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Modeling of raindrop size distributions from multiwavelength rain attenuation measurements

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Abstract. Techniques for modeling the distribution of raindrop sizes, in terms of the lognormal and modified gamma distribution, from multiwavelength rain attenuation measurements at millimeter and infrared wave bands have been demonstrated. In order to obtain the three-parameter lognormal distribution, three experimental measurements are required, which can be attenuations at two frequencies and rain rate. Three measurements are used to form three equations, which are treated as nonlinear and are solved using a numerical technique to obtain the distribution parameters. The gamma distribution is treated as a two-parameter distribution, taking a fixed value of 2 for the index. The two parameters of the gamma distribution are obtained from measurements of rain rate and infrared attenuation following the technique described by *Maitra and Gibbins* [1995]. From an analysis of two rain events it is found that the measured attenuations at millimeter wavelengths show somewhat better agreement with the calculated attenuations obtained from the lognormal distribution than from the modified gamma distribution. The gamma model gives higher number densities for small drops than those given by the lognormal model.

1. Introduction

The distribution of raindrop sizes has been a subject of continued interest with the availability of more data from different locations. Raindrop sizes can vary very rapidly in both space and time under the influence of different physical processes. The measurement of the drop size distribution (DSD) is important for many reasons and is most crucial for determining rain attenuations at millimeter wavelengths. Measurements of DSDs reveal the microstructure of rain and can be used to indicate the evolution of precipitation during a rain event. The detailed microstructure of rain is of fundamental importance for a wide range of meteorological [*Zawadzki and Antonio*, 1988] and radio applications.

The distribution of raindrops is not an easy function to quantify. It has been customary to give a measure of DSD in terms of a distribution function. The distribution specified by *Laws and Parsons* [1943] and negative exponential distributions, such as those given by *Marshall and Palmer* [1948] and *Joss et al.* [1968], are widely used for DSD models. However, in natural rain, the DSD is highly variable and is affected by a large number of multiplicative processes [*Mason*, 1971] for which the lognormal distribution may be a suitable candidate. Various workers have examined lognormal distributions for raindrops, particularly in view of the inadequacy of negative-exponential models in describing small drops [*Levin*, 1954; *Bradley and Stow*, 1974; *Barcley et al.*, 1978; *Fang and Chen*, 1982]. *Ajayi and Olsen* [1985] found a single lognormal model to be appropriate for the entire range of rain rates in a tropical location. Also, the lognormal distribution has been fitted to distrometer data obtained in the United Kingdom, which yielded better agreement with measured attenuations compared with those given by the Laws-Parson or Marshall-Palmer distributions [*Harden et al.*, 1978].

The suitability of a technique for DSD measurement depends on the type of application. Direct measurements using a distrometer have limitations, given that it gives a localized measurement and that the impact distrometer has a relatively large time constant (~1 min). Radar measurements of DSD are suitable for obtaining a picture throughout the vertical extent of rain. The average DSD over a certain

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path length is more appropriate for determining rain attenuations for millimeter waves propagating over the path length. Again, near-instantaneous measurements of DSD can reveal fast variations in DSD under various conditions of air turbulence. The near-instantaneous path-averaged DSD in terms of a negative-exponential function has been obtained by many workers from multifrequency observations at millimeter wavelengths [*Furuhama and Ihara*, 1981; *Ihara et al.*, 1984; *Manabe et al.*, 1984]. However, the negative-exponential model has been found inadequate to describe small raindrops.

In an earlier work, *Maitra and Gibbins* [1995] obtained the rain DSD in terms of a modified gamma distribution from measurements of infrared attenuation and rain rate. In the present work a technique is described to derive the DSD in terms of a three-parameter lognormal distribution from measurements of attenuations at two wavelengths and rainfall rate. The distribution parameters are modeled in terms of the rain rate. Two rain events have been analyzed to make a comparison between the lognormal and modified gamma distributions vis-à-vis their suitability for predicting rain attenuations and for describing raindrops over the entire range of sizes.

2. Theoretical Background

As mentioned earlier, the modified gamma function has been used to describe rain DSDs, which is expressed as follows:

$$N(D) = N_0 D^n \exp(-\Lambda D) \qquad (1)$$

where $N(D)$ is in $\mathrm{m^{-3}\ mm^{-1}}$ and D, the drop diameter, is in millimeters. The distribution parameters N_0, in $\mathrm{m^{-3}\ mm^{-n-1}}$, and Λ, per millimeter, depend on the rainfall rate. In the present case, n is taken to be 2. This value of n, in the range -1 to 4 [*Ulbrich*, 1983], provided the best agreement between measured and calculated attenuations in the present case, as determined in the previous study [*Maitra and Gibbins*, 1995].

The lognormal distribution used in the present study has the following form:

$$N(D) = \frac{N_T}{\sigma D\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln(D) - \mu}{\sigma}\right)^2\right] \qquad (2)$$

where N_T is the total number of drops of all sizes, μ is the mean of $\ln(D)$, and σ^2 is the variance.

The attenuation coefficient γ ($\mathrm{dB\ km^{-1}}$) is given by

$$\gamma = 4.343 \times 10^{-3} \int_0^\infty Q(D)N(D)\,dD \qquad (3)$$

where $Q(D)$ is the extinction coefficient in square millimeters.

The rainfall rate ($\mathrm{mm\ h^{-1}}$) is expressed in terms of the DSD as

$$R = 6\pi \times 10^{-4} \int_0^\infty v(D)D^3N(D)\,dD \qquad (4)$$

where the raindrop terminal velocity $v(D)$, in $\mathrm{m\ s^{-1}}$, is obtained from the relation [*Maitra and Gibbins*, 1995]

$$v(D) = 9.65 - 10.3\exp(-0.6D) + 0.65\exp(-7D) \qquad (5)$$

Combining (4) and (5) yields

$$R = N_0\Gamma(n+4) \times 10^{-2}\left\{\frac{1.819}{\Lambda^{n+4}} - \frac{1.942}{(\Lambda+0.6)^{n+4}} + \frac{0.123}{(\Lambda+7)^{n+4}}\right\} \qquad (6)$$

At optical wavelengths ($D \gg \lambda$) the extinction coefficient has the following simplified expression [*Deirmendjian*, 1969]:

$$Q(D) = \frac{1}{2}\pi D^2 \qquad (7)$$

which results in a simplified form of attenuation coefficient at optical wavelengths as follows:

$$\gamma_{opt} = 6.822 \times 10^{-3} \times \frac{N_0}{\Lambda^{n+3}}\Gamma(n+3) \qquad (8)$$

Measurements of R and γ_{opt} provide solutions to (6) and (8) for the modified gamma-distribution parameters N_0 and Γ for a particular value of n [*Maitra and Gibbins*, 1995].

For the lognormal distribution with three parameters in the distribution function, three experimental measurements are required, which can be attenuations at two frequencies (γ_1 and γ_2) and rainfall rate (R). Hence the following nonlinear equations are formed from these measurements:

$$\gamma_1 = 4.343 \times 10^{-3} \int_0^\infty Q_1(D)N(D)\,dD \qquad (9)$$

$$\gamma_2 = 4.343 \times 10^{-3} \int_0^\infty Q_2(D)N(D)\, dD \qquad (10)$$

$$R = 6\pi \times 10^{-4} \int_0^\infty v(D)D^3N(D)\, dD \qquad (11)$$

These three equations are to be solved to obtain three parameters, namely, N_T, σ, and μ. At millimeter wavelengths, however, (7) does not hold, and Mie scattering theory has to be invoked to obtain $Q(D)$. The solutions of three nonlinear equations are obtained using the modified Powell hybrid method to find a zero of N nonlinear functions in N variables [*Powell*, 1970]. Supplied with the initially guessed values of the unknown parameters, from the model of *Ajayi and Olsen* [1985] in the present case, this method gives final solutions with a specified tolerance through a finite number of iterations. However, solutions are not available for all sets of experimental observations. There may be several reasons for this; for example, the experimental measurements may not be sufficiently accurate to yield a realistic solution, or the number of iterations required to obtain the convergence for a solution may be higher than that specified. In the present case, solutions are mostly unavailable at low rain rates when attenuations are small and the relative accuracy of attenuation measurements is consequently much lower.

The present techniques indicate that the near-instantaneous DSD can be obtained in terms of a modified gamma function from measurements of infrared attenuation and rainfall rate and in terms of a lognormal function from measurements of attenuations at two wavelengths and rain rate.

3. Experimental Results

The experimental measurements utilized in this paper were made on the 500-m Millimetre-Wave Experimental Range at Chilbolton (MWERAC), in Hampshire (57°8′N, 1°26′W, elevation 84 m), operated by the Rutherford Appleton Laboratory. The range is described in detail by *Gibbins et al.* [1987]. The present work is based on rain attenuation measurements at 37, 57, and 97 GHz and 10.6 μm over the 500-m line-of-sight link. The rainfall rate considered in the analyses is the mean of measurements made with three rapid-response rain gauges, spaced 200 m apart along the link, with a minimum resolution of ~1 mm h^{-1} and an integration time of 10 s. The results presented are based on an analysis of two rain events, data inputs being 10-s samples of measurements of attenuations and rain rates, where rain rates exceeded 2 mm h^{-1}. The meteorological conditions prevailing during the two rain events are described by *Maitra and Gibbins* [1995].

As already noted, solutions for the lognormal distribution parameters were not obtained for all the measurement points with a particular pair of frequencies. Hence solutions obtained with measurements obtained at different pairs of frequencies, namely, 37 and 57 GHz, 37 GHz and 10.6 μm, 57 and 97 GHz, and 57 GHz and 10.6 μm, have been considered. The lognormal distribution parameters are fitted to appropriate relations in terms of the rain rate considering all the solutions to make it statistically more reliable and independent of any possible frequency-biased estimates. The tolerance of convergence for a solution is not more than 0.001. In fact, a lower value of tolerance is initially considered and subsequently increased up to 0.001 if a solution cannot be obtained at a lower tolerance.

3.1. Event 1

This rain event occurred on August 21, 1983, starting at 0830 UT. The lognormal distribution parameters, together with the fitted curves, are shown in Figure 1. The parameters of the two distributions are fitted to the following relations in terms of the rain rate:

Lognormal distribution parameters

$$N_T = 84R^{1.18}$$

$$\mu = 0.195 - 0.21 \ln R$$

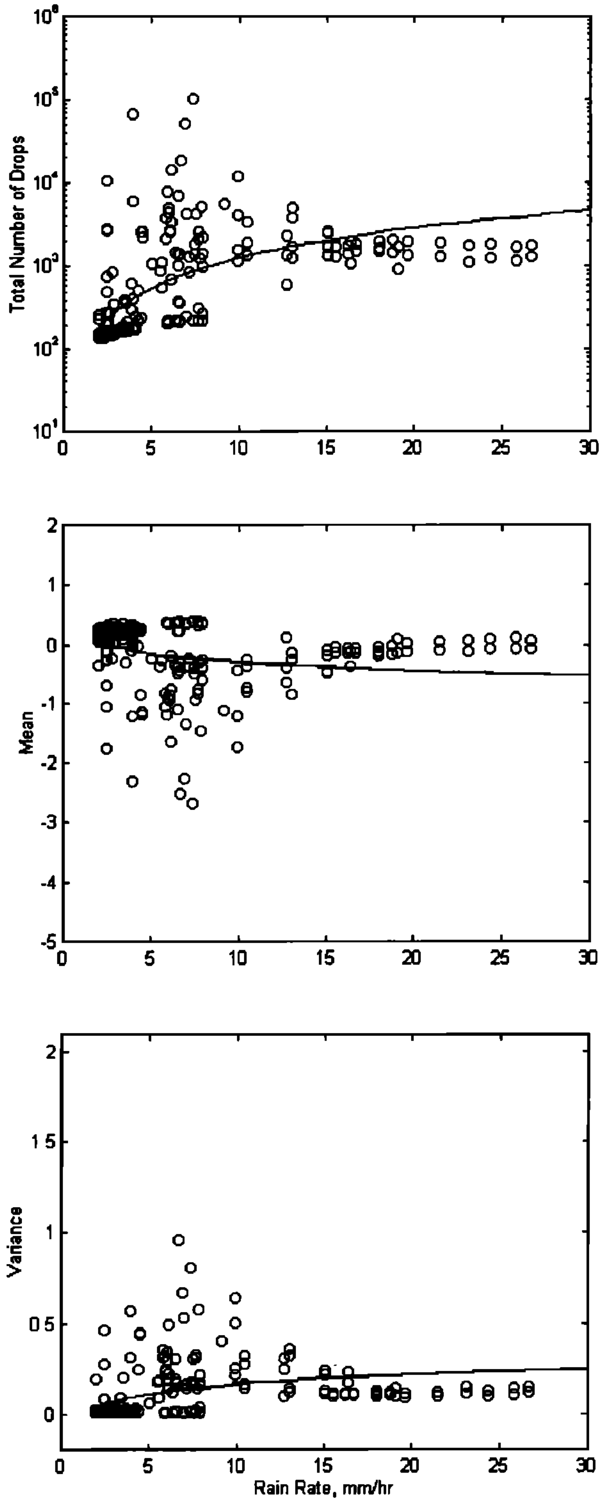
$$\sigma^2 = -0.012 + 0.078 \ln R$$

Gamma distribution parameters

$$\Lambda = 10R^{-0.31}$$

$$N_0 = 1.76 \times 10^6 R^{-1.2}$$

Attenuations are calculated using the above models for the DSD parameters and are shown in Figure 2, together with the power law curve fitted to measured attenuations. The circles indicate individual attenuation measurements. Calculated attenuations are obtained using Mie scattering theory. It can be seen that up to rain rates of 15 mm h^{-1}, the three curves obtained from power law fitting and from the lognormal and gamma distribution parameters agree reasonably well. For higher rain rates, however, these-



curves differ to some extent. It is observed that overall, the lognormal distribution gives a somewhat better fit to the experimental measurements than does the gamma distribution. For this event it appears that there are two regimes of DSD evolution, below and above a rain rate of about 15 mm h^{-1}. A single distribution model may therefore not be adequate for an event like this. To present more definitive evidence in support of the efficacy of the present technique, a comparison between measured attenuations at 97 GHz and those calculated with the derived lognormal parameters for this rain event is shown in Figure 3. The distribution parameters used for calculating 97-GHz attenuations at different times through the event are obtained from measurements with the pairs of frequencies 37 GHz and 10.6 μm and 57 GHz and 10.6 μm, again to indicate that the present technique works independently of the frequency of observations. The continuous line in Figure 3 is the line of equal attenuation. Good agreement is obtained between measured and calculated attenuations with a regression coefficient 1.05 and a correlation coefficient 0.98.

3.2. Event 2

The second rain event analyzed in the present study started at 0300 GMT on September 15, 1983. Scatterplots of the lognormal distribution parameters against rain rates, together with the fitted curves, are shown in Figure 4. The following relations are obtained for the best-fit curves for the distribution parameters:

Lognormal distribution parameters

$$N_T = 357R^{0.62}$$

$$\mu = -0.906 + 0.195 \ln R$$

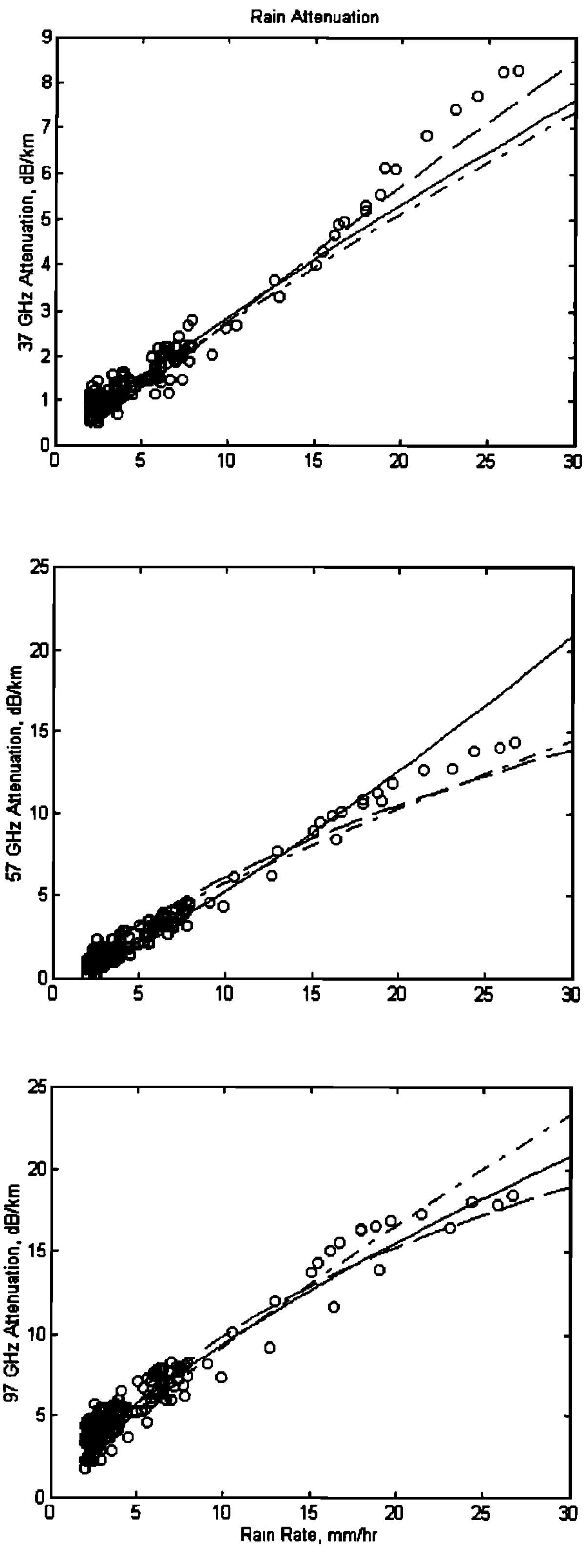
$$\sigma^2 = 0.377 - 0.0592 \ln R$$

Gamma distribution parameters

$$\Lambda = 7.6R^{-0.17}$$

$$N_0 = 2.8 \times 10^5 R^{-0.15}$$

Figure 1. (opposite) Rain rate dependence of lognormal distribution parameters during the rain event on August 21, 1983 (event 1). N_T (m^{-3} mm^{-1}) is the total number of drops, μ is the mean, and σ^2 is the variance. The circles are values of the parameters obtained at different time points; continuous lines are regression lines fitted to these values.



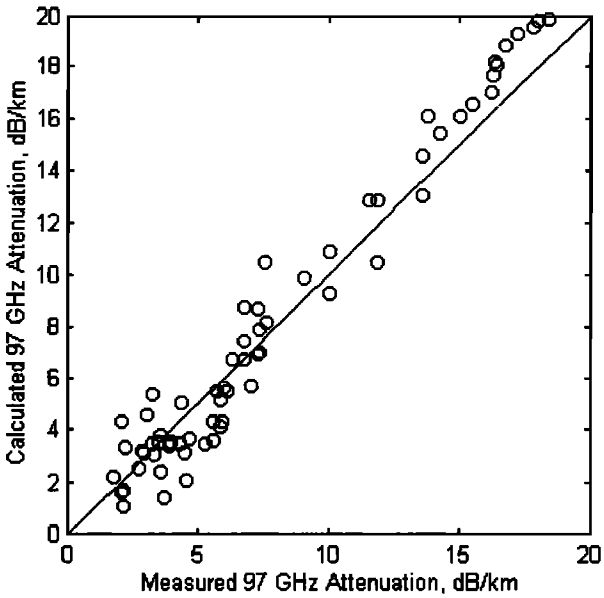


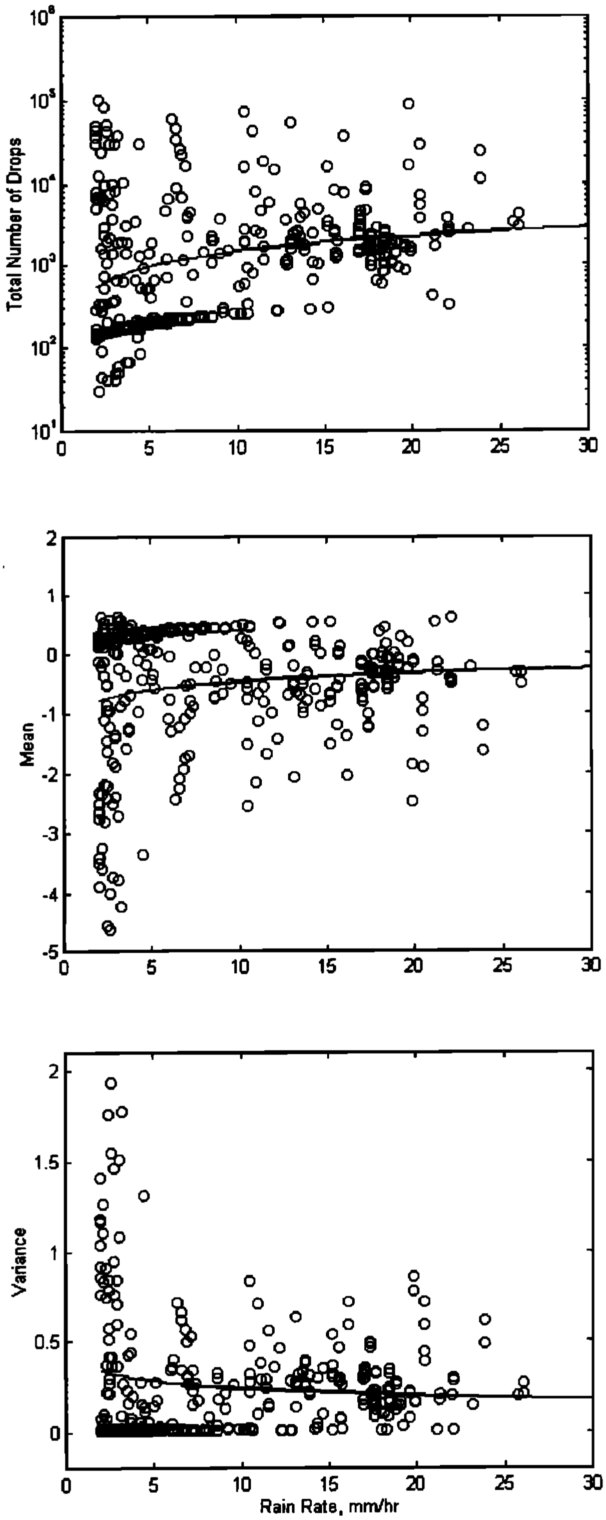
Figure 3. Comparison between measured attenuations at 97 GHz and those calculated with the lognormal distribution during event 1. The continuous line is the line of equal attenuation.

Figure 5 gives the attenuation curves calculated with the above DSD parameters, together with measured attenuations and the fitted power law curve. For this event also, the model-generated attenuations agree better with measured attenuations at rain rates up to 15 mm h^{-1} than at higher rain rates. Again, the lognormal model gives a somewhat better overall fit to the experimental measurements than does the modified gamma model. A comparison between measured and calculated attenuations at 97 GHz obtained during this event is shown in Figure 6. The agreement is again found to be very good, with a regression coefficient of 0.98 and a correlation coefficient of 0.98.

4. Discussion

Raindrop size distributions have been obtained in terms of two distribution functions, namely, the lognormal and modified gamma distributions. A numer-

Figure 2. (opposite) Attenuations against rain rates during event 1. Circles indicate measured data; solid lines indicate best-fit power law curves; short-dashed lines denote attenuations from the lognormal model; and long-dashed lines denote attenuations generated from the gamma model.

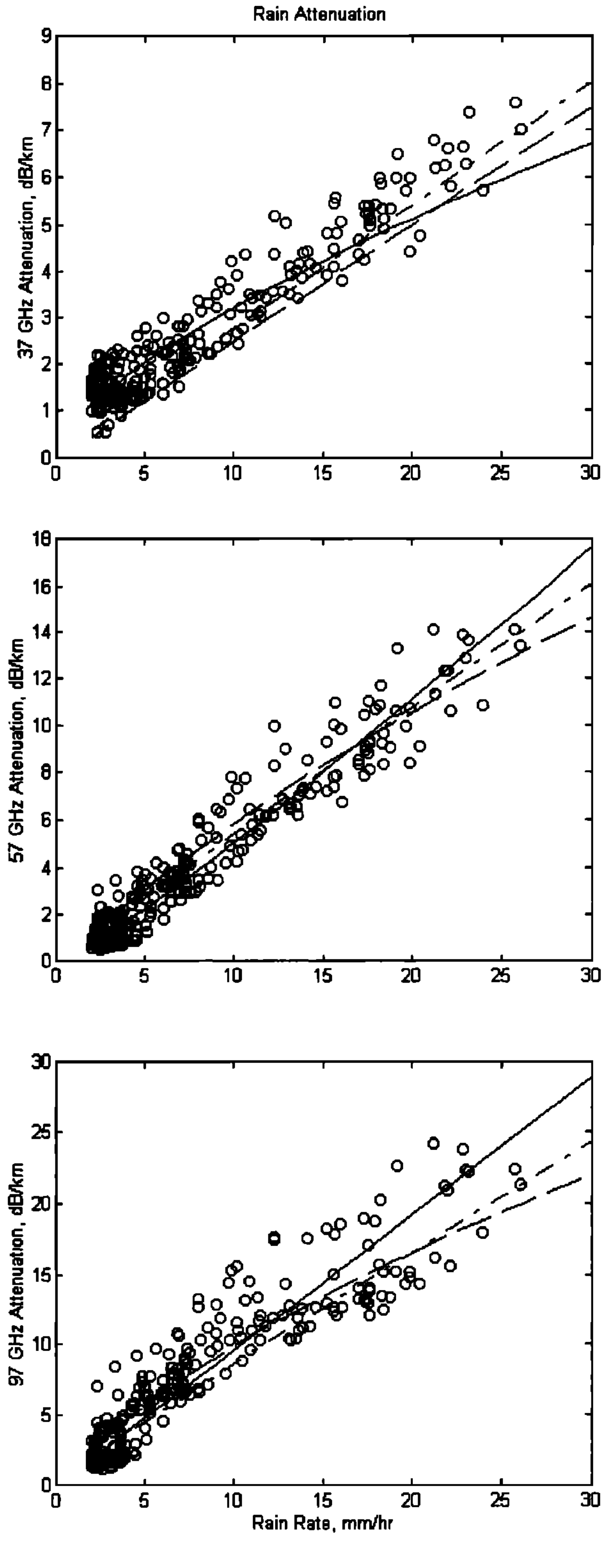


ical technique to solve a set of nonlinear equations has been utilized to obtain the three parameters in the lognormal distribution, from measurements of rain attenuations at two frequencies and rain rate. Two rain events have been analyzed to indicate the efficacy of the present technique. A large spread of DSD parameters about the fitted curves is observed at low to moderate rain rates (2–15 mm h^{-1}), while the parameters show less variability at higher rain rates. *Kozu* [1991] also reported large variability in DSD parameters at rain rates below 15 mm h^{-1} and less variability at higher rain rates, indicating that lower rain rates result from varying precipitation processes, whereas the physical processes causing intense rain rates may be relatively similar. A good overall agreement, however, is found between attenuations generated from the lognormal model and experimental measurements for the present rain events.

The solution for DSD parameters is either not obtainable or unrealistic for a large number of data sets at low rain rates, where the relative accuracy of the attenuation measurements, with a precision of 0.5 dB, becomes much less. At higher values of attenuation, however, the availability of solutions improves substantially. For attenuation measurements exceeding 5 dB used in the analysis, about 75% of the data sets yielded solutions. Hence the model-generated curve is less influenced by data points at low rain rates than at high rain rates. On the other hand, the power law curve is influenced mostly by measurements at low rain rates since the number of data points is much higher at low rain rates than at high rain rates. This bias, in fact, causes the discrepancy between the attenuation curves generated by the models of DSD parameters and the fitted power law curves to be appreciable at high rain rates.

The two distribution models are compared at rain rates of 5 and 25 mm h^{-1} for the two events in Figure 7. It is seen that the modified gamma distribution overestimates the number of small drops and underestimates the number of large drops, compared with the lognormal distribution. Large number densities given by the gamma distribution at very small drop sizes (<0.5 mm) may not be realistic. A value of the

Figure 4. (opposite) Rain rate dependence of lognormal distribution parameters during the rain event on September 15, 1983 (event 2). N_T (m^{-3} mm^{-1}) is the total number of drops, μ is the mean, and σ^2 is the variance. The circles are values of the parameters obtained at different time points; continuous lines are regression lines fitted to these values.



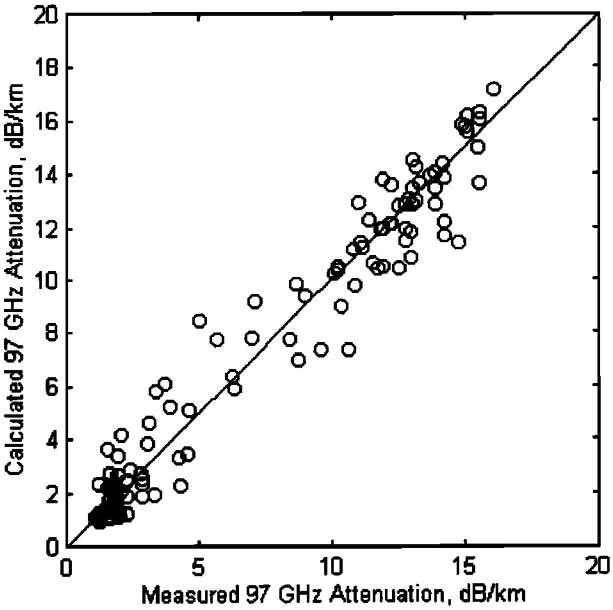


Figure 6. Comparison between measured attenuations at 97 GHz and those calculated with the lognormal distribution during event 2. The continuous line is the line of equal attenuation.

parameter n higher than 2 in the gamma distribution might have described the distribution of small raindrops more faithfully. In fact, instead of taking a fixed value of n, this parameter could be considered as a rain-dependent variable, treating the gamma model as a three-parameter distribution [*Ulbrich*, 1983; *Kozu*, 1991].

For event 1 the lognormal distribution gives a lower mode diameter for a higher rain rate, which is not indicated by the lognormal models for different locations reported by *Ajayi and Olsen* [1985]. Those models are, of course, based on statistical studies taking a large number of rain events. The present result might indicate the peculiarity of an individual event, which can be of interest to study the associated physical processes.

For the two events analyzed here, it is found that the mode diameters given by the lognormal distribu-

Figure 5. (opposite) Attenuations against rain rates during event 2. Circles indicate measured data; solid lines indicate best-fit power law curves; short-dashed lines indicate attenuations generated from the lognormal model; and long-dashed lines indicate attenuations generated from the gamma model.

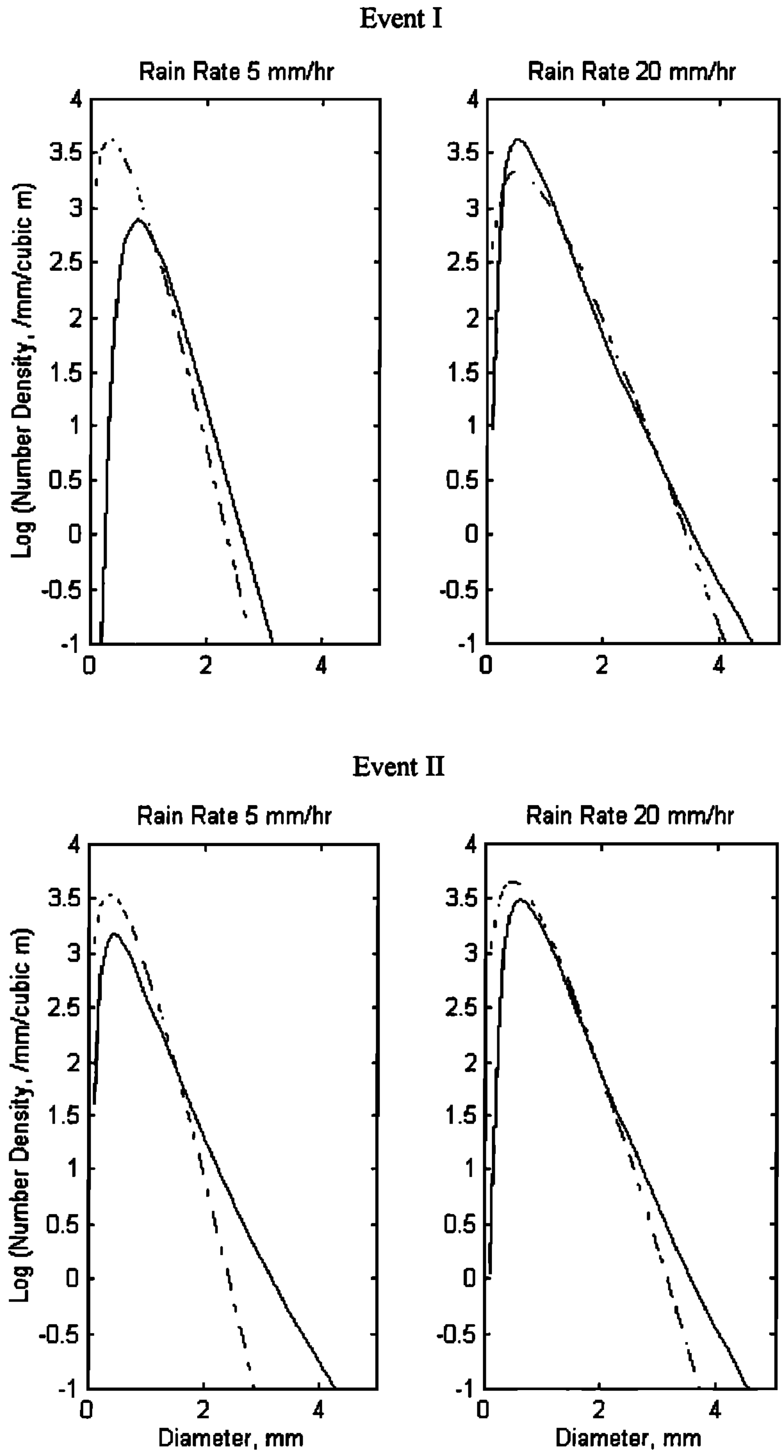


Figure 7. Comparison between the lognormal and gamma distribution model obtained at rain rates of 5 and 20 mm h^{-1} for events 1 and 2. Solid lines indicate lognormal model; and dashed lines indicate gamma model.

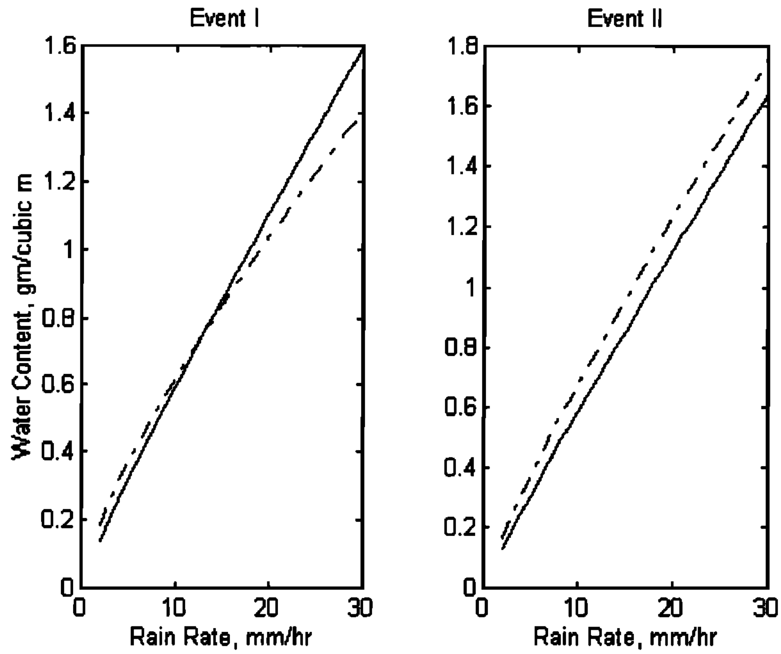


Figure 8. Variation of liquid water content with rain rate during the two rain events. Solid lines indicate lognormal model; dashed lines indicate gamma model.

tion are also smaller than those reported from distrometer observations [*Ajayi and Olsen*, 1985]. This could suggest that the present technique utilizing measurements with millimeter and optical propagation is capable of sensing very small drops, which is not possible with a distrometer.

To compare the two distributions further, the liquid water content W (g m^{-3}), at different rain rates, has been calculated for the two events using the following relation:

$$W = \frac{\pi}{6} \int_0^\infty D^3 N(D)\, dD \qquad (12)$$

The power law relations between water content and rain rate for the best-fit curves, shown in Figure 8, are given below.

Event 1

$$W_{\mathrm{LG}} = 0.075R^{0.9}$$

$$W_{\mathrm{GM}} = 0.11R^{0.75}$$

Event 2

$$W_{\mathrm{LG}} = 0.067R^{0.94}$$

$$W_{\mathrm{GM}} = 0.091R^{0.87}$$

where W_{LG} and W_{GM} denote the water content derived from the lognormal and gamma distributions, respectively. It is found that the two distributions give estimates of water content which agree well to within 10% up to a rain rate of 25 mm h^{-1}. This indicates that the distribution of drop sizes contributing most toward the water content is not significantly different for the two distributions, which is also evident from Figure 7.

5. Conclusions

A technique has been demonstrated to obtain a three-parameter lognormal distribution of raindrop sizes from measurements of rain attenuations at two frequencies and rain rate. The lognormal distribution is compared with the two-parameter gamma distribution with a fixed value of the index (n = 2) obtained from measurements of rain attenuation at infrared wavelength and rain rate. From the analysis of two rain events, it is found that the agreement between measured attenuations and calculated attenuations using the lognormal distribution is good and somewhat better than that using the gamma distribution. The gamma distribution yields higher number densities for small drops than those given by the lognormal distribution, which may not be realistic. However, the liquid water contents given by the two distributions are comparable.

Two individual rain events have been analyzed mainly to indicate the effectiveness of the technique to obtain near-instantaneous raindrop size distribution in terms of lognormal functions. A large number of rain events, however, need to be analyzed in order to obtain statistically reliable models of DSD parameters for long-term predictions and also to indicate the relative suitability of the lognormal and gamma functions for deriving rain attenuations and for faithfully describing raindrops over the entire range of sizes.

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