

# Misalignment Considerations in Laser Diode to Single-mode Fiber Excitation via Hemispherical Lens on the Fiber Tip

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## Summary

We report the theoretical investigation of the excitation efficiency of a laser diode to a single-mode fiber via a hemispherical lens on the fiber tip in presence of possible transverse and angular misalignments. The analysis takes care of the allowable aperture provided by the hemispherical lens. By employing ABCD matrix for refraction of paraxial rays by a hemispherical lens, we present analytical expressions of such coupling efficiencies and calculate the corresponding losses involving very little computations or even a pocket calculator. The results seem to be useful in the design of optimum launch optics to get maximum coupling efficiency.

## 1 Introduction

Microlenses are being fabricated on the fiber tip currently in order to maximize the source to single-mode fiber coupling efficiency [1–3]. These microlenses which are either hemispherical or conical in shape have the common advantage of being self-centred. Although the hyperbolic lens on the fiber tip has been found to be the most effective coupler [1–3], the easily fabricable hemispherical lens on the tip of the fiber is still commonly used worldwide [1]. Estimations of coupling efficiencies in case of laser diode to single-mode fiber excitation via hemispherical lens on the fiber tip have already been reported [1, 3] and these methods require mostly cumbersome numerical integrations involving long computational time. However, the analysis can be much simplified if the well known ABCD matrix for refraction by hemispherical lens is utilized. In fact very recently a realistic method, which takes care of the allowable aperture of the hemispherical lens and uses ABCD matrix, has been shown to estimate the practical situations excellently [4]. Now our object is to investigate the coupling losses for the said type of coupler in presence of possible transverse and angular misalignments. Such study, which is not available in the literature to the best of our knowledge, is important to estimate the sensitivity of this coupling device with respect to these two kinds of misalignments.

In this communication, we take care of the truncating aperture of the hemispherical lens through which coupling is allowed and employ the ABCD matrix for refrac-

tion to evaluate theoretically the coupling losses in separate cases of possible transverse offset and angular mismatch for laser diode to single-mode fiber excitation via hemispherical lens on the fiber tip.

## 2 Theory

The coupling scheme to be studied is shown in Fig. 1. Our analysis contains some usual approximations [1–3], namely Gaussian field distributions for both the source and the fiber, perfect matching of the polarization of the field of the fiber and that on the lens surface. The field  $\psi_u$  of the output of the laser diode at a distance  $u$  from the lens surface can be expressed as [5]

$$\psi_u = \exp\left[-\left(x^2/w_{1x}^2 + y^2/w_{1y}^2\right)\right] \exp\left[-jk_1(x^2 + y^2)/2R_1\right] \quad (1)$$

where  $w_{1x}$  and  $w_{1y}$  are spot sizes along two perpendicular directions  $X$  and  $Y$ ,  $R_1$  is the radius of curvature of the wavefront from the laser source and  $k_1$  is the wave number for the incident medium.

The fundamental mode  $\psi_f$  in the single-mode circular core fiber is given by [5]

$$\psi_f = \exp\left[-(x^2 + y^2)/w_f^2\right] \quad (2)$$

where the spot size  $w_f$  is approximated as [6]

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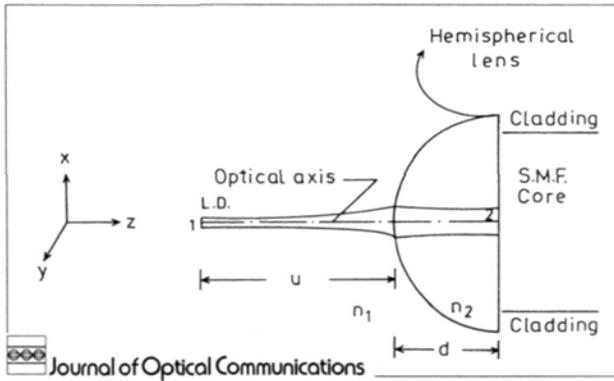


Fig. 1: Schematic diagram of a laser beam emitted from the input plane 1 and refracted through a hemispherical lens onto the plane 2, the end face of a single-mode fiber

$$w_f = a \left[ 0.65 + 1.619/V^{1.5} + 2.879/V^6 \right]. \quad (3)$$

Here 'a' is the core radius and 'V' is the normalized frequency given by  $k_0 a (n_{co}^2 - n_{cl}^2)^{1/2}$  with  $k_0$  being free space wave number and  $n_{co}$  and  $n_{cl}$  being refractive indices of the core and cladding, respectively.

The hemispherical lens transformed laser field  $\psi_v$  on the fiber plane 2 can be approximated as [5]

$$\psi_v = \exp \left[ - \left( x^2/w_{2x}^2 + y^2/w_{2y}^2 \right) \right] \exp \left[ -jk_2 \left( x^2/R_{2x} + y^2/R_{2y} \right) / 2 \right], \quad (4)$$

where  $k_2$  is the wave number in the lens and  $w_{2x}$  and  $w_{2y}$  are lens transformed spot sizes with  $R_{2x}$  and  $R_{2y}$  being corresponding radii of curvature in the X and Y directions, respectively. Again,  $w_{2x,2y}$  and  $R_{2x,2y}$  can be found in terms of  $w_{1x,1y}$  and  $R_1$  by utilizing the following relations [4, 7]

$$q_2 = (Aq_1 + B)/(Cq_1 + D) \quad (5)$$

where

$$1/q_{1,2} = 1/R_{1,2} - j\lambda_0 / (\pi w_{1,2}^2 n_{1,2}) \quad (6)$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ (1-n)/nR_L & 1/n \end{pmatrix} \begin{pmatrix} 1 & u \\ 0 & 1 \end{pmatrix} \quad (7)$$

$$w_{2x,2y}^2 = \frac{A_1^2 w_{1x,1y}^2 + (\lambda_1^2 B^2) / (\pi^2 w_{1x,1y}^2)}{n(A_1 D - BC_1)} \quad (8)$$

$$\frac{1}{R_{2x,2y}} = \frac{A_1 C_1 w_{1x,1y}^2 + (\lambda_1^2 B D) / (\pi^2 w_{1x,1y}^2)}{A_1^2 w_{1x,1y}^2 + (\lambda_1^2 B^2) / (\pi^2 w_{1x,1y}^2)}, \quad (9)$$

where the maximum depth (d) of the lens = its radius of curvature ( $R_L$ );  $n = n_2/n_1$ ;  $\lambda_1 = \lambda_0/n_1$ ;  $A_1 = A + B/R_1$ ;  $C_1 = C + D/R_1$ .

Further, the source to fiber coupling efficiency via the hemispherical lens on the fiber tip is given by the well known overlap integral [7]

$$\eta_0 = \frac{\left| \iint \psi_v \psi_f^* dx dy \right|^2}{\iint |\psi_v|^2 dx dy \iint |\psi_f|^2 dx dy}. \quad (10)$$

For a hemispherical lens the limiting radius  $\rho_c$  beyond which transmission is not allowed corresponds to the grazing angle of incidence and is obtained with explanations in [1] as

$$\rho_c = n_1 R_L / n_2. \quad (11)$$

Accordingly, the lens transmittivity factor T is given by [1]

$$T = \frac{\int_0^{\rho_c} |\psi_f t|^2 r dr}{\int_0^{\infty} |\psi_f|^2 r dr} \quad (12)$$

where the lens transmission coefficient t under paraxial approximation is given by

$$t = \frac{2(n_1 n_2)^{1/2}}{n_1 + n_2}. \quad (13)$$

Employing (2) and (11) in (12) we obtain

$$T = t^2 \left[ 1 - \exp(-2\rho_c^2 / w_f^2) \right]. \quad (14)$$

Clearly, the corrected efficiency  $\eta$  will be given by

$$\eta = \eta_0 T. \quad (15)$$

In order to obtain the coupling efficiency  $\eta_t$  in presence of transverse offset in the X-Y plane we assume that the centre of the fiber is shifted to a point having coordinates ( $d_1$ ,  $d_2$ ) as shown in Fig.2(a).

Accordingly, the fundamental mode in the fiber can be given by

$$\psi_f = \exp \left[ - \left( \frac{(x-d_1)^2 + (y-d_2)^2}{w_f^2} \right) \right]. \quad (16)$$

Employing (4), (10), (15) and (16) we obtain [8]

$$\eta_t = \eta \exp \left[ \frac{2d_1^2}{w_f^2} \left\{ \frac{w_{2x}^2 (w_{2x}^2 + w_f^2)}{(w_f^2 + w_{2x}^2)^2 + (k_2^2 w_{2x}^4 w_f^4) / 4R_{2x}^2} - 1 \right\} \right] \exp \left[ \frac{2d_2^2}{w_f^2} \left\{ \frac{w_{2y}^2 (w_{2y}^2 + w_f^2)}{(w_f^2 + w_{2y}^2)^2 + (k_2^2 w_{2y}^4 w_f^4) / 4R_{2y}^2} - 1 \right\} \right], \quad (17)$$

where

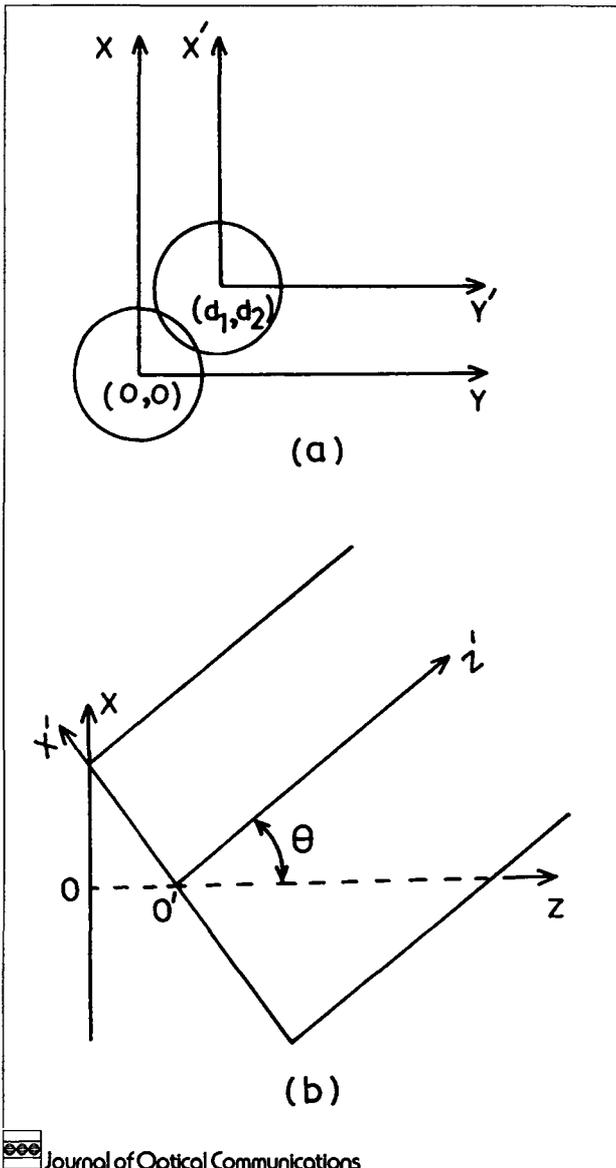


Fig. 2: (a) Transverse offset between the center of the fiber and the imaged laser spot; (b) angular mismatch between the end face of the fiber and the hemispherical lens transformed input face

$$\eta = \frac{4w_{2x}w_{2y}w_f^2T}{\left[ (w_f^2 + w_{2x}^2)^2 + \frac{k_2^2w_{2x}^4w_f^4}{4R_{2x}^2} \right]^{1/2} \left[ (w_f^2 + w_{2y}^2)^2 + \frac{k_2^2w_{2y}^4w_f^4}{4R_{2y}^2} \right]^{1/2}} \quad (18)$$

In Fig. 2(b) we show angular misalignment of very small angle  $\theta$  between the hemispherical lens transformed input face and the end face of the fiber. Following [8] we can express the lens transformed laser field on the fiber as

$$\Psi_v = \exp \left[ - \left( \frac{x'^2}{w_{2x}^2} + \frac{y'^2}{w_{2y}^2} \right) \right] \exp \left[ - \frac{jk_2}{2} \left( \frac{x'^2}{R_{2x}} + \frac{y'^2}{R_{2y}} \right) \right] \exp[jk_2x'\theta] \quad (19)$$

The fundamental mode of the fiber is then written as

$$\Psi_f = \exp \left[ - (x'^2 + y'^2) / w_f^2 \right] \quad (20)$$

Using (10), (15), (19) and (20) we obtain the coupling efficiency  $\eta_a$  in presence of small angular mismatch  $\theta$  as below [8]

$$\eta_a = \eta \exp \left[ - \frac{k_2^2\theta^2}{2} \left( \frac{(w_f^2 + w_{2x}^2)w_{2x}^2w_f^2}{(w_f^2 + w_{2x}^2)^2 + (k_2^2w_{2x}^4w_f^4)/4R_{2x}^2} \right) \right] \quad (21)$$

The analytical expressions for  $\eta_i$  and  $\eta_a$  thus obtained can be employed to investigate the coupling losses for this type of coupler in presence of said types of misalignments.

### 3 Results and discussions

In order to estimate the coupling losses in presence of possible misalignments in case of hemispherical lens on the fiber tip, we employ parameters similar to those taken by [3], namely a laser diode of wavelength  $1.5 \mu\text{m}$  with  $w_{1x} = 0.843 \mu\text{m}$ ,  $w_{1y} = 0.857 \mu\text{m}$  and a single-mode fiber of core diameter  $7.3 \mu\text{m}$  with  $w_f = 4.794 \mu\text{m}$ . Following [3], we take the refractive index (n) and the transmission coefficient (t) of the lens as 1.55 and 1, respectively. In this connection, it is relevant to mention that the value of t as found from (13) is 0.9765 which is close to unity. Also, for each radius of the hemispherical lens, the value u is optimized for maximum coupling efficiency.

We present the variation of coupling loss against the transverse misalignment in the X direction in case of hemispherical lens on the fiber tip for three different radii of curvature ( $R_1$ )  $5 \mu\text{m}$ ,  $6 \mu\text{m}$  and  $7 \mu\text{m}$  in Figs 3(a) and 4(a) corresponding to the planar and spherical wave models respectively for the incident wavefront. It may be recalled [4] that the chosen lens radii are of practical importance since they correspond to optimum coupling. In Figs. 3(b) and 4(b) which correspond to the same two wave models respectively for the incident wavefront, we represent the variation of coupling loss against the angular mismatch for the same coupling system for the said radii of curvature. It is seen from both the Figs. 3(a) and 3(b) that in absence of these two misalignments the found coupling efficiencies in the planar wave model for the incident wavefront are 58.85%, 66.46% and 69.36% for lens radii  $5 \mu\text{m}$ ,  $6 \mu\text{m}$  and  $7 \mu\text{m}$ , respectively with the corresponding respective losses being 2.30 dB, 1.77 dB and 1.59 dB. Further, it is observed from both the Figs. 4(a) and 4(b) that in absence of these two misalignments the coupling efficiencies in the spherical wave model for the incident wavefront are 58.05%, 66.61% and 68.98% for the same lens radii in order and the corresponding losses are 2.36 dB, 1.77 dB and 1.61 dB, respectively. These results in absence of the misalignments, which we have shown in our earlier work [4] to estimate the practical situation [3] excellently are used as reference points in our present study of coupling losses due to misalignments. From Figs. 3(a) and 4(a) it is seen that for transverse misalignment of  $10 \mu\text{m}$  the cou-

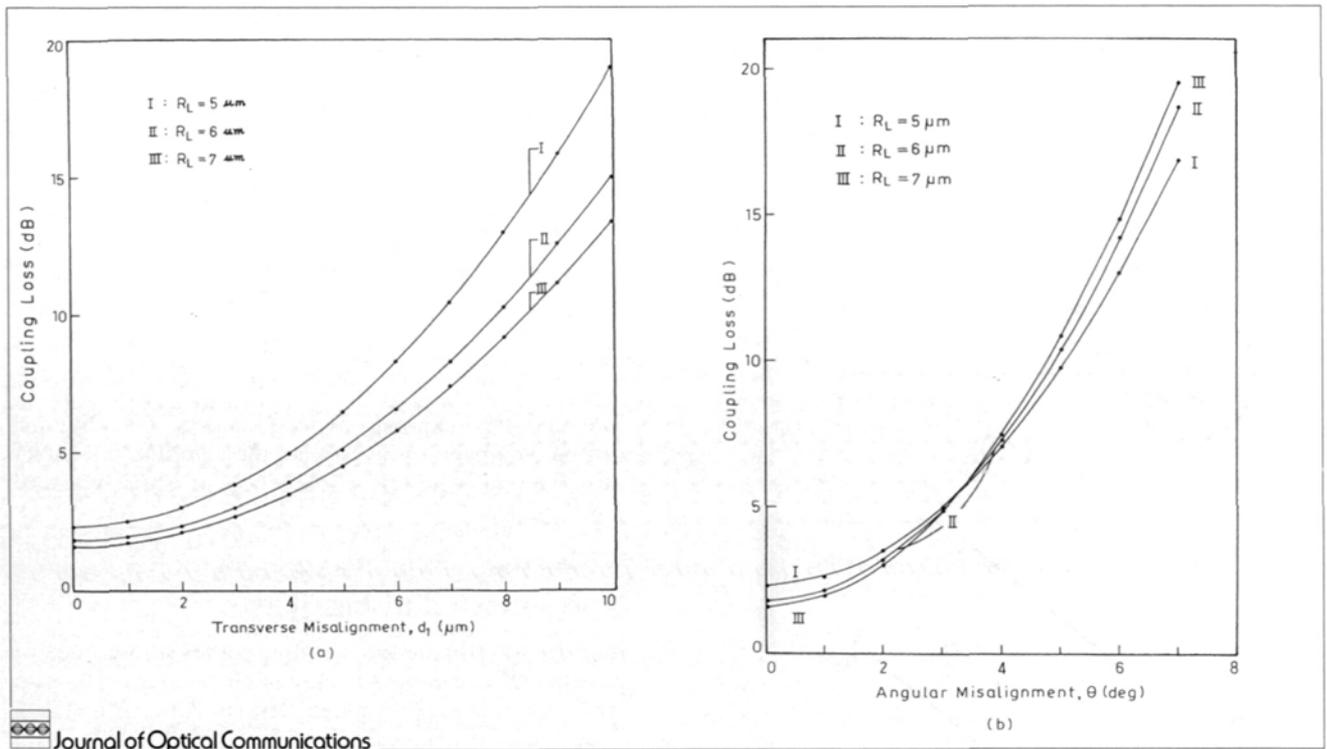


Fig. 3: Variation of coupling loss in the planar wave model for the incident wavefront against (a) transverse offset along X direction and (b) angular mismatch for three lens radii: (I)  $5 \mu\text{m}$  (II)  $6 \mu\text{m}$  (III)  $7 \mu\text{m}$

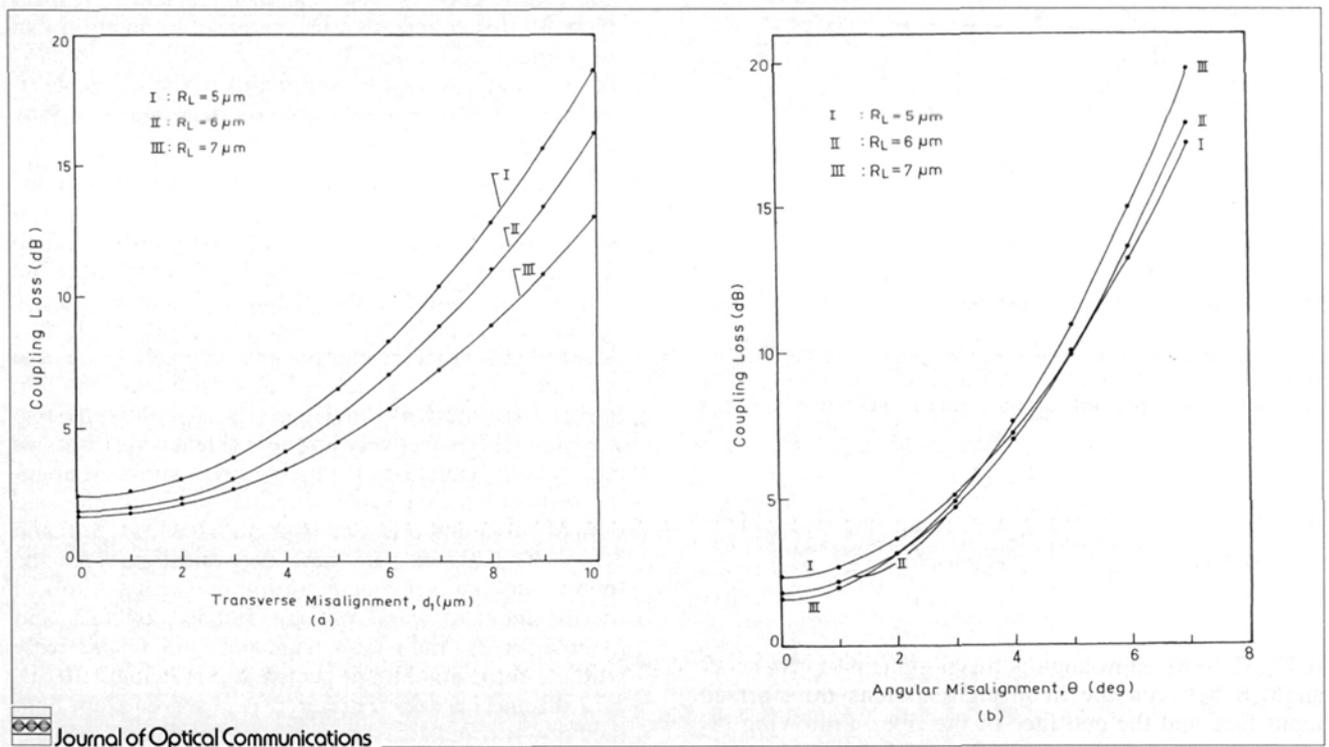


Fig. 4: Variation of coupling loss in the spherical wave model for the incident wave front against (a) transverse offset along X direction and (b) angular mismatch for three lens radii (I)  $5 \mu\text{m}$  (II)  $6 \mu\text{m}$  (III)  $7 \mu\text{m}$

pling losses are 19.02 dB, 15.09 dB and 13.42 dB in the planar wavefront model and 18.70 dB, 16.20 dB and 13.01 dB in the spherical wavefront model corresponding to lens radii  $5 \mu\text{m}$ ,  $6 \mu\text{m}$  and  $7 \mu\text{m}$ , respectively. This clearly indicates that tolerance with respect

to transverse misalignment increases with the increase in lens radius. From Figs. 3(b) and 4(b) it is found that for angular misalignment of  $7^\circ$  the coupling losses are 16.86 dB, 18.70 dB and 19.55 dB in the planar wavefront model and 17.19 dB, 17.91 dB and 19.85 dB in

the spherical wavefront model for the same lens radii in order. It is, therefore, clearly evident that tolerance with respect to angular mismatch increases with the decrease in lens radii for values of angular mismatch larger than  $5^\circ$  and  $3^\circ$  in the spherical and planar wavefront models, respectively. It is also found that though the hemispherical lens of radius  $7\ \mu\text{m}$  on the fiber tip produces maximum excitation efficiency in absence of the said two types of misalignments, the behaviour of the lens of radius  $6\ \mu\text{m}$ , which corresponds to a bit less efficiency, is moderate with respect to the transverse offset and a wide range of angular mismatch region. Moreover, in the region of angular misalignment from about  $2^\circ$  to  $5^\circ$  in the spherical wavemodel and at about  $3^\circ$  in the planar wave model, the hemispherical lens of  $R_L = 6\ \mu\text{m}$  shows most tolerance. Finally, from the point of view of constructional difficulties of lenses of radii more than  $6\ \mu\text{m}$  on the fiber tip [3] and the above observations, it becomes apparent that the hemispherical lens of radius  $6\ \mu\text{m}$  is the most suitable one for such type of coupler. Accordingly, the results should be of much help in the study of the sensitivity of coupling via such lenses with reference to the above kinds of misalignments and the design of such couplers as well.

#### 4 Conclusions

The coupling loss of laser diode to single-mode fiber excitation via hemispherical lens on the fiber tip in presence of possible transverse and angular misalignments is investigated for lens radii which correspond to optimum coupling. The hemispherical lens of radius  $6\ \mu\text{m}$

is found to be the most suitable one in this regard. On the basis of the application of ABCD matrix for hemispherical lens along with the considerations for its allowable aperture for coupling, analytical formulations for such losses are derived. The analysis is very simple and the concerned calculations are executable even by a pocket calculator. Such observations are useful for designing suitable hemispherical lens on the fiber tip.

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