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Modeling Of Fibre-reinforced Magneto-thermoelastic Plate With Heat Sources

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Abstract

In the present study, a novel mathematical model of magneto–thermoelasticity has been formulated to investigate the transient phenomena for a fibre–reinforced anisotropic thick plate having a heat source in the context of Green–Naghdi theory of thermoelasticity. The upper surface of the plate is free of traction having a prescribed surface temperature while the lower surface rests in a rigid foundation and is thermally insulated. Employing Laplace and Fourier transforms as tools, the problem has been solved analytically and the inversion of the Laplace–Fourier double transform is carried out using suitable numerical techniques. According to the graphical representations corresponding to the numerical results, conclusions about the new theory have been constructed. Excellent predictive capability is demonstrated due to the influence of the magnetic field and the presence of reinforcement also.

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1. Introduction

Fibre–reinforced materials have extensive applications in aerospace and automotive fields, as well as in sailboats, and notably in modern bicycles and motorcycles, where their high strength–to–weight ratio is of great importance. Materials such as resins reinforced by strong aligned fibres exhibit highly anisotropic elastic behavior in the sense that their elastic moduli for extension in the fibre direction are frequently the order of 50 or more times greater than their elastic moduli in transverse extension or in shear. The mechanical behavior of many fibre-reinforced composite materials is adequately modeled by the theory of linear elasticity for transversely isotropic materials, with the preferred direction coinciding with the fiber direction [1]. In such composites, the fibers are usually arranged in parallel straight lines. However, other configurations are used. An example is that of circumferential reinforcement, for which the fibers are arranged in concentric circles, giving strength and stiffness in the tangential (or hoop) direction.

In the present contribution, the two-dimensional problem of generalized thermoelasticity for a fibre–reinforced anisotropic thick plate containing a heat source is formulated under the influence of magnetic field where the heat conduction is considered employing the Green Naghdi models of generalized thermoelasticity. The governing equations are solved by incorporating Laplace–Fourier double transform. The inversion of the double transform is carried

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out numerically. Numerical inversion of Laplace transform is done by using a method based on Fourier series expansion technique [2]. The numerical results for the field quantities are given in the physical domain and illustrated graphically to study the effect of magnetic field and reinforcement. The results obtained in this paper may offer a theoretical basis and meaningful suggestions in the designing of various fibre–reinforced anisotropic thermoelastic elements to attain special engineering requirements.

2. Formulation of the Problem

We consider an infinite fibre–reinforced anisotropic thermally conducting thick plate with spatially varying heat source at an uniform reference temperature T_0 in the undisturbed state where $z-$axis is the direction of anisotropy. The adjacent space is assumed to be permeated by a uniform magnetic field $\mathbf{H}(0,0,H_0)$ acting parallel to the boundary $y = 0$. This produces an induced magnetic field $\mathbf{h}(0,0,h)$ and induced electric field $\mathbf{E}(E_1,E_2,0)$ which satisfies the linearized equations of electro-magnetism and are valid for slowly moving media. The upper surface of this medium is traction free and subjected to a known temperature distribution. The lower surface of the plate is laid down on a rigid foundation and is thermally insulated. Let the faces of the plate be the planes $x = \pm h$, referred to a rectangular set of cartesian co-ordinates axes. We shall consider two dimensional deformation of the plate parallel to xy plane.

The constitutive equation for a fibre-reinforced linearly thermoelastic anisotropic medium whose preferred direction is that of a unit vector $\mathbf{a}$ is [3]

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta_{ij} + a_i a_j e_{kk}) + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki}) - \beta a_k a_m e_{km} a_i a_j - \beta_{ij}(T - T_0)\delta_{ij}, \quad (1)$$

where σ_{ij} ar the stress components, e_{ij} are the strain components, λ, μ_T are the elastic constants; α, β, $(\mu_L - \mu_T)$ are reinforcement parameters, δ_{ij} are the kronecker delta, β_{ij} are the coefficient of linear thermal expansion, T is the temperature over the reference temperature T_0, $\widetilde{a} \equiv (a_1,a_2,a_3)$, $a_1^2 + a_2^2 + a_3^2 = 1$. Now the constitutive relations in the present case are

$$\sigma_{xx} = (\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta)\frac{\partial u}{\partial x} + (\lambda + \alpha)\frac{\partial v}{\partial y} - \beta_{11}(T - T_0), \quad (2)$$

$$\sigma_{yy} = (\lambda + 2\mu_T)\frac{\partial v}{\partial y} + (\lambda + \alpha)\frac{\partial u}{\partial x} - \beta_{22}(T - T_0), \quad (3)$$

$$\sigma_{xy} = \mu_T\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right). \quad (4)$$

The equations of motion along x and y directions can be obtained as

$$(A_{11} + \rho R_H^2)\frac{\partial^2 u}{\partial x^2} + (A_{12} + \rho R_H^2)\frac{\partial^2 v}{\partial x \partial y} + A_{13}\frac{\partial^2 u}{\partial y^2} - \beta_{11}\frac{\partial T}{\partial x} = \rho\left(1 + \frac{R_H^2}{c^2}\right)\frac{\partial^2 u}{\partial t^2}, \quad (5)$$

$$(A_{22} + \rho R_H^2)\frac{\partial^2 v}{\partial y^2} + (A_{12} + \rho R_H^2)\frac{\partial^2 u}{\partial x \partial y} + A_{13}\frac{\partial^2 v}{\partial x^2} - \beta_{22}\frac{\partial T}{\partial y} = \rho\left(1 + \frac{R_H^2}{c^2}\right)\frac{\partial^2 v}{\partial t^2}. \quad (6)$$

Heat equation for the present case can be considered by assuming the fibre direction $(1,0,0)$ as [4]

$$\left(K_{11}^\star\frac{\partial^2 T}{\partial x^2} + K_{22}^\star\frac{\partial^2 T}{\partial y^2}\right) + \chi\left(K_{11}\frac{\partial^2 \dot{T}}{\partial x^2} + K_{22}\frac{\partial^2 \dot{T}}{\partial y^2}\right) + \rho\dot{Q} = \rho C_v\ddot{T} + T_0\left(\beta_{11}\frac{\partial \ddot{u}}{\partial x} + \beta_{22}\frac{\partial \ddot{v}}{\partial y}\right), \quad (7)$$

where the Fourier law is defined as

$$\dot{\vec{q}} = -\chi K_{ij}\nabla\dot{T} - K_{ij}^\star\nabla T, \quad (8)$$

where ρ is the density, c_{ij}'s are the elastic constants for transversely isotropic material, K_{ii}'s are the thermal conductivities along x and y directions, $K_{ii}^\star$'s are the additional material constants for Green–Naghdi theories, β_{ii}'s $(i = 1,2)$ are the stress-temperature coefficients, c_v is the specific heat at constant volume, Q is the heat source, σ_{ij}, e_{ij} are the stress and strain components respectively. Here, dot represents the derivative with respect to time. Also, for $\chi = 0$,

eqn. (7) reduces to GN II model, whereas for $\chi = 1$, we have GN III model.
Introducing the following non–dimensional variables

$$x' = c_1\eta x, \quad y' = c_1\eta y, \quad u' = c_1\eta u, \quad v' = c_1\eta v, \quad t' = c_1^2\eta t, \quad \eta = \frac{\rho c_v}{K_{11}}, \quad \sigma'_{ij} = \frac{\sigma_{ij}}{\rho c_1^2}, \quad h' = \frac{h}{H_0},$$

$$c_1^2 = \frac{A_{11}}{\rho}, \quad \theta' = \frac{(T - T_0)\beta_{11}}{\rho c_1^2}, \quad q'_{x'} = \frac{q_x\beta_{11}}{\rho^2 c_v c_1^3}, \quad Q' = \frac{\rho\beta_{11}Q}{K_{11}c_1^4\eta^3 A_{11}},$$

omitting primes, the above equations can be rewritten in non–dimensional form as follows

$$\sigma_{xx} = \frac{\partial u}{\partial x} + B_1\frac{\partial v}{\partial y} - \theta, \tag{9}$$

$$\sigma_{yy} = B_2\frac{\partial v}{\partial y} + B_1\frac{\partial u}{\partial x} - B_3\theta, \tag{10}$$

$$\sigma_{xy} = B_4\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \tag{11}$$

$$B_5\frac{\partial^2 u}{\partial x^2} + B_6\frac{\partial^2 v}{\partial x\partial y} + B_4\frac{\partial^2 u}{\partial y^2} - \frac{\partial\theta}{\partial x} = \xi\frac{\partial^2 u}{\partial t^2}, \tag{12}$$

$$B_7\frac{\partial^2 v}{\partial y^2} + B_6\frac{\partial^2 u}{\partial x\partial y} + B_4\frac{\partial^2 v}{\partial x^2} - B_3\frac{\partial\theta}{\partial y} = \xi\frac{\partial^2 v}{\partial t^2}, \tag{13}$$

$$\left(\varepsilon_{21}\frac{\partial^2\theta}{\partial x^2} + \varepsilon_{22}\frac{\partial^2\theta}{\partial y^2}\right) + \left(\varepsilon_{31}\frac{\partial^2\dot\theta}{\partial x^2} + \varepsilon_{32}\frac{\partial^2\dot\theta}{\partial y^2}\right) + \dot Q = \ddot\theta + \left(\varepsilon_{11}\frac{\partial\ddot u}{\partial x} + \varepsilon_{12}\frac{\partial\ddot v}{\partial y}\right), \tag{14}$$

$$\dot q_x = -\left(\varepsilon_{21}\frac{\partial\theta}{\partial x} + \varepsilon_{31}\frac{\partial\dot\theta}{\partial x}\right), \tag{15}$$

where

$$A_{12} = \lambda + \alpha + \mu_T, \quad A_{13} = \mu_T, \quad A_{22} = \lambda + 2\mu_T, \quad B_1 = \frac{\lambda + \alpha}{A_{11}}, \quad B_2 = \frac{A_{22}}{A_{11}}, \quad B_3 = \frac{\beta_{22}}{\beta_{11}}, \quad B_4 = \frac{A_{13}}{A_{11}},$$

$$(\varepsilon_{21}, \varepsilon_{22}) = \frac{1}{\rho c_v c_1^2}(K^\star_{11}, K^\star_{22}), \quad (\varepsilon_{11}, \varepsilon_{12}) = \frac{T_0\beta_{11}}{\rho c_v A_{11}}(\beta_{11}, \beta_{22}), \quad (\varepsilon_{31}, \varepsilon_{32}) = \frac{\chi\eta}{\rho c_v}(K_{11}, K_{22}).$$

The above equations are solved subjected to the initial conditions

$$\theta = u = v = 0; \ \frac{\partial\theta}{\partial t} = \frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = 0 \ \text{ at } \ t = 0. \tag{16}$$

The boundary conditions for the problem may be taken as

$$\begin{cases} \theta(h, y, t) = \theta_0 H(g - |y|), \\ \sigma_{xx}(h, y, t) = 0, \\ \sigma_{xy}(h, y, t) = 0, \\ u(-h, y, t) = 0, \\ v(-h, y, t) = 0, \\ q_x(-h, y, t) = 0. \end{cases} \tag{17}$$

Laplace–Fourier double transform of a function $f(x, y, t)$ is given by

$$\begin{aligned} \overline{f}(x, y, p) &= \int_0^\infty f(x, y, t)e^{-pt}dt, \quad \mathrm{Re}(p) > 0, \\ \widehat{\overline{f}}(x, q, p) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\overline{f}(x, y, p)e^{-iqy}dy. \end{aligned} \tag{18}$$

Applying Laplace–Fourier double transform to eqs. (9)-(15), we have

$$\widehat{\overline{\sigma}}_{xx} = \frac{d\widehat{\overline{u}}}{dx} + B_1 iq\widehat{\overline{v}} - \widehat{\overline{\theta}}, \tag{19}$$

$$\widehat{\overline{\sigma}}_{yy} = B_1\frac{d\widehat{\overline{u}}}{dx} + iq\widehat{\overline{v}} - B_3\widehat{\overline{\theta}}, \tag{20}$$

$$\widehat{\overline{\sigma}}_{xy} = B_4\left(\frac{d\widehat{\overline{v}}}{dx} + iq\widehat{\overline{u}}\right), \tag{21}$$

$$\frac{d^2\widehat{\overline{u}}}{dx^2} - (B_4q^2 + \xi p^2)\widehat{\overline{u}} + B_6 iq\frac{d\widehat{\overline{v}}}{dx} = \frac{d\widehat{\overline{\theta}}}{dx}, \tag{22}$$

$$B_4\frac{d^2\widehat{\overline{v}}}{dx^2} - (B_7q^2 + \xi p^2)\widehat{\overline{v}} + B_6 iq\frac{d\widehat{\overline{u}}}{dx} = B_3 iq\widehat{\overline{\theta}}, \tag{23}$$

$$\varepsilon_{21}\frac{d^2\widehat{\overline{\theta}}}{dx^2} - (\varepsilon_{22}q^2 + pq^2 + p^2)\widehat{\overline{\theta}} + p\widehat{\overline{Q}} = p^2\left(\varepsilon_{11}\frac{d\widehat{\overline{u}}}{dx} + \varepsilon_{12} iq\widehat{\overline{v}}\right), \tag{24}$$

$$p\widehat{\overline{q}}_x = (\varepsilon_{21} + p\varepsilon_{31})\frac{d\widehat{\overline{\theta}}}{dx}, \tag{25}$$

where

$$c^2 = \frac{1}{\epsilon_0\mu_0}, \quad R_H^2 = \frac{\mu_0 H_0^2}{\rho}, \quad \eta = \frac{R_H^2}{c_1^2}, \quad B_5 = 1+\eta, \quad B_6 = B_1 + B_4 + \eta, \quad B_7 = B_2 + \eta, \quad \xi = 1 + \frac{R_H^2}{c^2}.$$

Eliminating $\widehat{\overline{u}}$ and $\widehat{\overline{\theta}}$ from (22)-(24), we get

$$A(q,p)\frac{d^6\widehat{\overline{v}}}{dx^6} + B(q,p)\frac{d^4\widehat{\overline{v}}}{dx^4} + C(q,p)\frac{d^2\widehat{\overline{v}}}{dx^2} + D(q,p)\widehat{\overline{v}} = F(x,q,p), \tag{26}$$

Eliminating $\widehat{\overline{u}}$ and $\widehat{\overline{v}}$ from (22)-(24), we get

$$A(q,p)\frac{d^6\widehat{\overline{\theta}}}{dx^6} + B(q,p)\frac{d^4\widehat{\overline{\theta}}}{dx^4} + C(q,p)\frac{d^2\widehat{\overline{\theta}}}{dx^2} + D(q,p)\widehat{\overline{\theta}} = F_1(x,q,p), \tag{27}$$

where

$$A(q,p) = iq\varepsilon_{21}B_4B_6,$$

$$B(q,p) = p^2\varepsilon_{11}B_4(B_3 - B_6)iq + (B_6^2q^2 - c_{41}B_4 - c_{42})B_6 iq\varepsilon_{21} - B_4(B_6 iqc_{43} + B_3p^2\varepsilon_{11}iq),$$

$$C(q,p) = c_{41}c_{42}B_6 iq\varepsilon_{21} - (B_6^2q^2c_{41}B_4 - c_{42})(B_6 iqc_{43} + B_3p^2\varepsilon_{11}iq) - p^2\varepsilon_{11}B_3B_4c_{41}iq$$
$$-iq(B_3 - B_6)(p^2\varepsilon_{11}c_{42} - p^2\varepsilon_{12}B_6q^2),$$

$$D(q,p) = B_3c_{41}iq(p^2\varepsilon_{11}c_{42} - p^2\varepsilon_{12}B_6q^2) - c_{41}c_{42}iq(B_6c_{43} + Bp^2\varepsilon_{11}),$$

$$F(x,q,p) = B_6pq^2(B_3 - B_6)\frac{d^2\widehat{\overline{Q}}}{dx^2} - B_3c_{41}B_6q^2p\widehat{\overline{Q}},$$

$$F_1(x,q,p) = -B_4B_6 ipq\frac{d^4\widehat{\overline{Q}}}{dx^4} - B_6 iqp(B_6^2q^2 - c_{41}B_4 - c_{42})\frac{d^2\widehat{\overline{Q}}}{dx^2} - B_6 ipqc_{41}c_{42}\widehat{\overline{Q}},$$

$$c_{41} = B_4q^2 + \xi p^2, \quad c_{42} = B_7q^2 + \xi p^2, \quad c_{43} = \varepsilon_{22}q^2 + pq^2 + p^2.$$

2.1. Spatially varying heat source

We take heat source $Q(x,y,t)$ in the following form

$$Q(x,y,t) = \frac{H(t)\cosh(bx)}{y^2+a^2},$$

where a and b are constants.
Then we have

$$\widehat{\overline{Q}}(x,q,p) = \sqrt{\frac{\pi}{2}}e^{-a|q|}\frac{\cosh(bx)}{ap},$$

Thus, we get the displacement components are given by

$$\widehat{\overline{u}} = \sum_{j=1}^{3}\frac{[1-B_6iqf_j(q,p)]A_j(q,p)k_je^{k_jx}}{k_j^2-k_4^2} - \sum_{j=1}^{3}\frac{[1-B_6iqf_{-j}(q,p)]A_{-j}(q,p)k_je^{-k_jx}}{k_j^2-k_4^2} + \frac{b\sinh(bx)[G_1(q,p)-B_6iqG(q,p)]}{b^2-k_4^2}, \tag{28}$$

$$\widehat{\overline{v}}(x,q,p) = \sum_{j=1}^{3} f_j(q,p)[A_j(q,p)e^{k_jx}+A_{-j}(q,p)e^{-k_jx}] + G(q,p)\cosh(bx), \tag{29}$$

$$\widehat{\overline{\theta}}(x,q,p)\sum_{j=1}^{3}[A_j(q,p)e^{k_jx}+A_{-j}(q,p)e^{-k_jx}] + G_1(q,p)\cosh(bx), \tag{30}$$

where

$$f_j(q,p) = \frac{(B_3iq-1)k_j^2 - iqB_3(B_4q^2+\xi p^2)}{(B_4k_j^2-c_{42})(k_j^2-B_4q^2-\xi p^2)-B_6iqk_j^2}.$$

$$G(q,p) = \frac{1}{a}\sqrt{\frac{\pi}{2}}\frac{e^{-a|q|}[(B_3-B_6)B_6pq^2b^2 - B_3c_{41}B_6q^2p]}{(b^2-k_1^2)(b^2-k_2^2)(b^2-k_3^2)},$$

$$G_1(q,p) = \frac{1}{a}\sqrt{\frac{\pi}{2}}\frac{e^{-a|q|}[-B_4B_6iqpb^4 - B_6ipq(B_6^2q^2-c_{41}B_4-c_{42})b^2 - B_6iqpc_{41}c_{42}]}{(b^2-k_1^2)(b^2-k_2^2)(b^2-k_3^2)},$$

where $k_4 = \sqrt{B_4q^2+\xi p^2}$ and k_j's and $-k_j$'s ($j=1,2,3$) are roots of the equation

$$A(q,p)k^6 + B(q,p)k^4 + C(q,p)k^2 + D(q,p) = 0. \tag{31}$$

Therefore, the stress components $\widehat{\overline{\sigma}}_{xx}$, $\widehat{\overline{\sigma}}_{yy}$, $\widehat{\overline{\sigma}}_{xy}$ can be obtained from eqs. (19)–(21) as follows

$$\widehat{\overline{\sigma}}_{xx} = \sum_{j=1}^{3}\left[\frac{\{1-B_6iqf_j(q,p)\}k_j^2}{k_j^2-k_4^2} + B_1iqf_j(q,p) - 1\right][A_j(q,p)e^{k_jx}+A_{-j}(q,p)e^{-k_jx}] + \left[\frac{b^2\{G_1(q,p)-B_6iqG(q,p)\}}{b^2-k_4^2} + B_1(q,p)iqG(q,p) - G_1(q,p)\right]\cosh(bx), \tag{32}$$

$$\widehat{\overline{\sigma}}_{yy} = \sum_{j=1}^{3}\left[B_2iqf_j(q,p) + \frac{B_1k_j^2\{1-B_6iqf_j(q,p)\}}{k_j^2-k_4^2} - B_3\right][A_j(q,p)e^{k_jx}+A_{-j}(q,p)e^{-k_jx}] + \left[\frac{B_1b^2\{G_1(q,p)-B_6iqG(q,p)\}}{b^2-k_4^2} + B_2iqG(q,p) - B_3G_1(q,p)\right]\cosh(bx), \tag{33}$$

$$\widehat{\overline{\sigma}}_{xy} = B_4\left\{\sum_{j=1}^{3}\left[f_j(q,p)k_j + \frac{iqk_jB_4\{1-B_6iqf_j(q,p)\}}{k_j^2-k_4^2}\right][A_j(q,p)e^{k_jx} - A_{-j}(q,p)e^{-k_jx}]\right.$$

$$\left[B_4 bG(q,p) + \frac{iqbB_4\{G_1(q,p) - B_6 iqG(q,p)\}}{b^2 - k_4^2}\right]\sinh(bx)\Bigg\}, \tag{34}$$

Now using the boundary conditions (17) we obtain

$$\sum_{j=1}^{3}[A_j(q,p)e^{k_jh} + A_{-j}(q,p)e^{-k_jh}] = \sqrt{\frac{2}{\pi}}\frac{\theta_0\sin(qg)}{pq} - G_1(q,p)\cosh(bh), \tag{35}$$

$$\sum_{j=1}^{3}[A_j(q,p)e^{k_jh} + A_{-j}(q,p)e^{-k_jh}]\Big[\frac{\{1 - B_6 iqf_j(q,p)\}k_j^2}{k_j^2 - k_4^2} + B_1 iqf_j(q,p) - 1\Big] =$$

$$-\Big[\frac{b^2\{G_1(q,p) - B_6 iqG(q,p)\}}{b^2 - k_4^2} + B_1 iqG(q,p) - G_1(q,p)\Big]\cosh(bh), \tag{36}$$

$$\sum_{j=1}^{3}[A_j(q,p)e^{k_jh} - A_{-j}(q,p)e^{-k_jh}]\Big[f_j(q,p)k_j + \frac{iqB_4k_j\{1 - B_6 iqf_j(q,p)\}}{k_j^2 - k_4^2}\Big] =$$

$$-\Big[B_4 bG(q,p) + \frac{iqB - 4b\{G_1(q,p) - B_6 iqG(q,p)\}}{b^2 - k_4^2}\Big]\sinh(bh), \tag{37}$$

$$\sum_{j=1}^{3} f_j(q,p)[A_j(q,p)e^{-k_jh} + A_{-j}(q,p)e^{k_jh}] = -G(q,p)\cosh(bh), \tag{38}$$

$$\sum_{j=1}^{3}\frac{\{1 - B_6 iqf_j(q,p)\}k_j}{k_j^2 - k_4^2}[A_j(q,p)e^{-k_jh} - A_{-j}(q,p)e^{k_jh}] = \frac{b\sinh(bh)\{G_1(q,p) - B_6 iqG(q,p)\}}{b^2 - k_4^2}, \tag{39}$$

$$\sum_{j=1}^{3}[A_j(q,p)e^{-k_jh} - A_{-j}(q,p)e^{k_jh}]k_j = bG_1(q,p)\sinh(bh). \tag{40}$$

Solution of the above system of linear equations (35)–(40) gives the unknown parameters $A_j(q,p)$ and $A_{-j}(q,p)$, $j = 1(1)3$.

3. Numerical results and discussions

In order to study the influence of the magnetic field and reinforcement on the wave propagation of the medium, we now present the results in the form of their graphical representations. The material constants are given by [5]

$$\rho = 2660\ \mathrm{Kg\cdot m^{-3}},\quad \lambda = 5.65\times10^{10}\ \mathrm{N\cdot m^{-2}},\quad \mu_T = 2.46\times10^{10}\ \mathrm{N\cdot m^{-2}},\quad T_0 = 293\ \mathrm{K},$$

$$\mu_L = 5.66\times10^{10}\ \mathrm{N\cdot m^{-2}},\quad \alpha = -1.28\times10^{10}\ \mathrm{N\cdot m^{-2}},\quad \beta = 0.015\times10^{-4}\ \mathrm{N\cdot m^{-2}},$$

$$\beta_{11} = 0.017\times10^{-4}\ \mathrm{K^{-1}},\quad H_0 = 10,\quad \varepsilon_0 = 0.3,\quad \beta_{22} = 0.015\times10^{-4}\ \mathrm{K^{-1}},\quad c_v = 0.787\times10^3\ \mathrm{J\cdot Kg^{-1}\cdot K^{-1}},$$

$$K_{11} = 0.0921\times10^3\ \mathrm{J\cdot m^{-1}\cdot s^{-1}\cdot K^{-1}},\quad K_{22} = 0.0963\times10^3\ \mathrm{J\cdot m^{-1}\cdot s^{-1}\cdot K^{-1}},$$

$$\mu_0 = 0.1,\quad b = 1,\quad a = 1,\quad g = 1,\quad h = 0.5,\quad \theta_0 = 1,$$

$$K_{11}^\star = 0.13\times10^3\ \mathrm{J\cdot m^{-1}\cdot s^{-1}\cdot K^{-1}},\quad K_{22}^\star = 0.13\times10^3\ \mathrm{J\cdot m^{-1}\cdot s^{-1}\cdot K^{-1}}.$$

To get the roots of the polynomial equation (31) in complex domain, we have used Laguerre's method. Here the numerical inversion of Laplace transform is done using a method based on Fourier series expansion technique [2].

In order to study the effect of reinforcement and magnetic field in the thermophysical quantities, figs. 1-3 have been plotted against the thickness of the plate for Green–Naghdi model III for $y = 0$ and $t = 0.25$. In these figures, the continuous lines represent the graphs corresponding to the absence of magnetic field ($H_0 = 0$), whereas the dotted lines represent the graphs correspond to the presence of magnetic field ($H_0 = 10$).

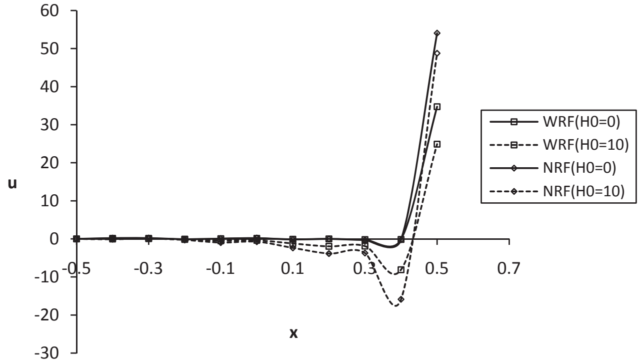

Fig. 1. Variation of u versus x for $H_0 = 0, 10$.

Fig 1 depicts the variation of the displacement component u against the thickness x for $t = 0.25$ for GN III model. From the figure, it is observed that, on the rigid base at $x = -0.5$, the displacement is zero, which satisfies the boundary condition of the problem. It is also observed that u attains the maximum value near the upper boundary of the plate in absence of reinforcement (NRF). Also, in absence of the magnetic field ($H_0 = 0$), the displacement almost disappears in $-0.5 \leqslant x < 0.3$ whereas due to the presence of magnetic field ($H_0 = 10$), u is prominently seen inside the plate.

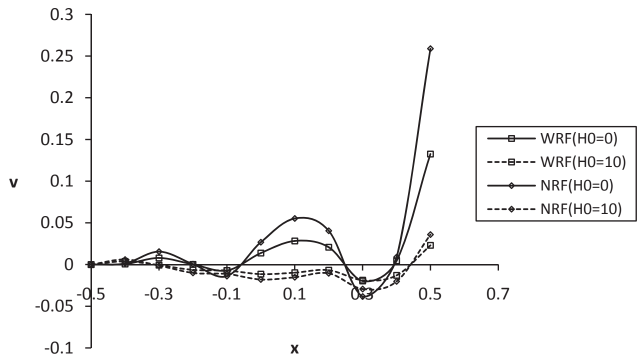

Fig. 2. Variation of v versus x for $H_0 = 0, 10$.

Fig 2 is plotted to study the variation of the displacement v against the thickness of the plate due to the presence and absence of magnetic field ($H_0 = 10, 0$) with reinforcement (WRF) and without reinforcement (NRF) also. As seem from fig 2, v disappears on the lower boundary of the plate validating the correctness of the numerical codes prepared in the problem. Here, the oscillatory nature in the variation of v is found and the magnitude of the peak of oscillation is larger due to the absence of magnetic field ($H_0 = 0$) compared to the presence of magnetic field ($H_0 = 10$). Also, the presence of reinforcement has a tendency to diminish the magnitude of the displacement v inside the plate.

In order to study the effect of reinforcement and magnetic field on the stress component σ_{xx} against the thickness, fig 3 has been plotted with the same set of parameters as mentioned above. As seen from the figure, σ_{xx} vanishes on the outer surface of the plate satisfying the mechanical boundary condition of the problem. Due to the presence of magnetic field, the stress component is compressive in nature near the lower boundary. Also, the presence of reinforcement has a tendency to decrease the magnitude of the stress component σ_{xx}.

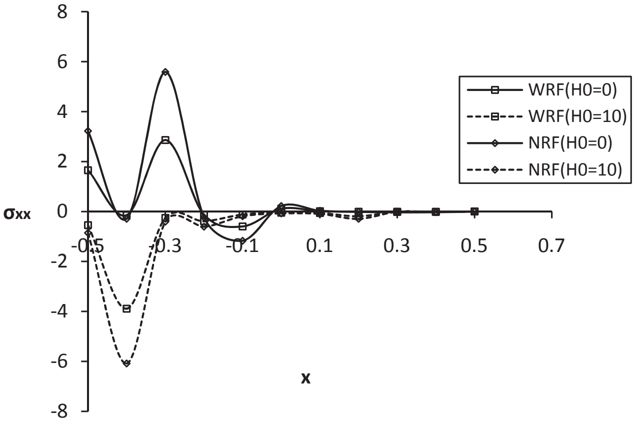

Fig. 3. Variation of σ_{xx} versus x for $H_0 = 0, 10$.

4. Conclusions

In the present analysis, a novel mathematical treatment has been presented to analyze the magneto-thermoelastic wave propagation in a fibre-reinforced thick plate subjected to a spatially varying heat source. Though the graphical representations are self-explanatory in exhibiting the different peculiarities which occur in the propagation of waves, yet the following remarks may be added.

1. Significant effect in the variation of the thermophysical quantities are found due to the presence of magnetic field. The magnitude of the displacement components show a decreasing effect due to the presence of magnetic field. Therefore, the effect of magnetic field should be taken into consideration.
2. Due to the presence of reinforcement, magnitude of the thermophysical quantities decay which indicates that reinforcement has a tendency in maintaining the smoothness of the profiles of the thermophysical quantities. So it is more advantageous to consider the effect of reinforcement in such problems of engineering.
3. Here, all the results in absence of magnetic field ($H_0 = 0$) and reinforcement (NRF) agree with the existing literature [6].

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