Available online at www.sciencedirect.com

ScienceDirect

Journal of Ocean Engineering and Science 4 (2019) 289–298

www.elsevier.com/locate/joes

Original Article

Memory-dependent magneto–thermoelasticity for perfectly conducting two-dimensional elastic solids with thermal shock

Sarhan Y. Atwa[a,*], Nantu Sarkar[b]

[a] *Department of Engineering Mathematics and Physics, Higher Institute of Engineering, El Shorouk Academy, Egypt*
[b] *Department of Applied Mathematics, University of Calcutta, Kolkata 700009, India*

Received 10 April 2019; received in revised form 4 May 2019; accepted 15 May 2019
Available online 21 May 2019

Abstract

Recently, Yu et al. (2014) proposed a new model in generalized thermoelasticity based on heat conduction with the *memory-dependent derivative*. The magneto–thermoelastic responses in a perfectly conducting thermoelastic solid half-space is investigated in the context of the above new theory. *Normal mode analysis* together with an *eigenvalue expansion* technique is used to solve the resulting non-dimensional coupled governing equations. The obtained solutions are then applied to a specific problem for thermoelastic half-space whose boundary is subjected to a time-dependent thermal shock and zero stress. The effects of the kernel function, time-delay parameter, magnetic field and thermoelastic coupling parameter on the variations of different field quantities inside the half-space are analyzed graphically. The results show that these parameters has significant influence on the variations of the considered variables.

Keywords: Magneto–thermoelasticity; Memory-dependent derivative; Time-delay; Normal mode analysis.

1. Introduction

Physical observations and results of the conventional coupled dynamic thermoelasticity theories involving infinite speed of thermal signals, which were based on the mixed parabolic-hyperbolic governing equations of Biot [1] are mismatched. The theory of generalized thermoelasticity with one relaxation time was firstly formulated in 1967 [2]. The heat equation associated this theory is hyperbolic and hence eliminates the paradox of infinite speeds of propagation inherent in both the uncoupled and the coupled theories of thermoelasticity. Several generalizations to the coupled theory are introduced. One can refer to Chandrasekharaiah [3] and Hetnarski and Ignaczak [4] for a review. Lord and Shulman [2] described the modern approaches to the analytical treatment of dynamical thermoelasticity. The effect of magnetic field on elastic media under thermal loadings attracted several researchers due to its applications in many fields such as nuclear reactors, geophysics, optics and medical fields and so on [5,6]. The propagation of electro–magneto–thermoelastic waves in an electrically and thermally conducting solid have been studied by many authors. Paria [7,8] discussed the propagation of plane magneto–thermoelastic waves in an isotropic unbounded medium under the influence of a uniform thermal field and with magnetic field acting transversely to the direction of propagation, Paria used the classical Fourier's law of heat conduction and neglected the electric displacement together with charge density, the coupling between the current density and the heat flow density and the coupling between the temperature gradient and the electric current. Willson [9] corrected an error in [7,8] and extended Paria's results by introducing a component of the magnetic field parallel to the direction of the propagation. Othman [10–13] discussed the effect of thermal relaxation with the effect of the magnetic field on generalized thermo-elastic medium.

In the last decade, considerable interest in fractional calculus has been stimulated by the applications in different areas of physics and engineering. Recently, some efforts have been

* Corresponding author.
E-mail address: srhan_1@yahoo.com (S.Y. Atwa).
URL: https://orcid.org/0000-0001-9144-4587 (N. Sarkar)

https://doi.org/10.1016/j.joes.2019.05.004

made to modify the classical Fourier law of heat conduction by using the fractional calculus [14–19]). Sherief [20] introduced the fractional order theory of thermoelasticity, Youssef [21] discussed the theory of fractional order generalized thermoelasticity and Yu et al. [22,23] introduced the fractional order generalized electro–magneto–thermoelasticity and the nonlocal thermoelasticity based on nonlocal heat conduction and nonlocal elasticity. The memory-dependent derivative is defined in an integral form of a common derivative with a kernel function on a slip in the interval. So this kind of definition is better than the fractional one for reflecting the memory effect (instantaneous change rate depends on the past state). It's definition is more intuitionistic for understanding the physical meaning and the corresponding memory-dependent differential equation has more expressive force. Wang and Li [24] introduced a memory-dependent derivative (MDD). Recently, an interesting application of memory-dependent derivative is given by Yu et al. [25]. They introduced the memory-dependent derivative (MDD) instead of fractional calculus, into the rate of heat flux in Lord–Shulman generalized thermoelasticity theory [2] to describe the memory dependence. Later, Ezzat et al. [26] introduced a novel magneto–thermoelasticity theory with memory-dependent derivative. Ezzat et al. [27] investigated generalized thermo-viscoelasticity with memory-dependent derivatives. Sarkar et al. [28] studied a two-dimensional magneto–thermoelastic problem based on a new two-temperature generalized thermoelasticity model with memory-dependent derivative. Recently, Mondal and Sarkar [29] studied the transient responses in a two-temperature thermoelastic infinite medium having cylindrical cavity due to moving heat source with memory-dependent derivative. Memory response in plane wave reflection in generalized magneto–thermoelasticity has been reported by Sarkar et al. [30].

The purpose of the present work is to study the effect of the memory-dependent derivative of the heat conduction law with the effect of the magneto–thermoelastic responses in a perfectly conducting thermoelastic solid half-space which investigated in the context of the above new theory. Normal mode analysis used with an eigenvalue expansion technique to solve the resulting non-dimensional coupled governing equations. The obtained solutions are then applied to a specific problem for thermoelastic half-space whose boundary is subjected a time-dependent thermal shock and zero stress. The effects of kernel function, time-delay parameter, magnetic field and thermoelastic coupling parameter on the variations of different field quantities inside the half-space are analyzed graphically.

2. Governing equations

In this work, we consider the problem of a homogeneous isotropic perfectly conducting thermoelastic solid occupying the region of a half-space in the theory of generalized magneto–thermoelasticity with a memory-dependent derivative [25,26]. A magnetic field with constant intensity, namely $H = (0, 0, H_0)$ acts parallel to the bounding plane (take as the direction of the z-axis). Due to the application of initial magnetic field $\mathbf{H_0}$, there results an induced magnetic field $\mathbf{h}$ and an induced electric field $\mathbf{E}$. The simplified linear equations of electrodynamics of a slowly moving medium for a homogeneous, isotropic and thermally conducting elastic solid are [7]

$$\nabla \times \mathbf{h} = \mathbf{J} + \dot{\mathbf{D}}, \tag{1}$$

$$\nabla \times \mathbf{E} = -\dot{\mathbf{B}}, \tag{2}$$

$$\nabla \cdot \mathbf{B} = \mathbf{0}, \quad \nabla \cdot \mathbf{D} = \mathbf{0}, \tag{3}$$

$$\mathbf{B} = \mu_{\mathbf{0}}\mathbf{H}, \quad \mathbf{D} = \varepsilon_{\mathbf{0}}\mathbf{E}. \tag{4}$$

The above system of coupled equations are supplemented by the Ohm's law [7] for media with finite conductivity, namely

$$\mathbf{J} = \sigma_{\mathbf{0}}(\mathbf{E} + \mu_{\mathbf{0}}\dot{\mathbf{u}} \times \mathbf{H}), \tag{5}$$

where $\mathbf{B}$ is the magnetic induction field vector, $\mathbf{J}$ is the current density vector, $\mathbf{D}$ is the electric displacement vector, μ_0 is the magnetic permeability, ε_0 is the electric permeability, σ_0 is the electric conductivity.

Since, in generalized thermoelasticity only the infinitesimal temperature derivations from the reference temperature are considered, therefore in the absence of the body force and internal heat source, the stress–strain–temperature relation is

$$\sigma_{ij} = 2\mu e_{ij} + \lambda e \delta_{ij} - \gamma(T - T_0)\delta_{ij}, \tag{6}$$

where

$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}),\ \ i, j = 1, 2, 3$ referee to a general coordinates.

Equation of motion is

$$\sigma_{ij,j} + F_i = \rho \ddot{u}_i, \tag{7}$$

where F_i (= ith component of $\mu_0(\mathbf{J} \times \mathbf{H})$) are the components of Lorentz force [7,8].

The heat conduction equation with the memory-dependent derivative operator is read from [26] as

$$K\nabla^2 T = \left(\rho C_E \dot{T} + \gamma T_0 \dot{e}\right) + \int_{t-\tau}^{t} K(t-\xi)\left(\rho C_E \frac{\partial^2 T}{\partial \xi^2} + \gamma T_0 \frac{\partial^2 e}{\partial \xi^2}\right) d\xi. \tag{8}$$

In the preceding equations, $\lambda,\ \mu$ are Lamé constants, ρ is the density, σ_{ij} are the components of the stress tensor, u_i are the components of the displacement vector, e_{ij} are the components of strain tensor, t is the time variable, K is the thermal conductivity, T is the absolute temperature, T_0 is the temperature of the medium in it's natural state, assumed to be such that $|(T - T_0)/T_0| << 1$, e is the cubical dilatation, γ is a material constant given by $\gamma = (3\lambda + 2\mu)\alpha_T$, where α_T is the coefficient of linear thermal expansion, C_E is the specific heat at constant strain and τ is the only *relaxation time parameter*. Also note that in the above equations, a comma followed by a suffix denotes *spatial derivative* and a superposed dot denotes the time-derivative.

3. Formulation of the problem

We consider a homogeneous isotropic perfectly conducting magneto–thermoelastic solid half-space occupying the region $\Omega = \{(x, y, z) : -\infty < x < \infty,\ \ 0 \le y < \infty,\ \ -\infty < z < \infty\}$. The body is initially at rest. For two-dimensional motion in xy-plane, the displacement components will have the form $u_x = u(x, y, t)$, $u_y = v(x, y, t)$, $u_z = 0$. Hence from Eq. (6), the non-zero stress components can be written as

$$\sigma_{xx} = (\lambda + 2\mu)u_{,x} + \lambda v_{,y} - \gamma(T - T_0), \tag{9}$$

$$\sigma_{yy} = (\lambda + 2\mu)v_{,y} + \lambda u_{,x} - \gamma(T - T_0), \tag{10}$$

$$\sigma_{zz} = \lambda(u_{,x} + v_{,y}) - \gamma(T - T_0), \tag{11}$$

$$\sigma_{xy} = \mu(u_{,y} + v_{,x}). \tag{12}$$

The components of the magnetic intensity vector $\mathbf{H}$ in the medium are

$$H_x = 0, \qquad H_y = 0, \qquad H_z = H_0 + h(x, y, t). \tag{13}$$

The electric intensity vector $\mathbf{E}$ is normal to both the magnetic intensity and the displacement vectors. Thus, it has the components

$$E_x = E_1, \qquad E_y = E_2, \qquad E_z = 0. \tag{14}$$

Since the current density vector $\mathbf{J}$ is parallel to the electric intensity vector $\mathbf{E}$, it has the components

$$J_x = J_1, \qquad J_y = J_2, \qquad J_z = 0. \tag{15}$$

Let the primary magnetic intensity field H_0 and also let the corresponding perturbation field h be small, so that its product with u_i and its derivatives can be neglected for linearization. Then using Eq. (4), Eqs. (1) and (2) can be written in the forms

$$\nabla \times \mathbf{h} = \mathbf{J} + \varepsilon_0 \dot{\mathbf{E}}, \tag{16}$$

$$\nabla \times \mathbf{E} = -\mu_0 \dot{\mathbf{h}}. \tag{17}$$

As the medium is a perfect conductive medium (i.e. $\sigma_0 \to \infty$), then Eq. (5) reduces to

$$\mathbf{E} = -\mu_0(\dot{\mathbf{u}} \times \mathbf{H}), \tag{18}$$

We assume now that the initial conditions are homogeneous. Then the relations (18) and (19) yield

$$\mathbf{E} = \mu_0 H_0(-\dot{v}, \dot{u}, 0), \tag{19}$$

$$\mathbf{h} = -H_0(0, 0, e), \tag{20}$$

where

$$e = u_{,x} + v_{,y}.$$

From the relations (17), (20) and (21), we obtained after some simple computation the current density vector $\mathbf{J}$ as

$$\mathbf{J} = -H_0\big(\varepsilon_0\mu_0\ddot{v} - e_{,y},\ e_{,x} - \varepsilon_0\mu_0\ddot{u},\ 0\big). \tag{21}$$

Using Eq. (22) in the relation $\mathbf{F_i} = \mu_0(\mathbf{J} \times \mathbf{H})_\mathbf{i}$, we get the non-zero components of Lorentz force as

$$F_x = \mu_0 H_0^2\big(e_{,x} - \varepsilon_0\mu_0\ddot{u}\big), \tag{22}$$

$$F_y = \mu_0 H_0^2\big(e_{,y} - \varepsilon_0\mu_0\ddot{v}\big), \tag{23}$$

Thus the electro–magneto–thermoelastic coupled problem is transformed into a thermoelastic coupled one and after getting the components of displacement, the induced electric and the induced magnetic field can be obtained from Eqs. (20) and (21) respectively.

From Eqs. (7), (10)–(13) and (23)–(25), we can write

$$(\lambda + \mu)e_{,x} + \mu\nabla^2 u - \gamma T_{,x} + \mu_0 H_0^2\big(e_{,x} - \varepsilon_0\mu_0\ddot{u}\big) = \rho\ddot{u}, \tag{24}$$

$$(\lambda + \mu)e_{,y} + \mu\nabla^2 v - \gamma T_{,y} + \mu_0 H_0^2\big(e_{,y} - \varepsilon_0\mu_0\ddot{v}\big) = \rho\ddot{v}. \tag{25}$$

To transform the above equations in non-dimensional forms, we define the following non-dimensional variables

$$(x', y') = c_0\eta(x, y),\ \ (u', v') = c_0\eta(u, v),\ \ (t', \tau') = c_0^2\eta(t, \tau),$$
$$\theta = \frac{\gamma(T - T_0)}{\rho c_0^2}, \sigma'_{ij} = \frac{\sigma_{ij}}{\mu},\ \ c_0^2 = c_1^2 + c_3^2,\ \ c_1^2 = \frac{\lambda + 2\mu}{\rho},$$
$$c_2^2 = \frac{\mu}{\rho},\ \ c_3^2 = \frac{\mu_0 H_0^2}{\rho},\ \ \eta = \frac{\rho C_E}{K}.$$

Eqs. (26) and (21) in the non-dimensional forms then reduce to (dropping the primes for simplicity)

$$\beta^2 u_{,xx} + u_{,yy} + (\beta^2 - 1)v_{,xy} - \beta^2\theta_{,x} = M\beta^2\ddot{u}, \tag{26}$$

$$\beta^2 v_{,yy} + v_{,xx} + (\beta^2 - 1)u_{,xy} - \beta^2\theta_{,y} = M\beta^2\ddot{v}, \tag{27}$$

where

$$\beta^2 = \frac{c_0^2}{c_2^2},\ \ M = \left(1 + \frac{c_3^2}{c^2}\right),\ \ c^2 = \frac{1}{\mu_0\varepsilon_0}.$$

The non-dimensional form of the heat conduction equation (8) can be written as

$$\nabla^2\theta = (1 + \tau D_\tau)(\dot{\theta} + \varepsilon\dot{e}), \tag{28}$$

where D_τ is the first order with memory-dependent derivative operator [25] defined as

$$D_\tau^1 f(x, t) = \frac{1}{\tau}\int_{t-\tau}^{t} K(t - \xi) f'(x, \xi) d\xi. \tag{29}$$

where τ is the time delay and $K(t - \xi)$ is the kernel function in which they can be chosen freely [25], $\varepsilon = \frac{\gamma^2 T_0}{\rho C_E(\lambda+2\mu)}$ is the dimensionless thermoelastic coupling parameter and $\nabla^2 \equiv (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})$. Following [28], we take the kernel function $K(t,$

ξ) freely as:

$$K(t,\xi) = 1 - \frac{2b}{\tau}(t-\xi) + \frac{a^2(t-\xi)^2}{\tau^2} = \begin{cases} 1 & \text{if} \quad a=b=0, \\ 1-\left(\dfrac{t-\xi}{\tau}\right) & \text{if} \quad a=0, b=\dfrac{1}{2}, \\ 1-(t-\xi) & \text{if} \quad a=0, b=\dfrac{\tau}{2}, \\ \left(1-\dfrac{t-\xi}{\tau}\right)^2 & \text{if} \quad a=b=1, \end{cases} \tag{30}$$

where a and b are constants.

The non-dimensional stress components become

$$\sigma_{xx} = \beta_0^2 u_{,x} + (\beta_0^2 - 2)v_{,y} - \beta^2\theta, \tag{31}$$

$$\sigma_{yy} = \beta_0^2 v_{,y} + (\beta_0^2 - 2)u_{,x} - \beta^2\theta, \tag{32}$$

$$\sigma_{zz} = (\beta_0^2 - 2)(u_{,x} + v_{,y}) - \beta^2\theta, \tag{33}$$

$$\sigma_{xy} = u_{,y} + v_{,x}, \tag{34}$$

where

$$\beta_0^2 = \frac{\lambda + 2\mu}{\mu} = \frac{c_1^2}{c_2^2}.$$

Differentiating Eq. (28) with respect to x and Eq. (29) with respect to y, we get

$$\nabla^2 u_{,x} + (\beta^2 - 1)e_{,xx} - \beta^2\theta_{,xx} = M\beta^2\ddot{u}_{,x}, \tag{35}$$

$$\nabla^2 v_{,y} + (\beta^2 - 1)e_{,yy} - \beta^2\theta_{,yy} = M\beta^2\ddot{u}_{,y}, \tag{36}$$

Adding Eqs. (35) and (36), we obtain

$$\nabla^2 e - \nabla^2\theta = M\ddot{e}. \tag{37}$$

Now, we introduce a function Φ defined as follows

$$\Phi = e - \theta. \tag{38}$$

Using the relations (38) in Eq. (37), we can write

$$\nabla^2\Phi = M(\ddot{\Phi} + \ddot{\theta}). \tag{39}$$

Again using the relations (38) in the heat conduction equation (30), we can write

$$\nabla^2\theta = (1 + \tau D_\tau)\left[(1+\varepsilon)\dot{\theta} + \varepsilon\dot{\Phi}\right]. \tag{40}$$

4. Normal mode analysis and solution of the problem

For the plane wave propagation along x-axis, one may seek the solution of Eqs. (39) and (40) in the following form [31]:

$$\left(\Phi, \theta, u, v, \sigma_{ij}\right)(x,y,t) = \left(\Phi^*, \theta^*, u^*, v^*, \sigma_{ij}^*\right)(y)\exp\{i(\zeta x - \omega t)\}, \tag{41}$$

where $i = \sqrt{-1}$, $\omega(>0)$ is the assigned angular frequency and ζ is the wave number in the x-direction.

Using Eq. (41), Eqs. (39) and (40) take the following forms

$$D^2\Phi^*(y) = C_1\Phi^* + C_2\theta^*, \tag{42}$$

$$D^2\theta^*(y) = D_1\Phi^* + D_2\theta^*, \tag{43}$$

where

$$D \equiv \frac{d}{dy}, \quad C_1 = \zeta^2 - M\omega^2, \quad C_2 = -M\omega^2,$$

$$D_1 = -i\varepsilon\omega[1 + G(\tau,\omega)], \quad D_2 = \zeta^2 - i\omega(1+\varepsilon)[1 + G(\tau,\omega)],$$

$$G(\tau,\omega) = \left[\exp(i\tau\omega)\{2a^2 - \tau^2\omega^2(a^2 - 2b + 1) - 2i\tau\omega(a^2 - b^2)\} - 2ib\tau\omega - 2a^2 + \tau^2\omega^2\right]/\tau^2\omega^2.$$

Eqs. (42) and (43) can be written in a matrix differential equation as follows [31]:

$$D\mathcal{V} = \mathcal{A}\mathcal{V}, \tag{44}$$

where

$$\mathcal{V} = \left(\Phi^*, \quad \theta^*, \quad D\Phi^*, \quad D\theta^*\right)^T,$$

$$\mathcal{A} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ C_1 & C_2 & 0 & 0 \\ D_1 & D_2 & 0 & 0 \end{pmatrix}.$$

Following the solution methodology through eigenvalue approach, as discussed in [31], the solutions for $\Phi^*(y)$ and $\theta^*(y)$, bounded as $y \to \infty$ can be written as

$$\Phi^*(y) = (\lambda_1^2 - D_2)A_1\exp(-\lambda_1 y) + (\lambda_2^2 - D_2)A_2\exp(-\lambda_2 y), \tag{45}$$

$$\theta^*(y) = D_1\left[A_1\exp(-\lambda_1 y) + A_2\exp(-\lambda_2 y)\right], \tag{46}$$

where $\pm\lambda_i$ $(i = 1, 2)$ are the eigenvalues of the coefficient matrix $\mathcal{A}$ with positive real parts given by

$$\lambda_i^2 = \frac{(C_1 + D_2) + (-1)^{i+1}\sqrt{(C_1 - D_2)^2 + 4C_2D_1}}{2}.$$

Using Eqs. (45) and (46), Eq. (38) gives

$$e^*(y) = (\lambda_1^2 + D_1 - D_2)A_1\exp(-\lambda_1 y) + (\lambda_2^2 + D_1 - D_2)A_2\exp(-\lambda_2 y). \tag{47}$$

To get the displacement $u^*(y)$, we use Eqs. (38), (41), (46) and (47) in Eq. (28) to obtain the following ordinary differential equation satisfied by $u^*(y)$

$$\left(D^2 - k^2\right)u^*(y) = i\zeta[D_1 - (\beta^2 - 1)(\lambda_1^2 - D_2)]A_1\exp(-\lambda_1 y) + i\zeta[D_1 - (\beta^2 - 1)(\lambda_2^2 - D_2)]A_2\exp(-\lambda_2 y), \tag{48}$$

where

$$k^2 = \left(\zeta^2 - M\beta^2\omega^2\right) \text{ and } \lambda_1^2 \neq \lambda_2^2 \neq k^2.$$

The solution of the above Eq. (48), bounded as $y \to \infty$ takes of the following form

$$u^*(y) = P\exp(-ky) + i\zeta\eta_1 A_1\exp(-\lambda_1 y) + i\zeta\eta_2 A_2\exp(-\lambda_2 y), \tag{49}$$

where

$$\eta_j = \frac{\left[D_1 - (\beta^2 - 1)(\lambda_j^2 - D_2)\right]}{(\lambda_j^2 - k^2)} \quad (j = 1, 2)$$

and $A_j \ (j = 1, 2)$, P are constants to be determined from the boundary conditions of the problem.

To get the displacement $v^*(y)$, we can use the following relation

$$Dv^*(y) = \left[e^*(y) - i\zeta u^*(y)\right]. \tag{50}$$

Substituting from Eqs. (47) and (49) in Eq. (50), we obtain

$$v^*(y) = \frac{i\zeta}{k} P\exp(-ky) - \gamma_1 A_1 \exp(-\lambda_1 y) - \gamma_2 A_2 \exp(-\lambda_2 y), \tag{51}$$

where

$$\gamma_j = \frac{(\lambda_j^2 + D_1 - D_2 + \zeta^2 \eta_j)}{\lambda_j} \quad (j = 1, 2).$$

In terms of Eq. (41), substituting from Eqs. (46), (47), (49) and (51) in Eqs. (31)–(34), respectively, we obtain after some simple computations, the stress components in the following forms

$$\sigma_{xx}^*(y) = 2i\zeta P\exp(-ky) + f_1 A_1 \exp(-\lambda_1 y) + f_2 A_2 \exp(-\lambda_2 y), \tag{52}$$

$$\sigma_{yy}^*(y) = -2i\zeta P\exp(-ky) + g_1 A_1 \exp(-\lambda_1 y) + g_2 A_2 \exp(-\lambda_2 y), \tag{53}$$

$$\sigma_{zz}^*(y) = h_1 A_1 \exp(-\lambda_1 y) + h_2 A_2 \exp(-\lambda_2 y), \tag{54}$$

$$\sigma_{xy}^*(y) = -\left(k + \frac{\zeta^2}{k}\right) P\exp(-ky) - i\zeta(\lambda_1\eta_1 + \gamma_1) A_1 \exp(-\lambda_1 y) - i\zeta(\lambda_2\eta_2 + \gamma_2) A_2 \exp(-\lambda_2 y), \tag{55}$$

where

$$f_j = [(\beta_0^2 - 2)(\lambda_j^2 - D_2) + D_1(\beta_0^2 - \beta^2 - 2) - 2\zeta^2 \eta_j],$$

$$g_j = [\beta_0^2(\lambda_j^2 - D_2) + D_1(\beta_0^2 - \beta^2) + 2\zeta^2 \eta_j],$$

and

$$h_j = [(\beta_0^2 - 2)(\lambda_j^2 - D_2) + D_1(\beta_0^2 - \beta^2 - 2)], \quad j = 1, 2.$$

5. Applications

In order to determine the constants $A_j \ (j = 1, 2)$ and P, we need to consider the following boundary conditions on the surfaces $y = 0$ of the half-space in non-dimensional form. In our present work, we consider the following boundary conditions as follows:

(a) thermal boundary condition: the surface $y = 0$ of the half-space is subjected to a time-dependent thermal shock in the following form

$$\theta(x, y, t) = f(x, t) \quad \text{on} \quad y = 0, \tag{56}$$

where

$$f(x, t) = \theta_0 H(|l| - x)\exp(-\varrho t),$$

$H(\cdot)$ is the Heaviside unit step function and θ_0 is a constant. This means that heat is applied on the surfaces of the plate on a narrow band of width $2l$ surrounding the x-axis to keep it at the temperature θ_0, while the rest of the surface is kept at zero temperature and $\varrho > 0$ is a constant.

(b) mechanical boundary conditions: the surface $y = 0$ of the half-space is traction free,

$$\sigma_{yy}(x, 0, t) = \sigma_{xy}(x, 0, t) = 0. \tag{57}$$

In this case, the constants $A_j \ (j = 1, 2)$ and P can be obtained in the following forms:

$$A_1 = \frac{f^*\left[g_2\left(k + \zeta^2/k\right) - 2\zeta^2(\lambda_2\eta_2 + \gamma_2)\right]}{D_1 \Delta^*},$$

$$A_2 = \frac{-f^*\left[g_1\left(k + \zeta^2/k\right) - 2\zeta^2(\lambda_1\eta_1 + \gamma_1)\right]}{D_1 \Delta^*},$$

$$P = \frac{i\zeta f^*[g_1(\lambda_2\eta_2 + \gamma_2) - g_2(\lambda_1\eta_1 + \gamma_1)]}{D_1 \Delta^*},$$

where

$$\Delta^* = \left(k + \frac{\zeta^2}{k}\right)(g_2 - g_1) + 2\zeta^2[(\lambda_1\eta_1 + \gamma_1) - (\lambda_2\eta_2 + \gamma_2)],$$

$$f^*(\zeta, \omega) = \frac{\sqrt{2}\theta_0 \sin(\zeta l)(1 + i\zeta\pi\delta(\zeta))}{\sqrt{\pi}\zeta(\varrho - i\omega)}.$$

6. Numerical results and discussions

In this section, we aim to illustrate the numerical results of the analytical expressions obtained in the previous section and elucidate the influence of time-delay τ on the behavior of the field quantities (Fig. 1). In order to interpret the numerical computations, we consider material properties of copper material for which the numerical constants of the problems are [26]:

$$\varepsilon = 0.0168, \ \ \tau = 0.05, \ \ \beta = \sqrt{3.5}, \ \ \beta_1 = 0.008.$$

The other constants of the problem may be taken as

$$\omega = 3, \ \ \zeta = 2, \ \ p^* = 50, \ \ l = 4, \ \ \theta_0 = 1, \ \ \varrho = 1.$$

Considering the above physical data, we have evaluated the numerical values of the real parts of the dimensionless field quantities with the help of a computer program developed by using MATLAB software. The computations were performed for a fixed value of time, namely $t = 0.1$ and for three different values of the time-delay, namely $\tau = 0.05, \ 0.005, \ 0.0005$. The numerical technique outlined above was used to obtain the displacement, temperature and the stress distributions as well as the current density and induced magnetic and electric fields. The results are displayed

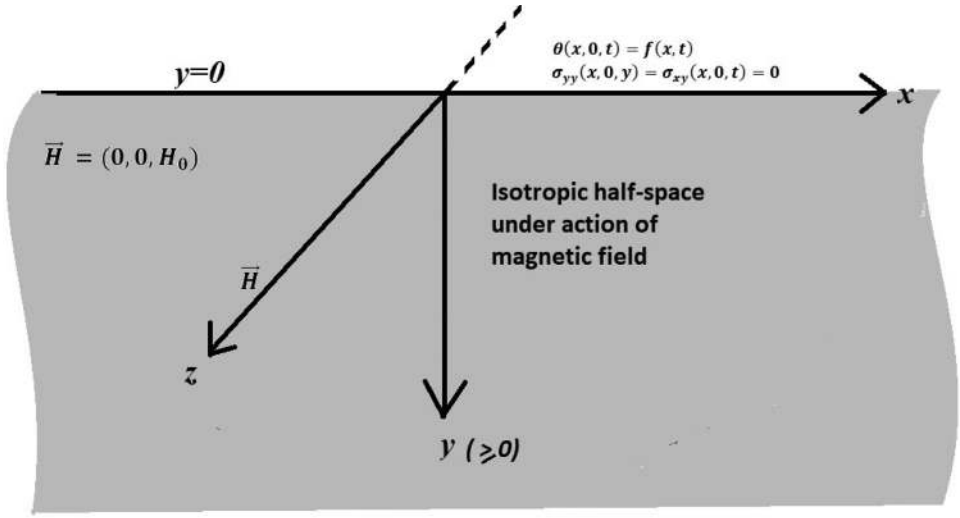

Fig. 1. Schematic of the present problem.

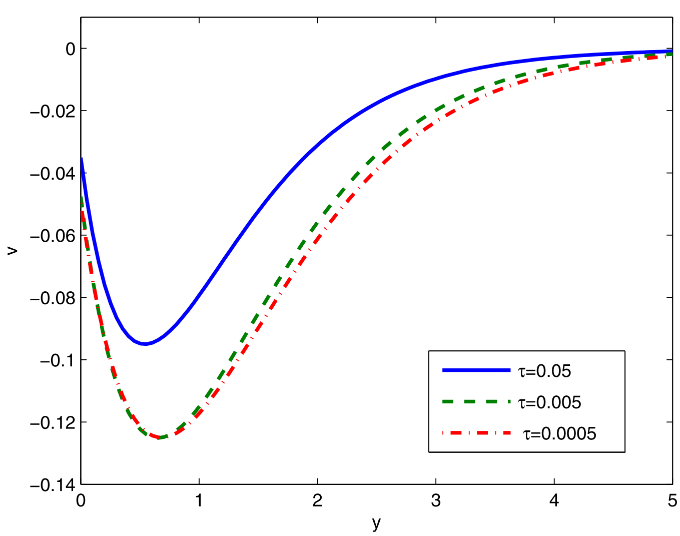

Fig. 2. The displacement component v for different τ when $K(t-\xi)=1-(t-\xi)$.

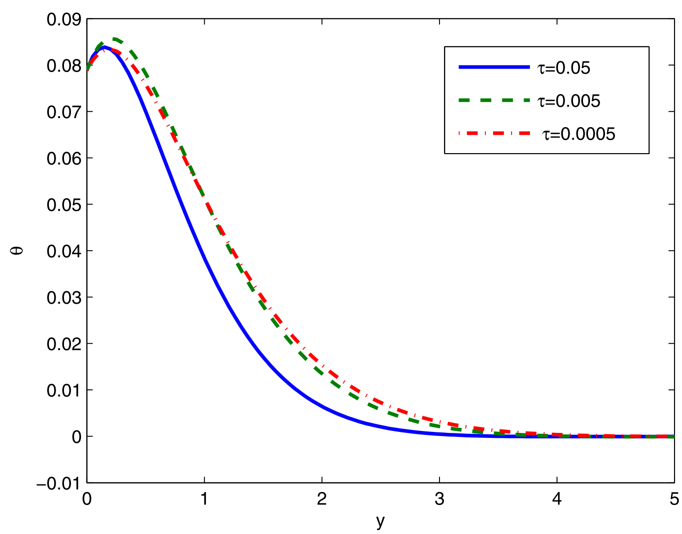

Fig. 3. The temperature θ for different τ when $K(t-\xi)=1-(t-\xi)$.

graphically for $x = 1.6,\ M = 1.4$ and in the case of the kernel function $K(t,\xi) = 1-(t-\xi)$ i.e. $a = 0,\ b = \tau/2$ where the solid line represents the solution obtained for $\tau = 0.05$, the dashed line for the value $\tau = 0.005$ and the dashed and dot line for the value $\tau = 0.0005$ as shown in Figs. 2–5.

Fig. 2 indicates the variation in the displacement distribution for different values of the time-delay parameter τ. We noticed that the displacement distribution has been affected by the time-delay, where the increasing of the value of the parameter causes decreasing in displacement distribution. Fig. 3 reveals the variation in the temperature for different values of the time-delay. We noticed that the temperature field has been affected by the time-delay, where the increasing of the value of the parameter causes decreasing in temperature. The thermal waves are continuous functions, smooth and reach to steady state depending on the value of time-delay, which means that the particles transport the heat to the other particles easily and this makes the decreasing rate of the temperature greater than the other ones. Figs. 4 and 5 display the stress distribution with distance for different values of τ. We observed that the stress field has the same behavior as the displacement distribution and the absolute value of the maximum stress decreases.

In Figs. 6–9, we study the influence of the kernel function $K(t,\xi) = [1-(t-\xi)/\tau]^2$ i.e. $a = b = 1$. The important phenomenon observed in all these figures that the solution of any of the considered function in the new generalized theory is restricted in a bounded region. Beyond this region, the variations of these distributions do not take place. This means that the solutions according to the new generalized magneto–thermoelasticity theory with MDD exhibit the behavior of finite speeds of wave propagation.

In Figs. 10–13, we discuss the effect of the non-dimensional time parameter t on the distributions of the

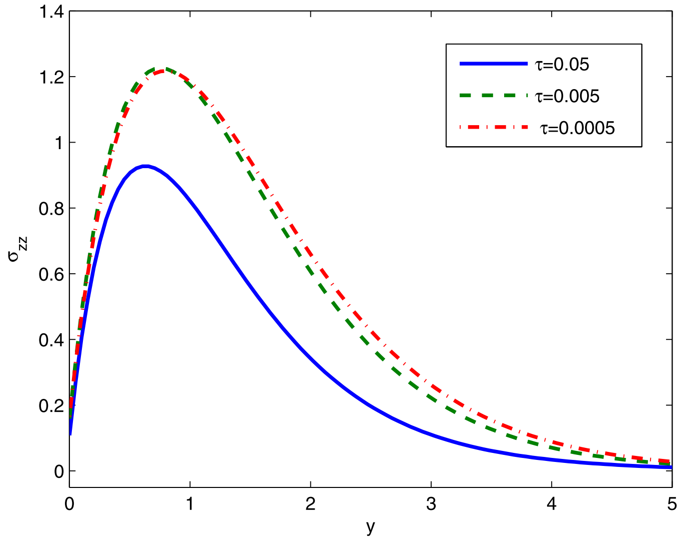

Fig. 4. The stress component σ_{zz} for different τ when $K(t-\xi) = 1-(t-\xi)$.

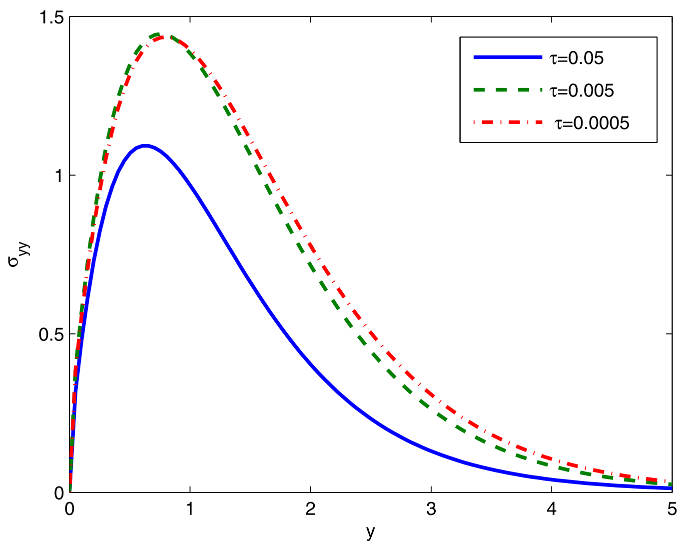

Fig. 5. The stress component σ_{yy} for different τ when $K(t-\xi) = 1-(t-\xi)$.

field variables of interest. For this purpose, we have taken three values of the t, namely $\tau = 0.1,\ \ 0.05,\ \ 0.01$ when $\tau = 0.05,\ K(t,\xi) = 1-(t-\xi)/\tau$. The solid line represents the solution obtained in the value of time $t = 0.1$, the dashed line for the value $t = 0.05$ and the dashed and dot line for the value $t = 0.01$.

Figs. 14–17 are plotted with the aim to show the effect of different kernel function on the distributions of various field variables of interest when $\tau = 0.005,\ t = 0.1$. Here the solid line represents the solution obtained for the case of the kernel function $K(t,\xi) = 1$ i.e. $a = b = 0$; the dashed line for the case of the kernel function $K(t,\xi) = 1-(t-\xi)/\tau$ i.e. $a = 0,\ b = 1/2$; the dashed and dot line for the case of the kernel function $K(t,\xi) = 1-(t-\xi)$ i.e. $a = 0,\ b = \tau/2$ and the dot line for the case of the kernel function $K(t,\xi) =$

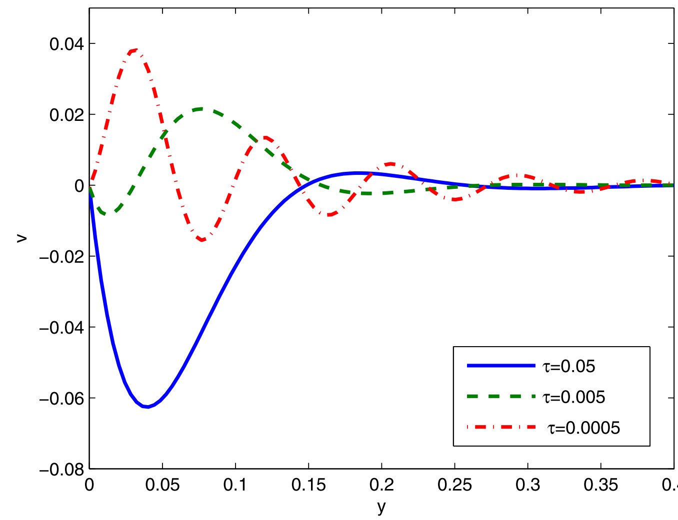

Fig. 6. The displacement component v for different τ when $K(t-\xi) = (1-(t-\xi)/\tau)^2$.

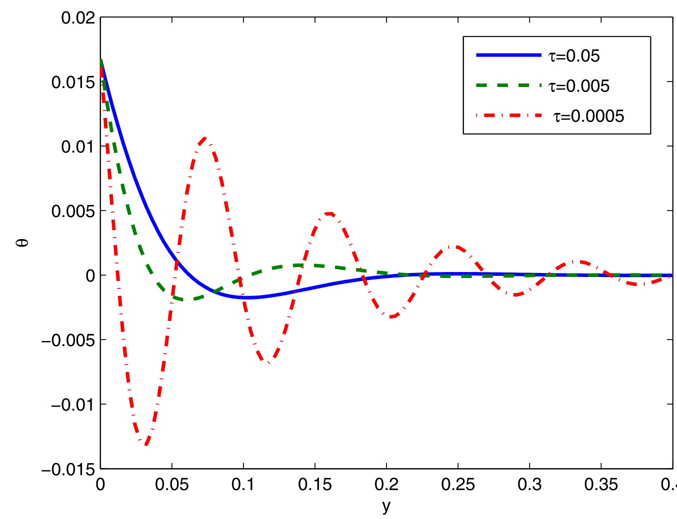

Fig. 7. The temperature θ for different τ when $K(t-\xi) = (1-(t-\xi)/\tau)^2$.

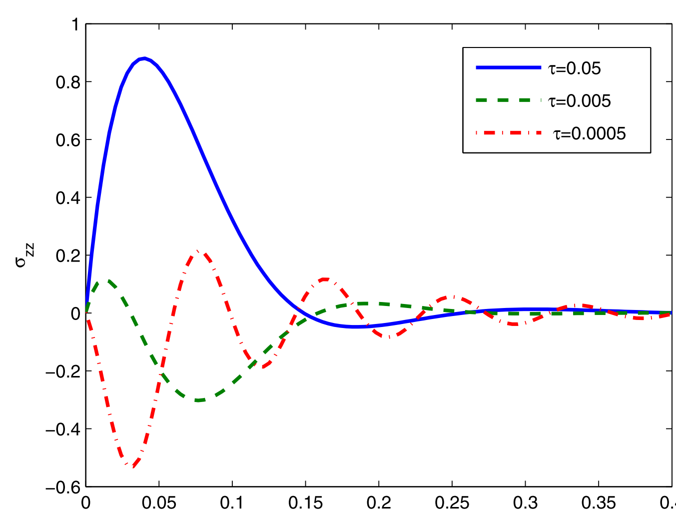

Fig. 8. The stress component σ_{zz} for different τ when $K(t-\xi) = (1-(t-\xi)/\tau)^2$.

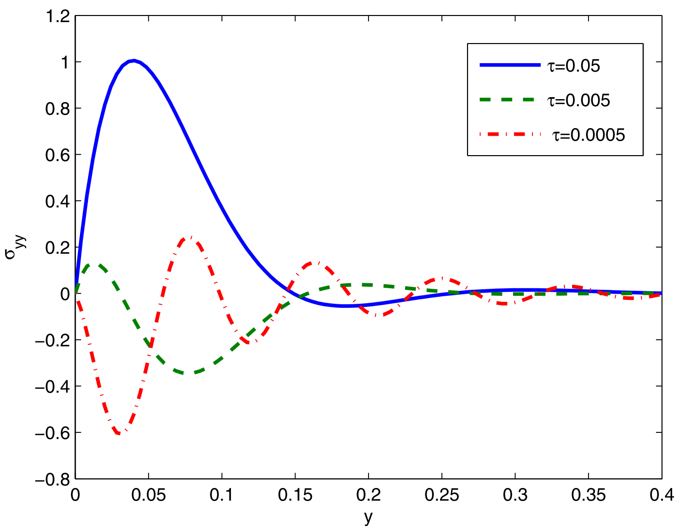

Fig. 9. The stress component σ_{yy} for different τ when $K(t-\xi)=(1-(t-\xi)/\tau)^2$.

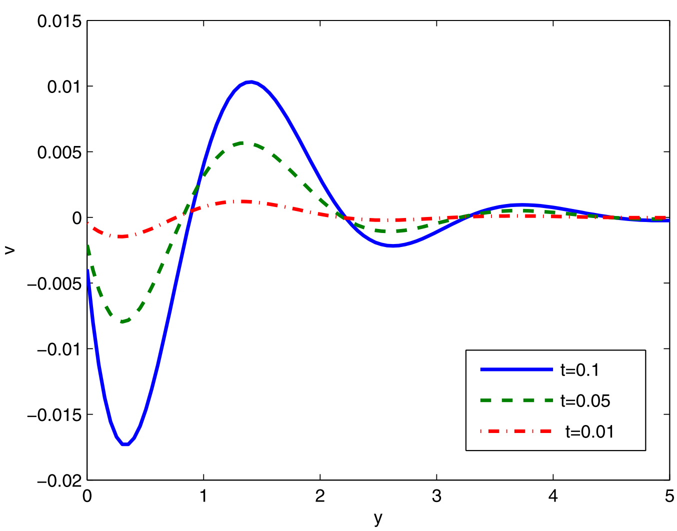

Fig. 10. The displacement component v for different t when $\tau=0.05, K(t-\xi)=1-(t-\xi)/\tau$.

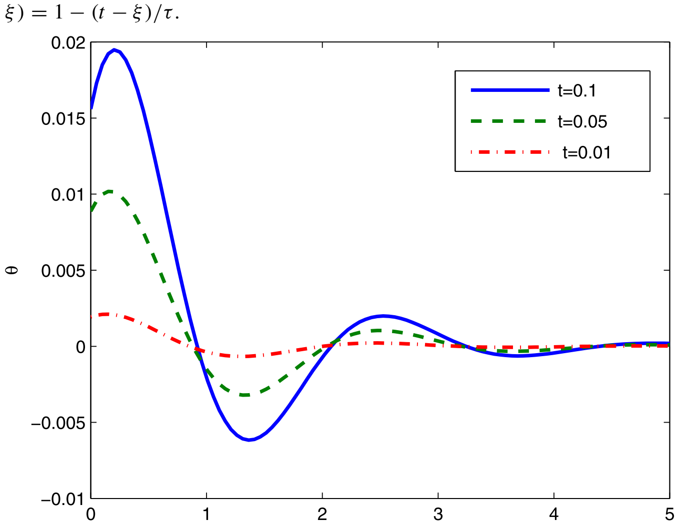

Fig. 11. The temperature θ for different t when $\tau=0.05, K(t-\xi)=1-(t-\xi)/\tau$.

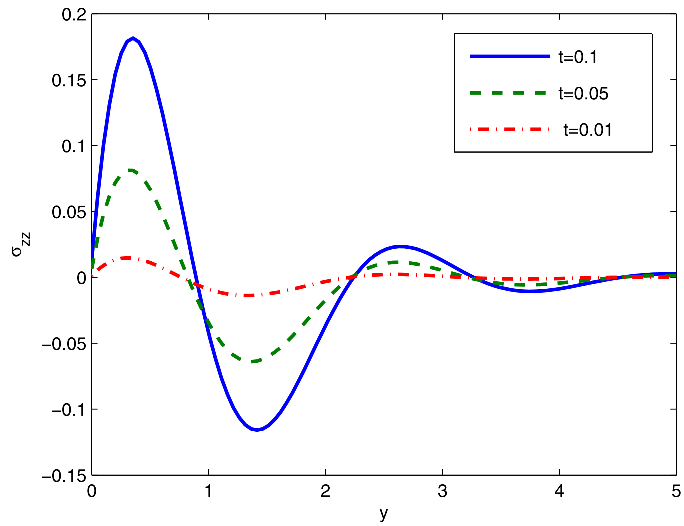

Fig. 12. The stress component σ_{zz} for different t when $\tau=0.05, K(t-\xi)=1-(t-\xi)/\tau$.

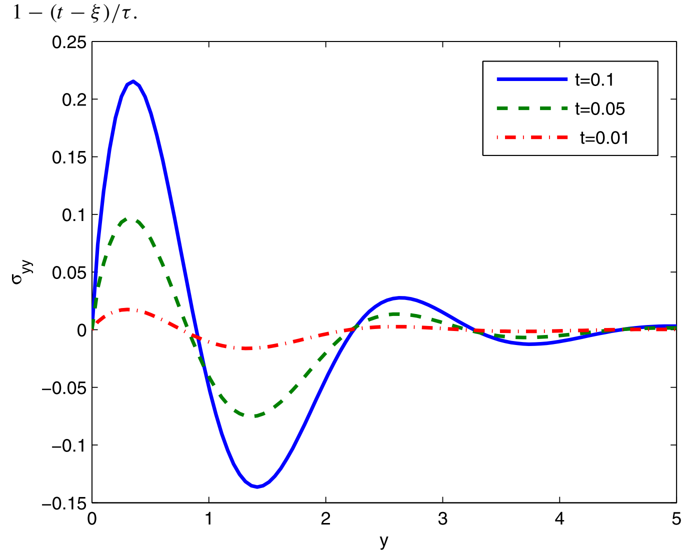

Fig. 13. The stress component σ_{yy} for different t when $\tau=0.05, K(t-\xi)=1-(t-\xi)/\tau$.

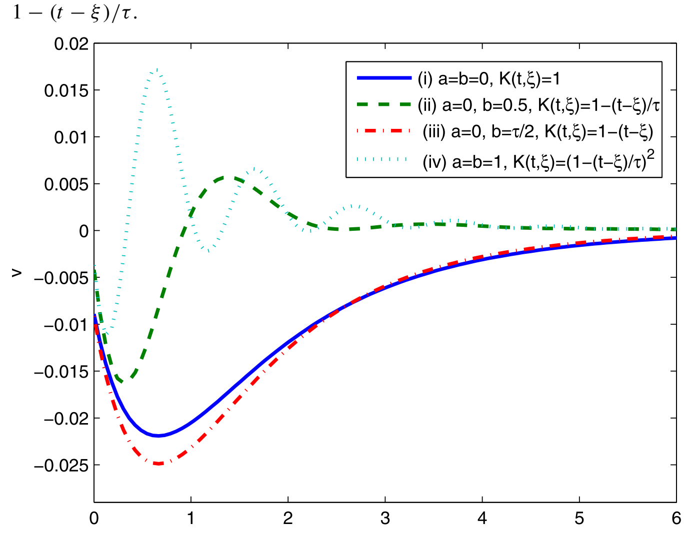

Fig. 14. The displacement component v for different forms of the kernel function $K(t-\xi)$ and time-delay $\tau=0.05$.

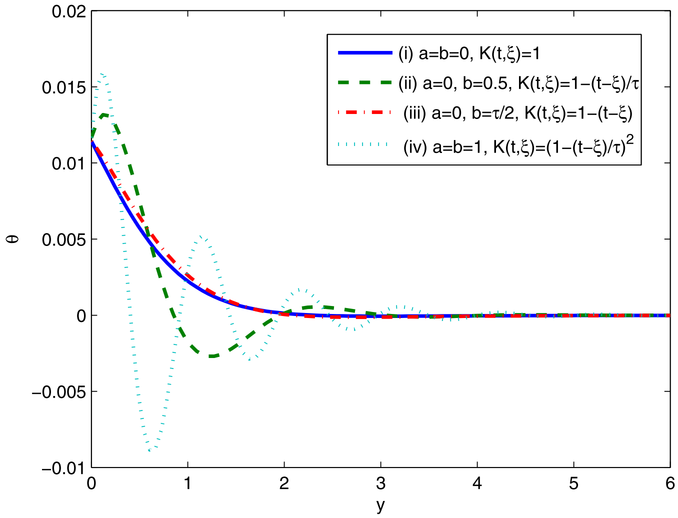

Fig. 15. The temperature θ for different forms of the kernel function $K(t-\xi)$ and time-delay $\tau = 0.05$.

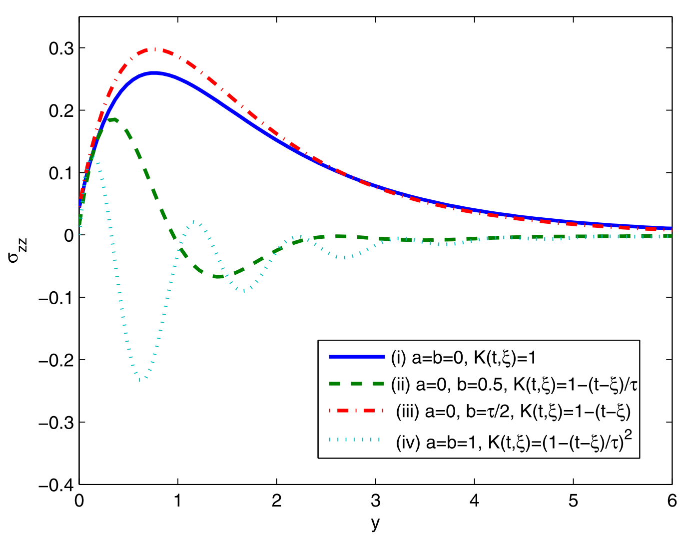

Fig. 16. The stress component σ_{zz} for different forms of the kernel function $K(t-\xi)$ and time-delay $\tau = 0.05$.

$[1-(t-\xi)/\tau]^2$ i.e. $a = b = 1$. The important phenomenon observed in these figures that the solution to all fields considered vanishes identically outside a bounded region of space surrounding the heat source at a distance from it equal to a particular value of x depending only on the choice of t and is the location of the wavefront. This demonstrates clearly the solution of new generalized case ($\tau > 0$). The waves propagate with infinite speeds, so the value of any of the functions is not identically zero (though it may be very small) for any large value of x. This result is very important since the new theory may preserve the advantage of the generalized theory, i.e. the response to the thermal and mechanical effects does not reach infinity instantaneously but remains in the bounded region of space that expands with the passing of time.

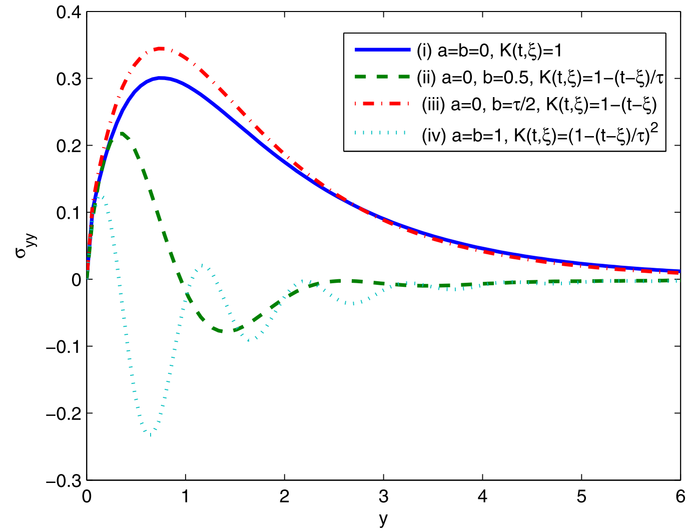

Fig. 17. The stress component σ_{yy} for different forms of the kernel function $K(t-\xi)$ and time-delay $\tau = 0.05$.

7. Concluding remarks

The main goal of this work is to introduce a new mathematical model for Fourier law of heat conduction with memory-dependent derivative and includes the magneto–thermoelastic. According to this new theory, we have to construct a new classification for materials according to a time-delay and kernel function where these variables become new indicator of its ability to conduct heat in conducting medium. This model enables us to improve the efficiency of a thermoelastic material figure-of-merit. The result provides a motivation to investigate conducting thermoelectric materials as a new class of applicable thermoelectric materials. The numerical results lead us to conclude that:

1. All the physical field quantities approach zero as the distance × approaches infinity.
2. The time-delay parameter and kernel function can be selected arbitrarily according to the necessity of the applications. They have a strong effect on the distributions of all physical field quantities, indicating its capability to conduct heat through the medium.

Funding

The authors received no financial support for the research.

Conflict of interest

The authors declare that they have no conflict of interest.

Acknowledgments

The authors would like to thank the Editor and the referees for their suggestions and comments to improve the quality of this manuscript.

References

[1] M. Biot, J. Appl. Phys. 27 (1956) 240–253.
[2] H. Lord, Y.A. Shulman, J. Mech. Phys. Solids. 15 (1967) 299–309.
[3] D.S. Chandrasekharaiah, Appl. Mech. Rev. 51 (1998) 705–729.
[4] R.B. Hetnarski, J. Ignaczak, J. Therm. Stress. 22 (1999) 451–476.
[5] L. Knopoff, J. Geophys. Res. 60 (1955) 441–456.
[6] S. Kaliski, J. Petykiewicz, Proc. Vib. Probl. 4 (1959) 1–12.
[7] G. Paria, Math. Proc. Camb. Phil. Soc. 58 (1962) 527–531.
[8] G. Paria, Adv. Appl. Mech. 10 (1967) 73–112.
[9] A. Willson, Math. Proc. Camb. Phil. Soc. 59 (1963) 483–488.
[10] M.A. Ezzat, M.I.A. Othman, Int. J. Eng. Sci. 38 (2000) 107–120.
[11] M.I.A. Othman, Multi. Mod. Mater. Struct. 5 (2009) 235–242.
[12] M.I.A. Othman, Multi. Mod. Mater. Struct. 5 (2009) 43–58.
[13] M.I.A. Othman, J. Appl. Mech. Tech. Phys. 57 (2016) 108–116.
[14] Y.Z. Povstenko, J. Therm. Stress. 28 (2005) 83–102.
[15] A.S. El-Karamany, M.A. Ezzat, Math. Mech. Solids. 16 (2011) 334–346.
[16] M.A. Ezzat, Physica B 405 (2010) 4188–4194.
[17] M.A. Ezzat, A.S. El-Karaman, Z. Angew. Math. Phys. 62 (2011) 937–952.
[18] M.A. Ezzat, A.S. El-Karaman, J. Appl. Polym. Sci. 124 (2012) 2187–2199.
[19] M. Bachher, N. Sarkar, A. Lahiri, Int. J. Mech. Sci. 89 (2014) 84–91.
[20] H.H. Sherief, Int. J. Sol. Struct. 47 (2010) 269–275.
[21] H.M. Youssef, J. Heat Transf. 132 (2010) 61301.
[22] Y.J. Yu, X.G. Tian, T.J. Lu, Eur. J. Mech. A-Solid 42 (2013) 188–202.
[23] Y.J. Yu, X.G. Tian, Q.L. Xiong, Eur. J. Mech. A-Solid 60 (2016) 238–253.
[24] J.L. Wang, H.F. Li, Comput. Math. Appl. 62 (2011) 1562–1567.
[25] Y.J. Yu, W. Hu, X.G. Tian, Int. J. Eng. Sci. 81 (2014) 123–134.
[26] M.A. Ezzat, A.S. El-Karamany, A.A. El-Bary, J. Electromagn. Wave 29 (2015) 1018–1031.
[27] M.A. Ezzat, A.S. El-Karamany, A.A. El-Bary, Int. J. Mech. Sci. 89 (2014) 470–475.
[28] N. Sarkar, D. Ghosh, A. Lahiri, Mech. Adv. Mater. Struct. (2018), doi:10.1080/15376494.2018.1432784.
[29] S. Mondal, N. Sarkar, J. Appl. Math. Mech. (2019), doi:10.1002/zamm.201800343.
[30] N. Sarkar, S. De, N. Sarkar, J. Electromagn. Wave 33 (2019) 1354–1374.
[31] N. Sarkar, A. Lahiri, Int. J. Eng. Sci. 51 (2012) 310–325.