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Method of measuring loaded Q-factor of single-ended cavity resonators using reflection bridge

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A bridge method of determining the loaded Q-factor and coupling coefficient of single-ended cavity resonators is described. The measuring system consists of a hybrid-T or "magic T" bridge with one of the collinear arms terminated by the cavity and the other by a precision attenuator followed by a variable precision short. The experimental procedure essentially measures the reflection coefficient of a cavity resonator as a function of frequency by balancing the bridge, using an oscillator with variable frequency amplitude modulation as the source.

INTRODUCTION

The methods of measuring Q-factors of reflection cavities can be broadly divided into two categories—(a) dynamic methods and (b) impedance measurement methods. The dynamic methods which use swept frequency sources are particularly suitable for high Q measurements where residual instability of the signal source creates serious problems. The transient decay method avoids this problem but the measurement of short time intervals becomes progressively difficult as the Q-value decreases.[1] The methods of impedance measurement require a stable frequency source as an appreciable period of time is necessary for accumulation of data. Of these the method where the VSWR of the cavity is measured as a function of frequency is perhaps the most used one.

In all these methods one has to identify the half-power points on both sides of resonance. In the case of cavities where accurate location of these points is difficult these methods are not suitable. Such problems were faced with low-Q post-coupled Gunn oscillator cavities and were solved by developing a method where determination of the half-power points would not be essential. This method, in which the reflection coefficient of the cavity is determined as a function of frequency by a reflection bridge arrangement has a number of advantages over the reflectometer technique[2,3] particularly for low-Q cavities with very small or large coupling factor, and is equally suitable for highly overcoupled or undercoupled high-Q cavities. Some experimental results obtained with a few cavities are presented. Finally the accuracy of the method is also discussed.

I. THEORY

The reflection coefficient of a one-port cavity resonator near resonance is given by[4]

$$\Gamma(\omega) = (\Gamma_0 - jQ_L\delta)/(1 + jQ_L\delta), \qquad (1)$$

where Q_L is the loaded Q-factor,

$$\delta = 2\Delta\omega/\omega_0, \quad \omega = \omega_0 \pm \Delta\omega,$$

Γ_0 is the reflection coefficient at resonance $= (\beta - 1)/(\beta + 1)$. β is the cavity coupling factor $= Q_U/Q_E$, and Q_U, Q_E are the unloaded and external Q-factors, respectively. Equation (1) can be written in the form

$$10 \log_{10}|\Gamma/\Gamma_0|^2 = 10[\log_{10}(1 + \chi^2/|\Gamma_0|^2) - \log_{10}(1 + \chi^2)], \qquad (2)$$

where $\chi = Q_L\delta$.

We observe that the location of the half power points is dependent on Γ_0. To determine Q_L for a particular Γ_0 one can calculate χ corresponding to 3-dB bandwidth, or calculate the relative power level corresponding to a chosen χ. But in either case, whether Q_L is low or high, if Γ_0 is large enough, it may not be possible to locate the half-power points accurately. Equation (2) suggests that if we measure Γ as a function of frequency, to calculate Q_L we need only Γ_0 and any particular off-resonance Γ for a known offset frequency. In practice the left hand side of Eq. (2) is plotted as a function of Γ_0 with $Q_L\delta$ as parameter. As explained in the next section, experimentally one measures the left hand side as a function of frequency through resonance. For the measured value of Γ_0 and a chosen value of $Q_L\delta$ one calculates from Eq. (2) or finds from the theoretical plot the corresponding value of $10 \log_{10}|\Gamma/\Gamma_0|^2$. Then from the experimental data one determines the corresponding $2\Delta f$ and hence Q_L from the chosen $Q_L\delta$. The coupling coefficient β is determined from Γ_0, where $\Gamma_0 = (\beta - 1)/(\beta + 1)$.

II. EXPERIMENTAL SETUP AND PROCEDURE

The experimental arrangement is shown in Fig. 1. The source frequency can be changed by a particular interval using a calibrated spectrum analyzer. However, it was found more convenient to use AM on the carrier tuned to cavity resonance and use the upper and lower sidebands as the off resonance signals. It should be pointed out here that the reflectometer arrangement of Ashley and Palka[2] would require amplitude stabilisation over a wide sweep range for low-Q measurement as well as broadband detectors, while the arrangement of Watanabe[3] requires monitoring of input power level.

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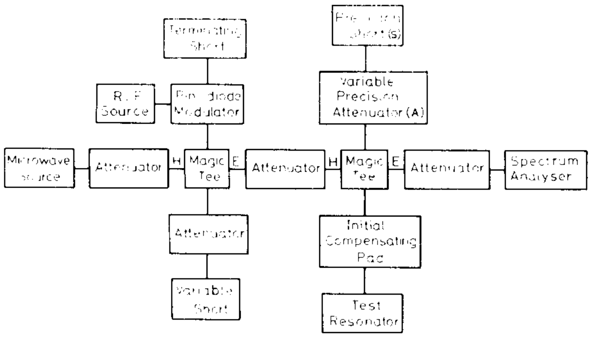

FIG. 1. Setup for the measurement of resonator Q.

The present arrangement avoids all these requirements as the reflection coefficient is measured by balancing a bridge.

The steps in determining Q_L are as follows:

(a) With an unmodulated carrier and the cavity detuned the bridge is balanced to produce a null by varying the precision attenuator A and the variable short S. Let the corresponding readings be α' dB and l' cm. (Actually α' should be zero but is found to have a small value for low-Q cavities.)

(b) Next the cavity is tuned to resonance, at which the unbalanced output from the bridge is maximum. The resonance frequency f_0 is noted.

(c) The balance is restored to null the carrier. If α_0 dB and l_0 cm are the attenuator and short readings then

$$\Gamma_0 = -10^{-(\alpha_0-\alpha')/10} \exp(-j4\pi|l_0 - l'|/\lambda_g), \qquad (3)$$

which at once indicates whether the cavity is overcoupled or undercoupled.

(d) The amplitude modulation of the carrier by an rf source is now switched on. The modulation frequency is gradually increased from a suitable low value and at each setting the upper and lower sidebands are nulled by balancing the bridge. Let α dB and l cm be the readings corresponding to any particular offset frequency Δf. In practice the α-values are usually not identical for the upper and lower sidebands as the cavity is almost never exactly tuned to the carrier. The corrected resonance frequency is obtained from the maximum value of α when α is plotted against Δf. For high-Q cavities where tuning is very sharp this asymmetry is negligibly small.

(e) We note that

$$10 \log_{10}|\Gamma/\Gamma_0|^2 = 2(\alpha_0 - \alpha). \qquad (4)$$

With a particular chosen value of $Q_L\delta$ and the measured Γ_0, we calculate from Eqs. (2) and (4) the corresponding value of $\alpha_0 - \alpha$, and hence α. From the experimental curve of α vs Δf we then determine the corresponding bandwidth $2\Delta f$, and hence Q_L from the $Q_L\delta$ chosen. The calculations are repeated for various values of $Q_L\delta$ and an average is taken.

III. RESULTS

Measurements were carried out on two types of cavity resonators: cavity A—an absorption type X-band wavemeter (Hewlett-Packard, type 530A) with one waveguide port shorted; cavity B—waveguide (WR-90) resonator of a centered post-mounted Gunn diode oscillator with the post gap shorted and for post diameters of (I) 3.175, (II) 2.381, and (III) 1.588 mm. The results are given in Table I.

TABLE I. Measured values of the cavity parameters for different cavities with calculated errors in Q_L and β.

Cavity type	f_0 (GHz)	Γ_0	β	Q_L[a]	% error $\Delta Q_L/Q_L$	% error $\Delta\beta/\beta$
A	8.0	$0.81 \angle 180°$	0.10	10960	30.8	11.0
B(I)	8.0	$0.21 \angle 6°$	1.56	384	4.1	−1.0
B(II)	8.0	$0.46 \angle -9°$	2.70	277	8.0	−3.0
B(III)	8.0	$0.59 \angle -8°$	3.88	191	11.9	−4.0

[a] Q_L—Average Q_L calculated using five values of $Q_L\delta$.

APPENDIX: ERROR ANALYSIS

In determining the accuracy of the method we have not taken into account the error arising out of the unequal power split between the two collinear arms of the hybrid-T. It was, however, experimentally verified that this inequality in our case was negligibly small. The only source of error in a particular measurement is in the two readings of the attenuator to determine Γ_0 and Γ for a particular offset frequency. For an attenuator dial reading error of $\Delta\alpha$, the fractional error in Q_L is

$$\Delta Q_L/Q_L = [1 + \chi^2 + 2|\Gamma_0|^2 + 2|\Gamma_0|^2/\chi^2] \times \Delta\alpha/4.343(1 - |\Gamma_0|^2). \qquad (5)$$

This is minimum for $\chi^2 = \sqrt{2}|\Gamma_0|$, in which case

$$\Delta Q_L/Q_L = 0.23(1 + \sqrt{2}|\Gamma_0|)^2 \cdot \Delta\alpha/(1 - |\Gamma_0|^2). \qquad (6)$$

The relative error in β is given by

$$\Delta\beta/\beta = -0.115(\beta - 1/\beta)\Delta\alpha. \qquad (7)$$

The minimum possible errors associated with our measurement of Q_L as computed from Eq. (6) and the error in β for $\Delta\alpha = 0.1$ dB are given in Table I.

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