, we can assume $\bar{f}_0^{(l)} = 0$ for any admissible value of l.

Substituting the perturbation expansions for n, u, and ϕ, according to the law as given in (34), into Equations (14), (15), (17), and (29) and collecting the terms of different powers of ϵ, we get a sequence of equations of different orders. From each equation of a particular order, one can generate another sequence of equations for different harmonics by changing the values of a.

A. First order ($l = 1$)

It is simple to check that the zeroth harmonic ($a = 0$) equations for the equation of continuity of ions, the equation of motion of ion fluid, and the Poisson equation are identically satisfied due to the consistency condition (36). So, at this order, we shall first of all consider the equations of the first harmonic ($a = 1$).

Solving the first harmonic ($a = 1$) equations of the equation of continuity of ions and the equation of motion of ions for the unknowns $n_1^{(1)}$ and $u_1^{(1)}$, we get

$$n_1^{(1)} = \frac{k^2}{W^2}\phi_1^{(1)}, \quad u_1^{(1)} = \frac{k\omega}{W^2}\phi_1^{(1)}, \tag{38}$$

where $W^2 = \omega^2 - \sigma\gamma k^2$.

From the first harmonic ($a = 1$) equation of the Poisson equation, we get

$$n_1^{(1)} - (k^2 + h_1)\phi_1^{(1)} = 0. \tag{39}$$

This equation and the first equation of (38) give the following linearized dispersion relation of the IA waves

$$\frac{\omega^2}{k^2} = \frac{1}{k^2 + h_1} + \gamma\sigma. \tag{40}$$

B. Second order ($l = 2$)

1. First harmonic ($a = 1$)

Solving the first harmonic ($a = 1$) equations of the equation of continuity of ions and the equation of motion of ions for the unknowns $n_1^{(2)}$ and $u_1^{(2)}$, we get

$$n_1^{(2)} = \frac{k^2}{W^2}\phi_1^{(2)} + \frac{2ik\omega(V_g k - \omega)}{W^4}\frac{\partial\phi_1^{(1)}}{\partial\xi}, \tag{41}$$

$$u_1^{(2)} = \frac{k\omega}{W^2}\phi_1^{(2)} + \frac{i(V_g k - \omega)(\omega^2 + \sigma\gamma k^2)}{W^4}\frac{\partial\phi_1^{(1)}}{\partial\xi}. \tag{42}$$

From the first harmonic ($a = 1$) equation of the Poisson equation, we get

$$n_1^{(2)} - (k^2 + h_1)\phi_1^{(2)} = -2ik\frac{\partial\phi_1^{(1)}}{\partial\xi}. \tag{43}$$

Substituting the expression (41) for $n_1^{(2)}$, in Equation (43), we get

$$-\frac{k^2(k^2+h_1)}{W^2}\left\{\frac{\omega^2}{k^2}-\left(\gamma\sigma+\frac{1}{k^2+h_1}\right)\right\}\phi_1^{(2)}$$
$$+\frac{2i\omega k^2}{W^4}\left(V_g-\frac{\omega^2-W^4}{\omega k}\right)\frac{\partial\phi_1^{(1)}}{\partial\xi}=0. \quad (44)$$

The first term of this equation is equal to zero due to the linear dispersion relation (40) of the IA waves, and the second term of the same equation can be made equal to zero if

$$V_g=\frac{\omega^2-W^4}{\omega k}=\frac{k}{\omega}\left[\frac{h_1}{(k^2+h_1)^2}+\sigma\gamma\right], \quad (45)$$

where we have used the linear dispersion relation (40) to simplify Equation (45).

Now, differentiating the linear dispersion relation (40) with respect to k, we get

$$\frac{\partial\omega}{\partial k}=\frac{k}{\omega}\left[\frac{h_1}{(k^2+h_1)^2}+\sigma\gamma\right]. \quad (46)$$

From Equations (45) and (46), we get

$$V_g=\frac{\partial\omega}{\partial k}, \quad (47)$$

and consequently, Equation (44) is identically satisfied if V_g is the group velocity of the IA wave.

2. Second harmonic (a = 2)

Solving the second harmonic ($a=2$) equations of the equation of continuity of ions, the equation of motion of ions and the Poisson equation for the unknowns $\phi_2^{(2)}$, $n_2^{(2)}$, and $u_2^{(2)}$, we get

$$(\phi_2^{(2)},n_2^{(2)},u_2^{(2)})=(A_\phi,A_n,A_u)\left[\phi_1^{(1)}\right]^2, \quad (48)$$

where

$$A_\phi=-\frac{h_2}{3k^2}+\frac{k^2\omega^2}{2W^6}+g_1\sigma\gamma\frac{k^4}{6W^6}, \quad (49)$$

$$A_n=(4k^2+h_1)A_\phi+h_2, \quad (50)$$

$$A_u=\frac{\omega}{k}\left[A_n-\frac{k^4}{W^4}\right], \quad (51)$$

and $g_1=\gamma-2$.

3. Zeroth harmonic (a = 0)

Solving the zeroth harmonic ($a=0$) equations of the equation of continuity of ions, the equation of motion of ions, and the Poisson equation for the unknowns $\phi_0^{(2)}$, $n_0^{(2)}$, and $u_0^{(2)}$, we get

$$(\phi_0^{(2)},n_0^{(2)},u_0^{(2)})=(B_\phi,B_n,B_w)|\phi_1^{(1)}|^2, \quad (52)$$

where

$$B_\phi=\frac{g_1\gamma\sigma k^4+k^2\omega(2kV_g+\omega)-2h_2W^4\left(V_g^2-\gamma\sigma\right)}{W^4\left[h_1\left(V_g^2-\sigma\gamma\right)-1\right]}, \quad (53)$$

$$B_n=h_1B_\phi+2h_2, \quad (54)$$

$$B_u=V_gB_n-\frac{2\omega k^3}{W^4}. \quad (55)$$

C. Third order (*l* = 3): First harmonic (*a* = 1)

Solving the equation of continuity of ions and the equation of motion of ions for the unknowns $n_1^{(3)}$ and $u_1^{(3)}$, we can express $n_1^{(3)}$ and $u_1^{(3)}$ as a function of $\phi_1^{(1)}$, $\phi_1^{(2)}$, and $\phi_1^{(3)}$ along with their different derivatives with respect to ξ and τ. In particular, $n_1^{(3)}$ can be expressed as

$$n_1^{(3)}=\frac{k^2}{W^2}\phi_1^{(3)}+2ik\frac{\omega(V_gk-\omega)}{W^4}\frac{\partial\phi_1^{(2)}}{\partial\xi}-i\frac{2k^2\omega}{W^4}\frac{\partial\phi_1^{(1)}}{\partial\tau}$$
$$-\frac{(V_gk-\omega)}{W^6}\left(3V_gk\omega^2-3\sigma\gamma k^2\omega-\omega^3+\sigma\gamma V_gk^3\right)\frac{\partial^2\phi_1^{(1)}}{\partial\xi^2}$$
$$+\left[2\frac{k^3\omega}{W^4}(A_u+B_u)+\frac{k^2}{W^4}\left(\omega^2+\sigma\gamma g_1k^2\right)(A_n+B_n)\right]$$
$$\times|\phi_1^{(1)}|^2\phi_1^{(1)}. \quad (56)$$

From the Poisson equation, we get

$$n_1^{(3)}-\left(k^2+h_1\right)\phi_1^{(3)}=-2ik\frac{\partial\phi_1^{(2)}}{\partial\xi}-\frac{\partial^2\phi_1^{(1)}}{\partial\xi^2}$$
$$+\left\{2h_2(A_\phi+B_\phi)+3h_3\right\}|\phi_1^{(1)}|^2\phi_1^{(1)}. \quad (57)$$

Now, eliminating $n_1^{(3)}$ from Equations (57) and (56), we get

$$-\frac{k^2(k^2+h_1)}{W^2}\left\{\frac{\omega^2}{k^2}-\left(\gamma\sigma+\frac{1}{k^2+h_1}\right)\right\}\phi_1^{(3)}$$
$$+\frac{2i\omega k^2}{W^4}\left(V_g-\frac{\omega^2-W^4}{\omega k}\right)\frac{\partial\phi_1^{(2)}}{\partial\xi}-i\frac{2k^2\omega}{W^4}\frac{\partial\phi_1^{(1)}}{\partial\tau}$$
$$+\left[1-\frac{k^4\left(V_g-\frac{\omega}{k}\right)}{W^6}\left(3V_g\frac{\omega^2}{k^2}-3\gamma\sigma\frac{\omega}{k}-\frac{\omega^3}{k^3}+\gamma\sigma V_g\right)\right]$$
$$\times\frac{\partial^2\phi_1^{(1)}}{\partial\xi^2}+\left[2\frac{k^3\omega}{W^4}(A_u+B_u)+\frac{k^2}{W^4}\left(\omega^2+\sigma\gamma g_1k^2\right)\right.$$
$$\left.\times(A_n+B_n)-3h_3-2h_2(A_\phi+B_\phi)\right]|\phi_1^{(1)}|^2\phi_1^{(1)}=0. \quad (58)$$

Using the linear dispersion relation (40) and the expression of V_g as given in Equation (45), Equation (58) can be written as

$$i\frac{\partial\phi_1^{(1)}}{\partial\tau}+P\frac{\partial^2\phi_1^{(1)}}{\partial\xi^2}+Q|\phi_1^{(1)}|^2\phi_1^{(1)}=0, \quad (59)$$

where

$$P=-\frac{W^4}{2k^2\omega}\left[1-\frac{k^4}{W^6}\left(V_g-\frac{\omega}{k}\right)\right.$$
$$\left.\times\left(3V_g\frac{\omega^2}{k^2}-3\gamma\sigma\frac{\omega}{k}-\frac{\omega^3}{k^3}+\gamma\sigma V_g\right)\right], \quad (60)$$

$$Q = -\frac{W^4}{2k^2\omega}\left[2\frac{k^3\omega}{W^4}(A_u + B_u) + \frac{k^2}{W^4}(\omega^2 + \sigma\gamma g_1 k^2)\right.$$
$$\left.\times (A_n + B_n) - 3h_3 - 2h_2(A_\phi + B_\phi)\right]. \quad (61)$$

IV. MODULATIONAL INSTABILITY

Now, to study the stability of the modulated IA waves represented by the NLSE (59), it is important to observe that

$$\phi_1^{(1)} = \phi_0 e^{i\Delta\tau} \quad (62)$$

is a steady state solution of the NLSE (59) if

$$\Delta = Q|\phi_0|^2, \quad (63)$$

where ϕ_0 is a constant.

To study the modulational instability of IA waves, we decompose $\phi_1^{(1)}$ as

$$\phi_1^{(1)} = (\phi_0 + \delta\phi)e^{i\Delta\tau}, \quad (64)$$

where $|\delta\phi| \ll |\phi_0|$.

Substituting (64) into Equation (59), using Equation (63), and finally linearizing the equation with respect to the perturbed quantity $\delta\phi$, we get the following equation:

$$i\frac{\partial\delta\phi}{\partial\tau} + P\frac{\partial^2\delta\phi}{\partial\xi^2} + Q|\phi_0|^2(\delta\phi + \overline{\delta\phi}) = 0, \quad (65)$$

where $\overline{\delta\phi}$ is the complex conjugate of $\delta\phi$.

Substituting $\delta\phi = U + iV$ into Equation (65) and then separating into real and imaginary parts, we obtain the following two coupled equations:

$$-\frac{\partial V}{\partial\tau} + P\frac{\partial^2 U}{\partial\xi^2} + 2QU|\phi_0|^2 = 0 \quad (66)$$

and

$$\frac{\partial U}{\partial\tau} + P\frac{\partial^2 V}{\partial\xi^2} = 0, \quad (67)$$

where we have assumed that U and V are real functions of ξ and τ.

Substituting

$$U = U_0\exp[i(K\xi - \Omega\tau)] + c.c., \quad (68)$$
$$V = V_0\exp[i(K\xi - \Omega\tau)] + c.c., \quad (69)$$

into Equations (66) and (67), we get the following equations:

$$i\Omega V_0 + (-PK^2 + 2Q|\phi_0|^2)U_0 = 0 \quad (70)$$

and

$$i\Omega U_0 + PK^2 V_0 = 0. \quad (71)$$

For the non trivial solution of the above linear equations for the unknown quantities U_0 and V_0, we get

$$\Omega^2 = [PK^2]^2\left(1 - \frac{2Q|\phi_0|^2}{PK^2}\right). \quad (72)$$

If $1 - \frac{2Q|\phi_0|^2}{PK^2} \geq 0$, then from the relation (72), one can get real values of Ω, and consequently, IA wave is modulationally stable. From the expression of Ω^2 as given in Equation (72), we see that Ω^2 is strictly positive for $PQ < 0$. Therefore, the IA wave is always modulationally stable for all $PQ < 0$. On the other hand, if $PQ > 0$, then $\Omega^2 \geq 0$ or $\Omega^2 < 0$ according to whether $K \geq K_c$ or $K < K_c$, where K_c is given by the equation

$$K_c = \sqrt{\frac{2Q|\phi_0|^2}{P}}. \quad (73)$$

Therefore, we see that IA wave is modulationally stable, i.e., $\Omega^2 \geq 0$ when either $PQ < 0$ or $K \geq K_c$ whenever $PQ > 0$.

On the other hand, if $PQ > 0$ and $K < K_c$, then $\Omega^2 < 0$, and all the roots of Equation (72) for the unknown Ω are purely imaginary. Consequently, the IA wave is modulationally unstable, and the growth rate of instability Γ $(= Im(\Omega))$ is given by the following equation:

$$\Gamma^2 = [PK^2]^2\left(\frac{2Q|\phi_0|^2}{PK^2} - 1\right). \quad (74)$$

For given values of P and Q, the growth rate of instability Γ attains its maximum value Γ_{max} at $K = \frac{K_c}{\sqrt{2}} = \sqrt{\frac{Q|\phi_0|^2}{P}}$ and the maximum growth rate of instability Γ_{max} is given by

$$\Gamma_{max} = |Q||\phi_0|^2. \quad (75)$$

Again, we can write Equation (72) as

$$\Omega^2 = [PK^2](PK^2 - 2Q|\phi_0|^2). \quad (76)$$

From this equation, we see that $\Omega^2 = 0$ if $P = 0$. Although we see that $\Omega^2 = [PK^2]^2$, if $Q = 0$, the analysis is erroneous because for $Q = 0$, it is not possible to study the MI of IA waves with the help of the present NLSE. For $Q = 0$, Equation (59) loses its nonlinearity, and consequently, a modified NLSE is necessary to study the MI of IA waves.

V. SUMMARY AND DISCUSSIONS

A NLSE has been derived to study the MI of IA waves in a multi-species collisionless unmagnetized plasma consisting of warm adiabatic ions and two distinct populations of electrons, one due to Cairns distributed energetic electrons[23] which generates the energetic electrons, and the other is the isothermal electrons. The instability condition and the maximum growth rate of instability are analytically derived. The instability condition has been investigated numerically. We have investigated the effect of each parameter of the present plasma system on the maximum growth rate of instability. β_e, n_{sc}, σ_{sc}, and σ are the four basic parameters of the system. The observations of the present problem regarding the instability conditions and the maximum growth rate of instability with respect to the different parameters of the system can be summarized as follows:

1. It is simple to check that $P<0$ for any set of values of the parameters involved in the system if k is restricted by the inequality: $0<k<4$. Again, it is simple to check that the phase velocity $\frac{\omega}{k}$ and also the group velocity $V_g=\frac{\partial\omega}{\partial k}$ are decreasing functions of k for any value of the parameters involved in the system, and at $k=4$, the phase velocity and the group velocity of the IA wave assume a very small numerical value, and consequently, the value of k is restricted by $0<k<4$. For this restriction of k, we have $P<0$ for any value of the parameters involved in the system.
2. As $P<0$, from the expression of Ω^2 as given in Equation (72), we see that Ω^2 is strictly positive for $Q>0$. Therefore, the IA wave is modulationally stable for all $Q>0$. On the other hand, if $Q<0$, then $\Omega^2\geq 0$ or $\Omega^2<0$ according to whether $K\geq K_c$ or $K<K_c$.
3. Now, it is simple to check that if Q is a function of k, β_e, n_{sc}, σ_{sc}, σ, and γ, i.e., $Q=Q(k,\beta_e,n_{sc},\sigma_{sc},\sigma,\gamma)$. Therefore, Q is a function of k and β_e for fixed values of n_{sc}, σ_{sc}, σ, and γ, and consequently, $Q=0$ gives a functional relation between k and β_e. This functional relation between k and β_e is plotted in Figure 1 for the other values of the parameters as mentioned in Figure 1, i.e., in Figure 1, k is plotted against the nonthermal parameter β_e when $Q=0$ for $\gamma=3$, $\sigma=0.001$, and $n_{sc}=0.2$, and for two different values of σ_{sc}, viz., Figure 1(a) for $\sigma_{sc}=0.08$ and Figure 1(b) for $\sigma_{sc}=0.15$. In each figure, the curve $Q=0$ divides the entire region into two parts, viz., $Q>0$ and $Q<0$ as shown in the figure. In the region $Q>0$, $\Omega^2>0$, and consequently, IA wave is modulationally stable for any point (β_e,k) that lies in the region $Q>0$.
4. In Figure 2, k is plotted against the nonthermal parameter β_e when $Q=0$ for $\gamma=3$, $\sigma=0.001$, and $\sigma_{sc}=0.6$ and for two different values of n_{sc}, viz., Figure 2(a) for $n_{sc}=0.1$ and Figure 2(b) for $n_{sc}=0.5$. In each figure, the curve $Q=0$ divides the entire region into two parts, viz., $Q>0$ and $Q<0$ as shown in the figure. In the region $Q>0$, $\Omega^2>0$, and consequently, the IA wave is modulationally stable for any point (β_e,k) that lies in the region $Q>0$.
5. In Figures 1 and 2, the region $Q<0$ can be divided into two parts $K>K_c$ and $K<K_c$ where $\Omega^2>0$ or $\Omega^2<0$ according to whether $K>K_c$ or $K<K_c$ and $\Omega^2=0$ along the curve $K=K_c$.
6. From Figures 1 and 2, we see that the region $Q>0$ is bounded, and this region is bounded by the curves $k=0$, $\beta_e=0$, $\beta_e=0.6$, and the curve $Q=0$, whereas the region $Q<0$ can be made bounded if k is restricted by $0<k<4$, and this region is bounded by the curves $k=0$, $k=4$ $\beta_e=0.6$, and the curve $Q=0$.
7. Figures 3(a) and 3(b) are characteristically different. To explain this fact, in Figure 3(a), k is plotted against β_e when $Q=0$ for $\gamma=3$, $\sigma=0.001$, and $n_{sc}=0.2$ and for different values of σ_{sc} starting from $\sigma_{sc}=0.025$ and ending with $\sigma_{sc}=0.0875$, whereas in Figure 3(b), k is plotted against the nonthermal parameter β_e when $Q=0$ for $\gamma=3$, $\sigma=0.001$, and $n_{sc}=0.2$ and for different values of σ_{sc} starting from $\sigma_{sc}=0.1$ and ending with $\sigma_{sc}=0.5$. From Figure 3(a), we see that the region $Q>0$ decreases with increasing σ_{sc} for $0<\sigma_{sc}\leq 0.0875$ (approximately), whereas from Figure 3(b), we see that the region $Q>0$ increases with increasing σ_{sc} for $0.0875<\sigma_{sc}<1$. From Figure 3(b), we see that the curve $Q=0$ appears to intersect the axis of β_e for the increasing values of β_e and the curve $Q=0$ tends to intersect the axis of β_e at the point $\beta_e=\beta_r$. It is simple to check that β_r increases for increasing values of σ_{sc}, but there exists a critical value $\sigma_{sc}^{(c)}=0.556$ (approximately) of σ_{sc} such that β_r assumes

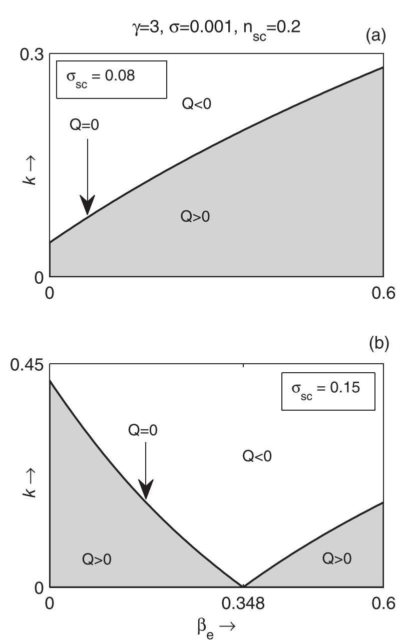

FIG. 1. k is plotted against β_e when $Q=0$ for the two values of σ_{sc}: (a) for $\sigma_{sc}=0.08$ and (b) for $\sigma_{sc}=0.15$.

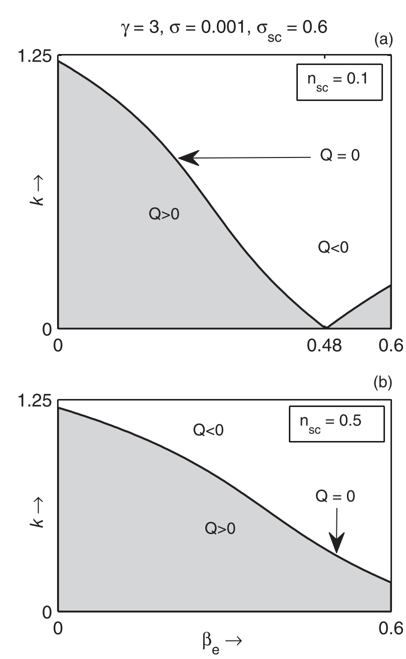

FIG. 2. k is plotted against β_e when $Q=0$ for the two values of n_{sc}: (a) for $n_{sc}=0.1$ and (b) for $n_{sc}=0.5$.

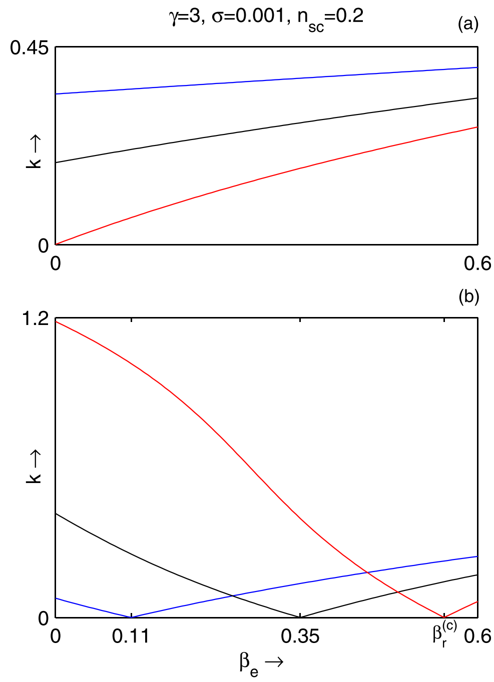

FIG. 3. k is plotted against β_e when $Q=0$ for different values of σ_{sc}: (a) Blue, black, and red curves correspond to $\sigma_{sc}=0.025$, $\sigma_{sc}=0.055$, and $\sigma_{sc}=0.0875$, respectively, and (b) blue, black, and red curves correspond to $\sigma_{sc}=0.1$, $\sigma_{sc}=0.15$, and $\sigma_{sc}=0.5$, respectively, and here, $\beta_r^{(c)}=0.552$.

its maximum value $\beta_r^{(c)}=0.552$; therefore, we have $\beta_r \leq \beta_r^{(c)}$ for $0.0875 < \sigma_{sc} \leq \sigma_{sc}^{(c)}$. So, β_r increases for increasing values of σ_{sc} lying within $0.0875 < \sigma_{sc} \leq \sigma_{sc}^{(c)}$, whereas β_r decreases for increasing values of σ_{sc} for $\sigma_{sc}^{(c)} < \sigma_{sc} < 1$.

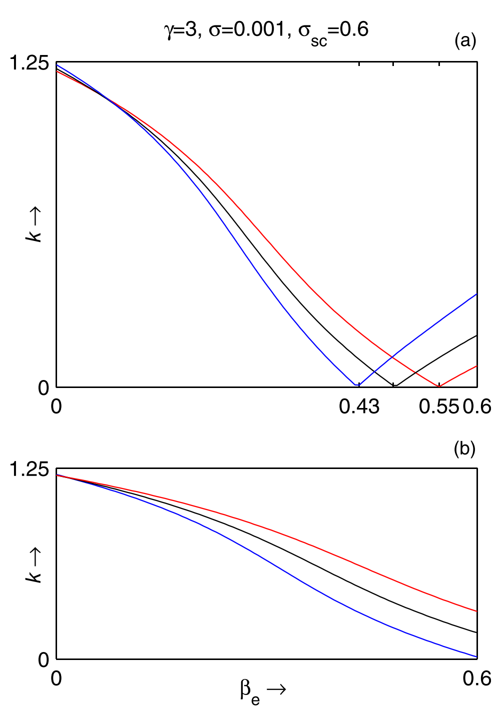

FIG. 4. k is plotted against β_e when $Q=0$ for different values of n_{sc}: (a) Blue, black, and red curves correspond to $n_{sc}=0.01$, $n_{sc}=0.1$, and $n_{sc}=0.2$, respectively, and (b) blue, black, and red curves correspond to $n_{sc}=0.3$, $n_{sc}=0.5$, and $n_{sc}=0.7$, respectively.

8. In Figures 4(a) and 4(b), we plot the curve $Q=0$ in the $\beta_e - k$ plane for three different values of n_{sc}. From Figure 4(a), we see that the region $Q>0$ increases with increasing n_{sc} for $0 < n_{sc} \leq 0.28$ (approximately), i.e., the stable region increases with increasing n_{sc} for $0 < n_{sc} \leq 0.28$ (approximately), and also we see that the curve $Q=0$ appears to intersect the axis of β_e for $0 < n_{sc} \leq 0.28$ (approximately), whereas from Figure 4(b), we see that the region $Q>0$ increases with increasing n_{sc} for $0.28 < n_{sc} \leq 1$, and consequently, the stable region increases with increasing n_{sc} for $0.28 < n_{sc} \leq 1$. So, we observe that the region $Q>0$ increases with increasing n_{sc}, i.e., the stable region increases with increasing n_{sc} for $0 < n_{sc} \leq 1$.
9. P, Q, and $\Gamma_{max}/|\phi_0|^2$ are plotted in Figures 5(a), 5(b), and 5(c), respectively, against k for $\gamma=3$, $\sigma=0.001$, $n_{sc}=0.25$, and $\sigma_{sc}=0.25$ and for different values of β_e. Here, blue, black, and red curves in each figure correspond to $\beta_e=0$, $\beta_e=0.3$, and $\beta_e=0.6$, respectively. From Figures 5(a) and 5(b), we see that $P<0$ and $Q<0$ in a particular region of the wave number as mentioned in the figures, and consequently, $PQ>0$ implies that the maximum modulational growth rate of instability is given by Equation (75). From Figure 5(c), we see that

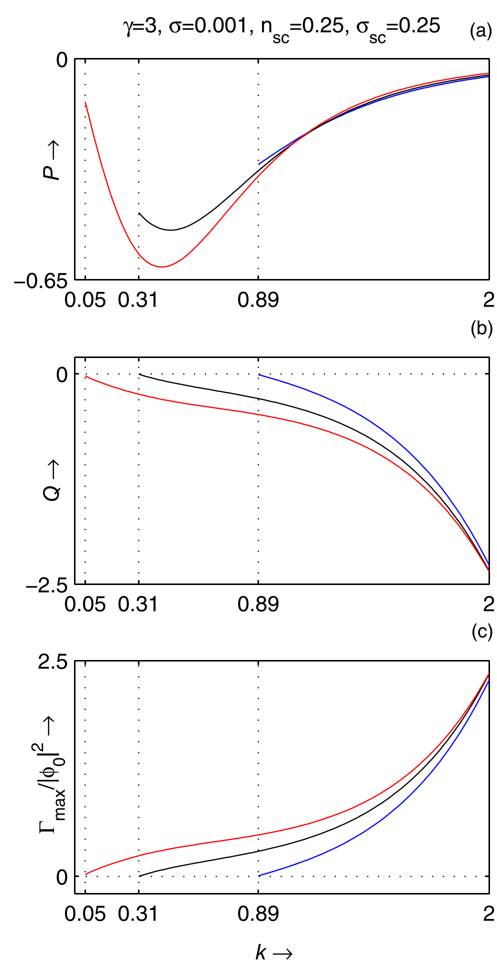

FIG. 5. P, Q, and $\Gamma_{max}/|\phi_0|^2$ are plotted in (a), (b), and (c), respectively, against k for different values of β_e. Blue, black, and red curves of each figure correspond to $\beta_e=0$, $\beta_e=0.3$, and $\beta_e=0.6$, respectively.

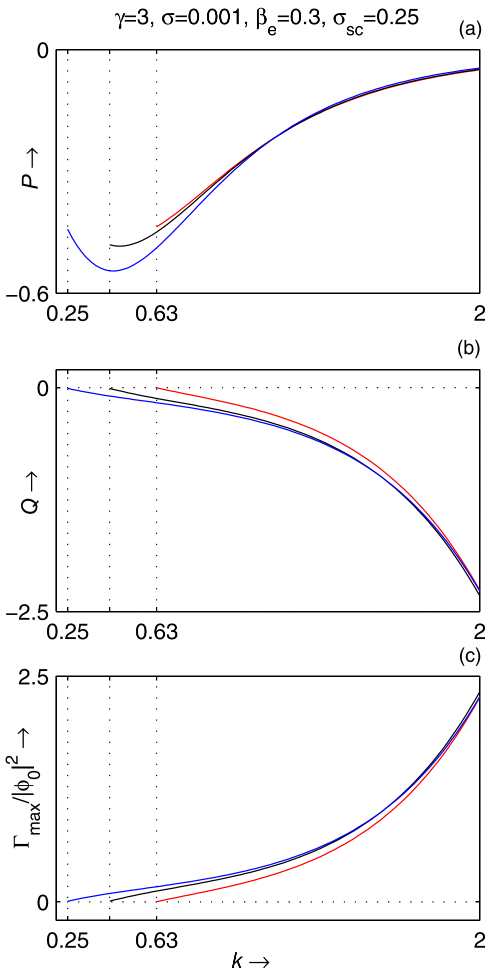

FIG. 6. P, Q, and $\Gamma_{max}/|\phi_0|^2$ are plotted in (a), (b), and (c), respectively, against k for different values of n_{sc}. Blue, black, and red curves of each figure correspond to $n_{sc}=0.1$, $n_{sc}=0.4$, and $n_{sc}=0.7$, respectively.

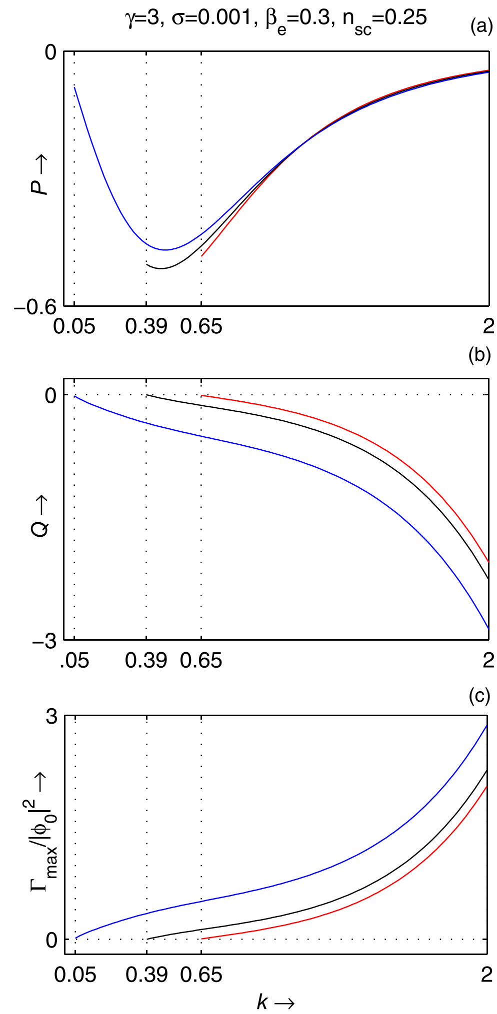

FIG. 7. P, Q, and $\Gamma_{max}/|\phi_0|^2$ are plotted in (a), (b), and (c), respectively, against k for different values of σ_{sc}. Blue, black, and red curves of each figure correspond to $\sigma_{sc}=0.1$, $\sigma_{sc}=0.3$, and $\sigma_{sc}=0.8$, respectively.

the maximum modulational growth rate of instability increases with increasing β_e and also the maximum modulational growth rate of instability strictly increases with the increasing wave number.

10. In Figures 6(a), 6(b), and 6(c), P, Q, and $\Gamma_{max}/|\phi_0|^2$ are, respectively, plotted against k for $\gamma=3$, $\sigma=0.001$, $\beta_e=0.3$, and $\sigma_{sc}=0.25$ and for different values of n_{sc}. Here, also blue, black, and red curves of each figure correspond to $n_{sc}=0.1$, $n_{sc}=0.4$, and $n_{sc}=0.7$, respectively. Figures 6(a) and 6(b) together show that PQ is strictly positive in a region of the wave number as mentioned in the figures. Figure 6(c) shows that the maximum modulational growth rate of instability strictly increases with the increasing wave number.
11. P, Q, and $\Gamma_{max}/|\phi_0|^2$ are plotted in Figures 7(a), 7(b), and 7(c), respectively, against k for $\gamma=3$, $\sigma=0.001$, $\beta_e=0.3$, and $n_{sc}=0.25$ and for different values of σ_{sc}. Here, the blue curve, black curve, and red curve of each figure correspond to $\sigma_{sc}=0.1$, $\sigma_{sc}=0.3$, and $\sigma_{sc}=0.8$. We see that the maximum modulational growth rate of instability strictly decreases with the increasing values of σ_{sc}, but the maximum modulational growth rate of instability increases with the increasing wave number.

In the steady state, under different conditions, the soliton solutions of the NLSE are bright envelop solitons[4–6] and dark (black and grey) envelop solitons.[4–6] In the present paper, we have considered only the modulational instability of the ion acoustic wave.

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