AIP Journal of Applied Physics

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Citation: Journal of Applied Physics **38**, 2422 (1967); doi: 10.1063/1.1709918
View online: http://dx.doi.org/10.1063/1.1709918
View Table of Contents: http://scitation.aip.org/content/aip/journal/jap/38/6?ver=pdfcov
Published by the AIP Publishing

specimens and dislocations acquired a jagged appearance because of pinning by defect clusters generated during irradiation.


## ACKNOWLEDGMENTS

The authors are indebted to Professor J. F. Breedis of the Department of Metallurgy, Massachusetts Institute of Technology, for many valuable discussions and suggestions. They acknowledge the use of the facilities of the Cyclotron Laboratory and thank Professor A. M. Bernstein as well as E. F. White, F. J. Fay, and W. M. Bucelewicz for their cooperation. R. L. Goss and L. I. Sudenfield of the Department of Metallurgy gave valuable assistance with the experimental work. Financial support by the National Aeronautics and Space Administration under Grant NsG-496 to the Center for Space Research, Massachusetts Institute of Technology, is acknowledged.


JOURNAL OF APPLIED PHYSICS VOLUME 38, NUMBER 6 MAY 1967

# Microwave Faraday Rotation in Nickel-Powder Artificial Dielectric—A Suggested Explanation

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(Received 17 October 1966)

It is shown that the anisotropic permeability of the magnetic particles constituting an artificial dielectric provides an explanation for the rotation of the plane of polarization and the variation of the attenuation of a plane-polarized electromagnetic signal propagating through the dielectric in the presence of a longitudinal magnetic field. The attenuation curve is found to be readily explainable from the nature of variation of the power transmitted into the particles. A more detailed agreement between experimentally observed rotation, attenuation, and computed values is shown to follow if Lewin's formulas for the permeability and permittivity of an artificial dielectric is assumed to be valid also for circularly polarized signals. The effect of the size and permeability of the metal particles on the figure of merit of rotation is also discussed.

## 1. INTRODUCTION

The properties of an artificial dielectric composed of magnetic metal powders in the presence of a steady magnetic field have been reported in a previous communication by Engineer, Nag, and Datta.[1] It has been found that the plane of polarization of a linearly polarized wave is rotated when propagating through the dielectric. The angle of rotation first increases with increasing magnetic field and after passing through a maximum finally decreases with the magnetic field. The plane-polarized wave for large magnetic fields is also converted to a nearly circularly polarized wave. The attenuation of the signal was found to decrease with the magnetic field after showing a small initial increase. In this article an explanation of these observed features is suggested.

## 2. SUGGESTED EXPLANATION

The nickel particles used in the experiment have a complex permeability tensor in the presence of a dc magnetic field. It is suggested that the observed facts may be explained by considering the effect of this tensor character of the permeability on the signal propagating through the dielectric. Since the conductivity of the particles is high, the propagation constants $\Gamma_\pm(=\alpha_\pm+j\beta_\pm)$ corresponding to the two directions of rotation of a circularly polarized wave may be written as

$$\alpha_\pm{}^2=\tfrac{1}{2}k[\mu_\pm''+(\mu_\pm''^2+\mu_\pm'^2)^{1/2}] \quad (1)$$

$$\beta_\pm{}^2=\tfrac{1}{2}k[(\mu_\pm''^2+\mu_\pm'^2)^{1/2}-\mu_\pm''], \quad (2)$$

where $k=\omega\mu_0\sigma$; $\omega$ is the angular frequency of the signal, $\mu_0$ is the free space permeability, $\sigma$ is the conductivity, and $\mu'$ and $\mu''$ are the real and imaginary parts of the scalar permeabilities. The $\pm$ sign corresponds to the clockwise and anticlockwise direction of rotation, respectively.

As $\mu_-'$ changes little and $\mu_-''$ is nearly equal to zero, $\Gamma_-$ remains practically stationary in magnitude and phase for changes in the dc magnetic field. On the other hand, $\alpha_+$ shows a resonance maximum and $\beta_+$ exhibits an oscillatory variation. Thus when a plane polarized wave is incident on the particles, the two counter-rotating circularly polarized waves which make up the incident signal will have two different transmission coefficients. Power transmitted into the metallic particles per unit area due to the positively rotating component of unit amplitude is given by

$$(P_t)_+=\frac{2\beta_0}{\omega\mu_0}\frac{(\beta_+/\beta_0)\mu_+'+(\alpha_+/\beta_0)\mu_+''}{[(\alpha_+/\beta_0)\mu_+'-(\beta_+/\beta_0)\mu_+'']^2+[(\beta_+/\beta_0)\mu_+'+(\alpha_+/\beta_0)\mu_+'']^2}, \quad (3)$$

where $j\beta_0$= free-space propagation constant.

[1] M. H. Engineer, A. N. Datta, and B. R. Nag, J. Appl. Phys. (to be published).

On calculating $(P_t)_+$ (shown in Fig. 1) with representative values of $\mu_+'$ and $\mu_+''$, one finds that $(P_t)_+$ after a small initial increase decreases rapidly with increasing magnetic field, attains a minimum at the resonance and then again increases slowly. Power transmitted into the metallic particles due to the counter rotating component is, however, approximately independent of the field. The attenuation of the signal transmitted through the dielectric is due to the power absorbed by the metallic particles and would thus vary with the magnetic field in the same fashion as $\{(P_t)_+ + (P_t)_-\}$ (as shown in Fig. 1). The plot agrees qualitatively with the experimental characteristic.

This agreement suggests that the rotation of the transmitted signal will also be due to the tensor character of the permeability of the particles. The phase and magnitude of the two counter-rotating circularly polarized fields scattered by the metallic particles will be different due to the anisotropic nature of the permeability tensor. The field transmitted through the artificial dielectric is the sum of all these scattered signals coming from the different scattering centers combined with the incident signal. One may thus expect that the output signal will be elliptically polarized with its principal axis rotated with respect to the initial direction of polarization, since the oppositely circularly polarized signals constituting the scattered signal have different amplitudes as well as different phase.

A detailed quantitative analysis of scattering from the spherical anisotropic particles is extremely complicated. However, an estimate of this rotation may be obtained if we assume that Lewin's formulas[2] for effective permeability and permittivity of a mixture hold even for circularly polarized fields. Lewin's expressions for

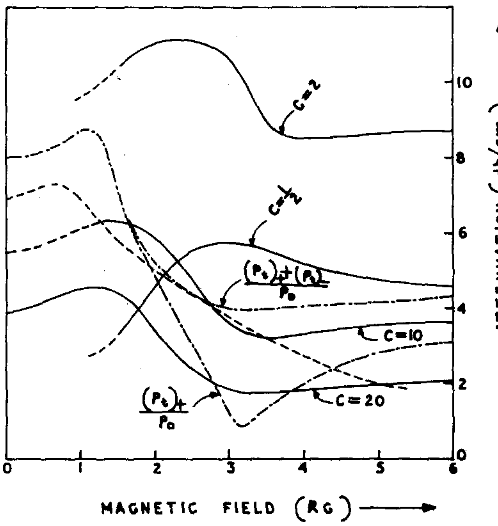


FIG. 1. Attenuation of the transmitted signal at different magnetic fields: (a) —— theoretical curves for different values of $c$, (b) – – – – experimental curve, (c) –·–·– relative attenuation (reduced $\frac{1}{4}$) obtained from Eq. (3).

[2] L. Lewin, J. Inst. Elec. Engrs. (London) **94**, 65 (1947).

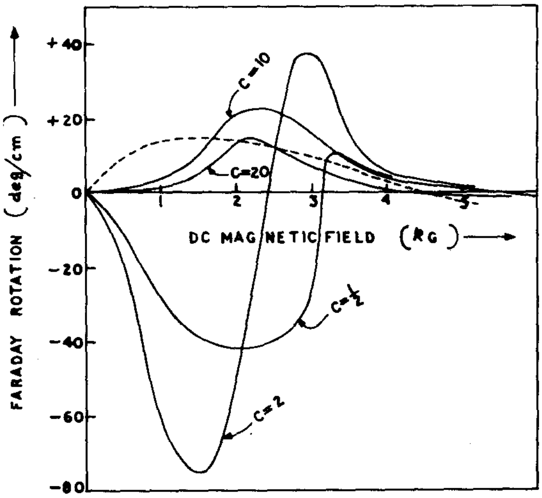


FIG. 2. Faraday rotation in the artificial dielectric at different magnetic fields: (a) —— theoretical curves for different values of $c$, (b) – – – – experimental curve.

the specific magnetic permeability and the specific inductive capacity $K$ of a mixture are

$$K = K_1\left\{1+\frac{3f}{[(K_P+2K_1)/(K_P-K_1)-f]}\right\} \qquad (4)$$

$$\mu = \mu_1\left\{1+\frac{3f}{[(\mu_P+2\mu_1)/(\mu_P-\mu_1)-f]}\right\}, \qquad (5)$$

where $f$ is the fractional volume of the particles in the mixture; $K_1$ and $\mu_1$ are respectively the dielectric constant and specific magnetic permeability of the medium in which the particles are embedded; $K_P$ and $\mu_P$ are the effective dielectric constant and the specific magnetic permeability of the particles and are given by

$$K_P = (\lambda/j\pi a)(K_2/\mu_2)^{1/2}, \qquad \mu_P = (\lambda/j\pi a)(\mu_2/K_2)^{1/2}, \quad (6)$$

where $K_2$ and $\mu_2$ are the complex dielectric constant and the specific magnetic permeability of the particles; $a$ is the radius of the particles and $\lambda$ is the free-space wavelength.

For circularly polarized waves the permeability of the particles is

$$\mu_2 = \mu_\pm' - j\mu_\pm''. \qquad (7)$$

Putting (7) in (6) and assuming that $\sigma \gg \omega\epsilon_0 K$ ($\epsilon_0$= free-space permittivity) one obtains

$$\mu_P = c[\mu_{i_\pm} - j\mu_{r_\pm}], \qquad (8)$$

where

$$(j\mu_2)^{1/2} = \mu_{r_\pm} + j\mu_{i_\pm};$$

$$\mu_{r_\pm} = \sqrt{2}^{-1}[(\mu_\pm''^2 + \mu_\pm'^2)^{1/2} + \mu_\pm'']^{1/2};$$

$$\mu_{i_\pm} = \sqrt{2}^{-1}[(\mu_\pm''^2 + \mu_\pm'^2)^{1/2} - \mu_\pm'']^{1/2};$$

$$c = \lambda(\omega\epsilon_0)^{1/2}/\pi a\sqrt{\sigma}.$$

Substituting the expression for $\mu_P$ in (5) we get

$$\mu = \mu_{m_\pm}' - j\mu_{m_\pm}''. \qquad (9)$$

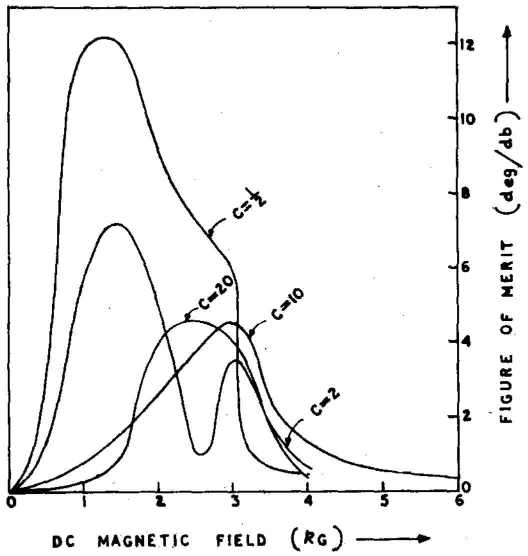


FIG. 3. Figure of merit as a function of the dc magnetic field.

For $\mu_1=1$ and $f=0.25$ (the experimental value)

$$\mu_{m\pm}'=2\frac{(c\mu_{i_\pm}+1)\,(c\mu_{i_\pm}+3)+c^2\mu_{r_\pm}^2}{(c\mu_{i_\pm}+3)^2+c^2\mu_{r_\pm}^2} \qquad (10)$$

$$\mu_{m_\pm}''=\frac{4c\mu_{r_\pm}}{(c\mu_{i_\pm}+3)^2+c^2\mu_{r_\pm}^2}. \qquad (11)$$

Although $K_P$ is much less than $K_2$, it is still very large compared to $K_1$. As a result the anisotropy in $K$ turns out to be insignificantly small and $K$ may be simplified to

$$K=K_1[1+3f/(1-f)]. \qquad (12)$$

The propagation constant $\Gamma_\pm=\alpha_\pm+j\beta_\pm$ through the mixture is, therefore, given by

$$\beta_\pm^2=\tfrac{1}{2}\omega^2\mu_0\epsilon_0K[(\mu_{m_\pm}'^2+\mu_{m_\pm}''^2)^{1/2}+\mu_{m_\pm}'] \qquad (13)$$

$$\alpha_\pm^2=\tfrac{1}{2}\omega^2\mu_0\epsilon_0K[(\mu_{m_\pm}'^2+\mu_{m_\pm}''^2)^{1/2}-\mu_{m_\pm}'], \qquad (14)$$

where $\mu_0$ is the free-space permeability. Equations (13) and (14) suggest that Faraday rotation would take place in an artificial dielectric and the angle of rotation and the amount of attenuation in the sample would vary with the dc magnetic field.

## 3. COMPARISON BETWEEN THEORY AND EXPERIMENT

Computed curves showing the variation of the angle of rotation and the attenuation with the magnetic field for typical values of the real and imaginary parts of the scalar permeability of a ferromagnetic material (taken from the data on ferrite) is shown in Figs. 1 and 2. On comparing these curves with those obtained from the experiment, one finds that there is excellent qualitative agreement if the value of $c$ is taken to be near 10 which is of the order of its experimental value. This agreement confirms that the suggestion here explains the basic physical mechanism of the observed rotation and attenuation. More exact analysis of the scattering problem is expected to bear out this conclusion, but the values of $\mu_P$ may be modified due to the spherical geometry of the particles.

## 4. DISCUSSION

In view of the above-mentioned agreement we were prompted to examine further Eqs. (10) and (11) for determining the optimum value of $c$ for large rotation and small attenuation. The calculated plots for four values of $c$ are shown in Figs. 1 and 2. It is observed that for large values of $c$ rotation for small magnetic filed is clockwise, but changes its sign for large fields. This nature, however, is completely reversed for small values of $c$. One also finds that there is an optimum value of $c$ (nearly 2 in this case) for which the rotation is maximum. On the other hand, for $c=\frac{1}{2}$ we get quite large rotation over a broad range of the magnetic field. It is also observed that for large values of $c$ the attenuation from an initial high value at low fields, falls to a low value at high magnetic fields, the fall is not, however, so marked, for small values of $c$. The attenuation has its maximum value for intermediate values of $c$ (nearly 2 in this case).

To assess the suitability of the sample as a rotator, the figure of merit for rotation, defined as the ratio of the angle of rotation in degrees to the attenuation in *db* per unit length has been computed for different magnetic fields using $c$ as a parameter. The curves are shown in Fig. 3. It is evident from these curves that there is an optimum value of $c$ (nearly 0.5) for which the highest figure of merit (nearly 12) can be achieved. Hence, for a given material, the size of the particles which determines the value of $c$ should be properly chosen to obtain the maximum value of the figure of merit. It is also evident that the maximum value, 12, indicated by the above analysis which has been based on the values of $\mu_\pm$ for ferrites may be improved for magnetic materials like Permalloy which have sharper magnetic resonance.

## ACKNOWLEDGMENT

The authors are grateful to Professor J. N. Bhar for his kind interest in the work.