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Citation: Phys. Plasmas **23**, 062124 (2016); doi: 10.1063/1.4954403
View online: http://dx.doi.org/10.1063/1.4954403
View Table of Contents: http://aip.scitation.org/toc/php/23/6
Published by the American Institute of Physics

PHYSICS OF PLASMAS 23, 062124 (2016)

Magnetosonic shock wave in collisional pair-ion plasma

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(Received 2 March 2016; accepted 9 June 2016; published online 24 June 2016)

Nonlinear propagation of magnetosonic shock wave has been studied in collisional magnetized pair-ion plasma. The masses of both ions are same but the temperatures are slightly different. Two fluid model has been taken to describe the model. Two different modes of the magnetosonic wave have been obtained. The dynamics of the nonlinear magnetosonic wave is governed by the Korteweg-de Vries Burgers' equation. It has been shown that the ion-ion collision is the source of dissipation that causes the Burgers' term which is responsible for the shock structures in equal mass pair-ion plasma. The numerical investigations reveal that the magnetosonic wave exhibits both oscillatory and monotonic shock structures depending on the strength of the dissipation. The nonlinear wave exhibited the oscillatory shock wave for strong magnetic field (weak dissipation) and monotonic shock wave for weak magnetic field (strong dissipation). The results have been discussed in the context of the fullerene pair-ion plasma experiments. *Published by AIP Publishing.*
[http://dx.doi.org/10.1063/1.4954403]

I. INTRODUCTION

Study of various nonlinear phenomena in pair plasma composed of positive and negative charge particle of equal masses has been exclusively studied theoretically[1–6] as well as experimentally.[7–10] In normal electron-ion plasma, space time asymmetry occurs due to the huge mass difference of electron and ion. This space-time asymmetry can be used to study the linear and nonlinear wave phenomena in short and long wave length limit. On contrary, such asymmetry in space-time collapse in pair plasma. Equality in mass makes the pair plasma space-time symmetric. This symmetry allows us to analyse new collective phenomena (linear and nonlinear) in pair plasma. Pair plasma composed of electron and positron is found to exist in astrophysical environment (pulsar magnetospheres, early universe, active galaxy, etc)[11–14] and also in inertial confinement fusion reactor using ultraintense lasers.[15] On the other hand, pair-ion plasma composed of positive and negative ions of equal masses have been developed by Oohara and Hatakeyama[16–19] by using positive and negative fullerene ions ($C_{60}^{\pm}$) in laboratory. Pair-ion plasma is used to study in nanotechnology as well as for the synthesis of dimers directly from carbon allotropes.[20]

Study of shock phenomena in plasma physics community has attracted growing attention in recent years. It is a well established idea that the shock is generated due to interplay between nonlinearity and the combined effects of dissipation and dispersion. Various authors[21–23] have studied the nonlinear shock wave phenomena in pair plasma taking the viscosity as a source of dissipation. Some authors have studied the landau damping, instability, and structures like soliton, shock, and wave modulation in pair-ion plasma, as well as electron-positron-ion plasmas.[24–29] Most of these investigations have been carried out in collision less limit. However, a collisional product C_{121} is produced when negative C_{60}^{-} ions collide with positive C_{60}^{+} ions and/or neutral fullerene in fullerene pair-ion plasma.[30,31] Thus, collisions play an important role in nonlinear collective processes of such pair-ion plasma. Though some of these investigations have been carried out in collisional limit, none of them have considered the magnetic field effect.

The aim of the present investigation is to study the nonlinear propagation of magnetosonic shock wave in a collisional pair-ion plasma composed of positive and negative fullerene ions. Taking the two fluid model for both ions (positive and negative), we have employed the well-known reductive perturbation technique (RPT) in finite amplitude wave theory. Two different modes (fast and slow) of magnetosonic wave have been observed. The propagation of magnetosonic shock wave is governed by the Korteweg-de Vries Burger (KdVB) in collisional pair-ion plasma. Due to the ion-ion collision, the dissipation occurs. In case of collisional unmagnetized pair-ion plasma, the nonlinear propagation of ion acoustic waves (IAW) has been governed by the Korteweg-de Vries (KdV) equation with a linear damping term.[26] In this case, the linear damping term appeared due to the ion-neutral collision. As the dissipation of the wave is weak, the nonlinear wave retained its solitonic shape with diminishing amplitude and width. On the other hand, collision between same species (viscosity) produced the strong dissipation and as a result both oscillatory and monotonic shock wave structures were shown to be formed.[22] Moreover, direct collision between positive (negative) and negative (positive) ions in the presence of plasma current (collective phenomena) can

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produce enormous dissipation that causes the ion acoustic shock wave in both oscillatory and monotonic form,[32] although they have investigated such results in the unmagnetized pair-ion plasma. In our case, we have studied the magnetosonic shock wave in one spatial dimension under the assumption that the external magnetic field has lied in $x-z$ plane making an angle θ with x-axis. Therefore, the propagation vector lied in perpendicular to z-axis, i.e., in $x-y$ plane. Here, the magnetic field plays an important role. For strong magnetic field, dispersion dominates over dissipation and yields an oscillatory shock structure in both modes, fast and slow. In case of weak magnetic field, dissipation dominates over dispersion and as a result monotonic shock wave structures are formed in both modes.

This paper is organized as follows: In Section II, the physical assumption and basic model is described. In Section III, the propagation characteristic of nonlinear magnetosonic wave has been discussed by deriving the KdVB equation using RPT. The numerical solutions of the KdVB equation are presented in Section IV. Finally, we have concluded our results in Section V.

II. PHYSICAL ASSUMPTION AND BASIC EQUATIONS

We have considered a homogeneous, magnetized, and fully collisional pair-ion plasma composed of positive and negative fullerene ions (C_{60}^{+} and C_{60}^{-}) (without electron). We have taken the quasi-neutral plasma, so in equilibrium, $n_{+0} = n_{-0} = n_0$ for singly charged ions, where $n_{\pm 0}$ is the equilibrium number density of positive (negative) ions. The external magnetic field lies in the $x-z$ plane making an angle θ with x axis. So the propagation vector lies in the perpendicular z axis, i.e., in the $x-y$ plane. Dissipation in the dynamical system has been taken through the collision between positive (negative) and negative (positive) fullerene ions. As per the experimental observation,[16–19] we have taken the mass of both ions as equal, $m_+ = m_- = m$(say) [where $m_\pm$ is the positive (negative) ion mass], as they are generated from the same source (fullerene ion source), whereas the temperatures are slightly different [range $(0.3-0.5)$eV], i.e., $T_+ \neq T_-$ [where $T_\pm$ is the positive (negative) ion temperature]. We define a new temperature variable $T = (T_+ + T_-)/2$.

The basic equations are momentum and continuity equations for positive $(+)$ and negative $(-)$ ion fluids in a collisional regime together with Maxwell's electromagnetic equations which can be expressed as

$$\frac{\partial n_\pm}{\partial t} + \nabla\cdot(n_\pm \mathbf{u}_\pm) = 0, \tag{1}$$

$$mn_\pm\left(\frac{\partial}{\partial t} + \mathbf{u}_\pm\cdot\nabla\right)\mathbf{u}_\pm = \pm e n_\pm\left(\mathbf{E} + \frac{1}{c}\mathbf{u}_\pm\times\mathbf{B}\right) - T_\pm\nabla n_\pm \pm m n_\pm \nu_\pm(\mathbf{u}_- - \mathbf{u}_+), \tag{2}$$

$$\nabla\times\mathbf{B} = \frac{4\pi e}{c}(n_+\mathbf{u}_+ - n_-\mathbf{u}_-), \tag{3}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla\times\mathbf{E}, \tag{4}$$

$$\nabla\cdot\mathbf{B} = 0, \tag{5}$$

where e is the magnitude of the electronic charge, $\mathbf{B}$ is the magnetic field, $\mathbf{E}$ is the electric field, $\mathbf{u}_\pm$ is the velocities, $n_\pm$ is the densities, $p_\pm$ is the pressure of positive (negative) ions, and $\nu_\pm$ is the positive(negative) ion - negative (positive) ion collision frequency. The equation of state is isotropic for both ions, i.e., $p_\pm = T_\pm n_\pm$. For the long wavelength limit, i.e., $k\rho_s \ll 1$ (where k is the wave number and ρ_s is the Larmor radius of ions), from the Poisson equation

$$\nabla\cdot\mathbf{E} = 4\pi e(n_+ - n_-), \tag{6}$$

the spatial variation of electric field may be ignored throughout the plasma, so that we can assume quasi neutrality condition, i.e., $n_+ \approx n_- = n$.

For the sake of simplicity of the basic equations (1)–(5), it is convenient to express all the variables in normalized form. For this purpose, we introduce the following normalization: $\hat{t} = \omega_c t$, $\hat{\nabla} = \rho_s\nabla$, $\hat{n} = n/n_0$, $\hat{\mathbf{u}}_\pm = \mathbf{u}_\pm/C_s$, $\hat{\mathbf{B}} = \mathbf{B}/B_0$, where B_0 is the magnitude of the magnetic field, the cyclotron frequency $\omega_c = eB_0/mc$, and the acoustic speed $C_s = (T/m)^{1/2}$. This defines the Larmor radius $\rho_s = C_s/\omega_c$. From the basic equations (1)–(5), eliminating $\mathbf{E}$ and $\mathbf{u}_-$, we have obtained the following normalized basic equations:

$$\frac{\partial \hat{n}}{\partial \hat{t}} + \hat{\nabla}\cdot(\hat{n}\hat{\mathbf{u}}) = 0, \tag{7}$$

$$\left(\frac{C_s}{V_A}\right)^2\frac{d\hat{\mathbf{u}}}{d\hat{t}} - \frac{d}{d\hat{t}}\left(\frac{1}{\hat{n}}\hat{\nabla}\times\hat{\mathbf{B}}\right) = \frac{1}{\hat{n}}(\hat{\nabla}\times\hat{\mathbf{B}})\times\hat{\mathbf{B}} + \frac{1}{\hat{n}}(\hat{\nabla}\times\hat{\mathbf{B}})\cdot\hat{\nabla} \times\left(\hat{\mathbf{u}} - 2\left(\frac{V_A}{C_s}\right)^2\frac{1}{\hat{n}}\hat{\nabla}\times\hat{\mathbf{B}}\right) - \left(\frac{C_s}{V_A}\right)^2\frac{1}{\hat{n}}\hat{\nabla}\hat{n}, \tag{8}$$

$$\frac{\partial \hat{\mathbf{B}}}{\partial \hat{t}} = \hat{\nabla}\times(\hat{\mathbf{u}}\times\hat{\mathbf{B}}) - \hat{\nabla}\times\frac{d\hat{\mathbf{u}}}{d\hat{t}} - 2\left(\frac{V_A}{C_s}\right)^2\left(\frac{\nu_+}{\omega_c}\right)\hat{\nabla}\times\left(\frac{1}{\hat{n}}\hat{\nabla}\times\hat{\mathbf{B}}\right), \tag{9}$$

$$\hat{\nabla}\cdot\hat{\mathbf{B}} = 0, \quad \frac{d}{d\hat{t}} \equiv \frac{\partial}{\partial\hat{t}} + \hat{\mathbf{u}}\cdot\hat{\nabla}, \tag{10}$$

where the positive ion velocity $\hat{\mathbf{u}}_+$ is denoted by $\hat{\mathbf{u}}$ and the speed of the Alfvén wave V_A is defined by $B_0/\sqrt{8\pi n_0 m}$. In rest part of the manuscript, we have used the normalized variables without hat for the sake of simplicity of notation.

In our present work, we have assumed the motion of the wave carried out only in one space variable, say in x, so $\partial/\partial y = \partial/\partial z = 0$. As a result, ion velocity and magnetic fields can be expressed as $\mathbf{u} = u_x(x,t)\hat{e}_x + u_y(x,t)\hat{e}_y + u_z(x,t)\hat{e}_z$ and $\mathbf{B} = B_x\hat{e}_x + B_y(x,t)\hat{e}_y + B_z(x,t)\hat{e}_z$, respectively, where B_x is a constant which can be shown easily from Equations (4) and (5). In case of 1D, it can be shown that the operator $(\nabla\times B)\cdot\nabla \equiv 0$. Therefore, using the above assumption, we have obtained the following x component normalized equations:

$$\frac{dn}{dt}+n\frac{\partial u_x}{\partial x}=0, \tag{11}$$

$$M^2\frac{du_x}{dt}+\frac{1}{n}\frac{\partial}{\partial x}\left(\frac{1}{2}\left(B_y^2+B_z^2\right)\right)+M^2\frac{1}{n}\frac{\partial n}{\partial x}=0, \tag{12}$$

$$M^2\frac{du_y}{dt}-\frac{B_x}{n}\frac{\partial B_y}{\partial x}+\frac{d}{dt}\left(\frac{1}{n}\frac{\partial B_z}{\partial x}\right)=0, \tag{13}$$

$$M^2\frac{du_z}{dt}-\frac{B_x}{n}\frac{\partial B_z}{\partial x}-\frac{d}{dt}\left(\frac{1}{n}\frac{\partial B_y}{\partial x}\right)=0, \tag{14}$$

$$\begin{aligned}&\frac{dB_y}{dt}+B_y\frac{\partial u_x}{\partial x}-B_x\frac{\partial u_y}{\partial x}-\frac{\partial}{\partial x}\left(\frac{du_z}{dt}\right)\\&-\left(\frac{2}{M^2}\right)\left(\frac{\nu_+}{\omega_c}\right)\frac{\partial}{\partial x}\left(\frac{1}{n}\frac{\partial B_y}{\partial x}\right)=0,\end{aligned} \tag{15}$$

$$\begin{aligned}&\frac{dB_z}{dt}+B_z\frac{\partial u_x}{\partial x}-B_x\frac{\partial u_z}{\partial x}+\frac{\partial}{\partial x}\left(\frac{du_y}{dt}\right)\\&-\left(\frac{2}{M^2}\right)\left(\frac{\nu_+}{\omega_c}\right)\frac{\partial}{\partial x}\left(\frac{1}{n}\frac{\partial B_z}{\partial x}\right)=0.\end{aligned} \tag{16}$$

In above equations, the operator $d/dt\equiv\partial/\partial t+u_x\cdot\nabla$ and the physical parameter $M=C_s/V_A$ is Alfvén-Mach number.

III. DERIVATION OF KORTEWEG-DE VRIES BURGERS' EQUATION

To study the propagation of nonlinear magnetosonic wave in magnetized, collisional pair-ion plasma from the basic model equations (11)–(16), we have introduced the following space and time scales:

$$\xi=\epsilon^{\frac{1}{2}}(x-\lambda t),\quad \tau=\epsilon^{\frac{3}{2}}t, \tag{17}$$

where the phase velocity λ of the propagated linear wave is normalized by acoustic speed C_s and strength of the nonlinearity is determined by the small dimensionless parameter ϵ. Here, we have assumed that the ratio between ion-ion collision frequency (ν_+) and cyclotron frequency is small which can be scaled as[33]

$$\frac{\nu_+}{\omega_c}\equiv\nu\epsilon^{\frac{1}{2}}, \tag{18}$$

to include the collisional effect and also to make the nonlinear perturbation consistent. So, the dynamical variables n, u_j, and $B_j\ (j=x,y,z)$ can be expanded in the power series of ϵ as

$$\begin{aligned}n&=1+\epsilon n^{(1)}+\epsilon^2n^{(2)}+\cdots,\\u_x&=0+\epsilon u_x^{(1)}+\epsilon^2u_x^{(2)}+\cdots,\\u_y&=0+\epsilon^{3/2}u_y^{(1)}+\epsilon^{5/2}u_y^{(2)}+\cdots,\\u_z&=0+\epsilon u_z^{(1)}+\epsilon^2u_z^{(2)}+\cdots,\\B_x&=\cos\theta,\\B_y&=0+\epsilon^{3/2}B_y^{(1)}+\epsilon^{5/2}B_y^{(2)}+\cdots,\\B_z&=\sin\theta+\epsilon B_z^{(1)}+\epsilon^2B_z^{(2)}+\cdots.\end{aligned} \tag{19}$$

Therefore, from the basic equations (11)–(16), with the help of stretched coordinate (17), scaling (18), and the perturbation expansion (19), we have obtained the following relations in the lowest powers of ϵ:

$$\begin{aligned}u_x^{(1)}-\lambda n^{(1)}&=0,\\M^2\lambda u_x^{(1)}-\sin\theta B_z^{(1)}-M^2n^{(1)}&=0,\\M^2\lambda u_y^{(1)}+\cos\theta B_y^{(1)}+\lambda\frac{\partial B_z^{(1)}}{\partial\xi}&=0,\\M^2\lambda u_z^{(1)}+\cos\theta B_z^{(1)}&=0,\\\lambda B_y^{(1)}+\cos\theta u_y^{(1)}-\lambda\frac{\partial u_z^{(1)}}{\partial\xi}&=0,\\\lambda B_z^{(1)}+\cos\theta u_z^{(1)}-\sin\theta u_x^{(1)}&=0.\end{aligned} \tag{20}$$

This set of equations in the above self-consistently determines the phase velocity of the linear wave

$$\lambda^2=\frac{(1+M^2)\pm\sqrt{(1+M^2)^2-4M^2\cos^2\theta}}{2M^2}. \tag{21}$$

In dimensional form, the above Equation (21) yields

$$\frac{\omega^2}{k^2}=\frac{\left(V_A^2+C_s^2\right)\pm\sqrt{\left(V_A^2+C_s^2\right)^2-4V_A^2C_s^2\cos^2\theta}}{2}, \tag{22}$$

which is the standard dispersion relation of magnetosonic wave propagating perpendicular to B_0 with two different modes of propagation, namely, "+" sign for fast mode and "−" sign for slow mode. In the limit $B_0\to0$, $v_a\to0$, the above magnetosonic wave (fast mode) turns into ion acoustic wave $(\omega^2=C_s^2k^2)$ for pair-ion plasma. In case of $C_s=0$, this magnetosonic wave (fast mode) turns into Alfvén wave $(\omega^2=V_A^2k^2)$ for cold plasma but in our case C_s cannot be zero.

The next highest power of ϵ yields the following second order equations:

$$\frac{\partial}{\partial\xi}\left(\lambda n^{(2)}-u_x^{(2)}\right)=\frac{\partial n^{(1)}}{\partial\tau}+\frac{\partial}{\partial\xi}\left(n^{(1)}u_x^{(1)}\right), \tag{23}$$

$$\begin{aligned}&\frac{\partial}{\partial\xi}\left(M^2\lambda u_x^{(2)}-\sin\theta B_z^{(2)}-M^2n^{(2)}\right)=M^2\frac{\partial u_x^{(1)}}{\partial\tau}\\&+M^2u_x^{(1)}\frac{\partial u_x^{(1)}}{\partial\xi}+B_z^{(1)}\frac{\partial B_z^{(1)}}{\partial\xi}-\sin\theta n^{(1)}\frac{\partial B_z^{(1)}}{\partial\xi}\\&-M^2n^{(1)}\frac{\partial n^{(1)}}{\partial\xi},\end{aligned} \tag{24}$$

$$\begin{aligned}&\frac{\partial}{\partial\xi}\left(M^2\lambda u_y^{(2)}+\cos\theta B_y^{(2)}+\lambda\frac{\partial B_z^{(2)}}{\partial\xi}\right)=M^2\frac{\partial u_y^{(1)}}{\partial\tau}\\&+M^2u_x^{(1)}\frac{\partial u_y^{(1)}}{\partial\xi}+\cos\theta n^{(1)}\frac{\partial B_y^{(1)}}{\partial\xi}+\frac{\partial^2B_z^{(1)}}{\partial\tau\partial\xi}\\&+u_x^{(1)}\frac{\partial^2B_z^{(1)}}{\partial\xi^2}+\lambda\frac{\partial}{\partial\xi}\left(n^{(1)}\frac{\partial B_z^{(1)}}{\partial\xi}\right),\end{aligned} \tag{25}$$

$$\frac{\partial}{\partial\xi}\left(M^2\lambda u_z^{(2)}+\cos\theta B_z^{(2)}\right)=M^2\frac{\partial u_z^{(1)}}{\partial\tau}$$
$$+M^2u_x^{(1)}\frac{\partial u_z^{(1)}}{\partial\xi}+\cos\theta n^{(1)}\frac{\partial B_z^{(1)}}{\partial\xi}+\lambda\frac{\partial^2 B_y^{(1)}}{\partial\xi^2}, \tag{26}$$

$$\frac{\partial}{\partial\xi}\left(\lambda B_y^{(2)}+\cos\theta u_y^{(2)}-\lambda\frac{\partial u_z^{(2)}}{\partial\xi}\right)=\frac{\partial B_y^{(1)}}{\partial\tau}$$
$$+u_x^{(1)}\frac{\partial B_y^{(1)}}{\partial\xi}+B_y^{(1)}\frac{\partial u_x^{(1)}}{\partial\xi}-\frac{\partial^2 u_z^{(1)}}{\partial\tau\partial\xi}$$
$$-\frac{\partial}{\partial\xi}\left(u_x^{(1)}\frac{\partial u_z^{(1)}}{\partial\xi}\right)-\frac{2\nu}{M^2}\frac{\partial^2 B_y^{(1)}}{\partial\xi^2}, \tag{27}$$

$$\frac{\partial}{\partial\xi}\left(\lambda B_z^{(2)}+\cos\theta u_z^{(2)}-\sin\theta u_x^{(2)}\right)=\frac{\partial B_z^{(1)}}{\partial\tau}$$
$$+u_x^{(1)}\frac{\partial B_z^{(1)}}{\partial\xi}+B_z^{(1)}\frac{\partial u_x^{(1)}}{\partial\xi}-\lambda\frac{\partial^2 u_y^{(1)}}{\partial\xi^2}-\frac{2\nu}{M^2}\frac{\partial^2 B_z^{(1)}}{\partial\xi^2}. \tag{28}$$

Finally, eliminating $n^{(2)}, u_i^{(2)}$, and $B_j^{(2)}$, where $(i = x, y, z)$ and $(j = y, z)$ from Equations (23)–(28), we have obtained the following Korteweg-de Vries Burgers' (KdVB) equation [with $n^{(1)} = \psi$]:

$$\frac{\partial\psi}{\partial\tau}+\alpha\psi\frac{\partial\psi}{\partial\xi}+\beta\frac{\partial^3\psi}{\partial\xi^3}=\gamma\frac{\partial^2\psi}{\partial\xi^2}, \tag{29}$$

where α, β, and γ are as follows:

$$\alpha=\frac{1}{\delta}[2\lambda^2(2M^2\lambda^2-M^2-1)]$$
$$+\frac{M^2\lambda^2}{\delta\sin^2\theta}(\lambda^2-1)^2(M^2\lambda^2-1), \tag{30}$$

$$\beta=\frac{\lambda^2(\lambda^2-1)}{\delta}, \tag{31}$$

and

$$\gamma=\frac{2\nu}{\delta}\lambda(\lambda^2-1), \tag{32}$$

where

$$\delta=\frac{2}{\lambda}(M^2\lambda^4-\cos^2\theta). \tag{33}$$

The nonlinear propagation of fast and slow magnetosonic wave is governed by the nonlinear evolution equation (29) in finite beta plasma. The dissipation term γ represents the Burgers' term in the Eq. (29) that is proportional to ν. The term γ varies due to the ion-ion collision. There is no Burgers' term in Eq. (29) in the absence of ion-ion collision and the Eq. (29) reduces to the KdV equation for nonlinear magnetosonic wave that possesses solitary waves. Therefore, the ion-ion collision is responsible for the Burgers' term in the Eq. (29) that possesses the magnetosonic acoustic shock solution similar to the solution obtained from viscosity.

However, the slow mode disappears in a finite beta plasma at $\theta = \pi/2$. In case of slow mode, for the square Alfvén-Mach number $M^2 \lesseqgtr 1$, $\lambda^2 \le 1$ and λ^2 attains its maximum value 1 at $\theta = 0$ for finite M. For the fast mode, $\lambda^2 \ge 1$ when $M^2 \lesseqgtr 1$ and λ^2 attains its minimum value 1 when $M \ge 1$ at $\theta = 0$. At $\theta = \pi/2$, λ^2 attains a value $1 + 1/M^2$ for finite M. Moreover, the expression δ is infinite at $\theta = \pi/2$ for the slow mode and the expression α is infinite at $\theta = 0$ for both the modes fast and slow. So that the nonlinear evolution equation (29) is not valid at $\theta = 0$ and $\theta = \pi/2$ in the above case where $\delta = 0, \infty$ and $\alpha = \infty$. Therefore, we shall exclude the points $\theta = 0$ and $\theta = \pi/2$ in our discussion to make our theory valid.

IV. MAGNETOSONIC SHOCK STRUCTURES

The Burgers' term γ in Eq. (29) implies the generation of magnetosonic shock wave in a pair-ion plasma in the presence of magnetic fields and ion-ion collision. The Eq. (29) is not completely integrable Hamiltonian system. So that the energy of Eq. (29) is not conserved. This indicates that the exact analytical solution of the Eq. (29) is not possible, though one can derive the approximate solution by using perturbation technique. However, we can study the nature of the analytical solution of Eq. (29) by using moving frame nonlinear analysis. Therefore, we have transformed the Eq. (29) into the moving wave frame $\zeta = U\tau - \xi$, where U is the shock wave velocity. Then the first integral of the transformed equation leads to the following equation:

$$\frac{d^2\psi}{d\zeta^2}=\frac{1}{\beta}\left[U\psi-\frac{\alpha}{2}\psi^2-\gamma\frac{d\psi}{d\zeta}\right], \tag{34}$$

subject to the boundary condition $\psi(\zeta)$, $\partial_\zeta\psi(\zeta)$, and $\partial_\zeta^2\psi(\zeta)$ all $\to 0$ as $|\zeta| \to \infty$. We recast the above Equation (34) in the following dynamical form:

$$\frac{d\psi}{d\zeta}=\phi,\quad \frac{d\phi}{d\zeta}=\frac{1}{\beta}\left[U\psi-\frac{\alpha}{2}\psi^2-\gamma\phi\right]. \tag{35}$$

In the $(\psi,\ \phi)$ plane, the Eq. (35) exhibits two fixed points, namely, $(\psi = 0,\ \phi = 0)$ and $(\psi = 2U/\alpha,\ \phi = 0)$. The first fixed point (0, 0) is always a saddle point. For the second fixed point, we have to analyze the nature of the point. For that, we have assumed the asymptotic behavior of the solution of the form $\sim\exp(p\zeta)$[34] of the linearized equation of Eq. (34) and obtained

$$p=\frac{\gamma}{2\beta}\left[1\pm\left(1-\frac{4U\beta}{\gamma^2}\right)^{1/2}\right].$$

From this equation, we can conclude that the fixed point $(2U/\alpha,\ 0)$ is a stable focus or stable nodes according as $\gamma^2 \lesseqgtr 4U\beta$. The stable focus corresponds to the oscillatory shock structure (dispersion dominates over dissipation) while the stable node corresponds to a monotonic shock wave (dissipation dominates over dispersion).

In order to do the numerical analysis of the simultaneous equation (35) with (0, 0) as an initial condition by the Runge-Kutta Fehlberg (RKF) method, we have taken the help of MATHEMATICA software based on finite difference scheme. To obtain the precise results in computation, we have taken the value of the following parameters as the

Alfvén-Mach number $M = 0.94$ and the normalized ion-ion collision frequency $\nu = 0.1$ (weak dissipation) for the low beta plasma (strong magnetic field). For the case of high beta plasma (weak magnetic field), we have taken $\nu = 1.0$ (strong dissipation). Since our calculation is valid in $(0, \pi/2)$ [range of θ], we have assumed an arbitrary angle $\theta = 45°$. The computational results have been depicted in Figs. 1–4. In case of weak dissipation ($\nu = 0.1$), the computational results show the oscillatory shock wave in both modes fast and slow which have been depicted in Figs. 1 and 3. We have observed from Figs. 1 and 3 that the small perturbation of the initial condition (0, 0) leads to the oscillatory shock structures transited corresponding to the second fixed point $(2U/\alpha, 0)$. On the other hand, for strong dissipation ($\nu = 1$), the results show the monotonic shock wave in both modes fast and slow as seen in Figs. 2 and 4. These figures show that the small perturbation of the initial condition (0, 0) transformed into a monotonic shock structure transited corresponding to the second fixed point $(2U/\alpha, 0)$. Note that the amplitude of both shock waves (oscillatory and monotonic) is positive as the nonlinear coefficient α is always positive.

Finally, to observe the effects of slight temperature differences on the shock structures in equal mass pair-ion plasma, we have plotted the shock strength as a function of temperature ratio $\sigma(= T_+/T_-)$. The variations of shock strength with σ are shown in Figs. 5 and 6. The shock strength (shock height) is given by the following relation:

$$\psi_{\zeta=+\infty} - \psi_{\zeta=-\infty} = \frac{2U}{\alpha}.$$

In the above, α is an implicit function of σ as can be seen from the expression (30). To obtain the explicit dependence of α on σ, we have expressed the temperature (T) and Mach number (M) in the following form:

$$T = \frac{(1+\sigma)}{2} T_- \quad \text{and} \quad M = \sqrt{\frac{1+\sigma}{2}} M_-,$$

where $M_- = C_{s-}/V_A$ and $C_{s-} = \sqrt{T_-/m}$ (negative ion acoustic speed). Inserting all these in (30), we have computed α (nonlinearity coefficient) and plotted the shock strength vs temperature ratio σ for fast and slow modes in Figs. 5 and 6, respectively. Both the figures show that the shock strength increases initially with the increase of temperature ratio (σ); at certain value of σ, it assumes a maximum and then decreases continuously with the increase of temperature ratio. This can be understood physically as follows. The particle thermal velocity increases with the increase of $T_+(T_-)$ which leads to the wave-particle interaction mechanism. This interaction prevents the formation of a shock wave and thereby reduces the shock strength in

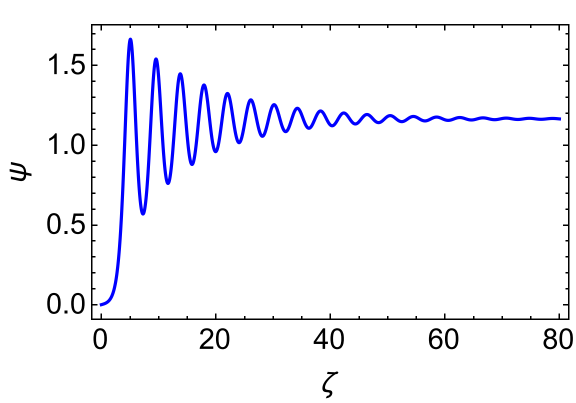

FIG. 1. Oscillatory shock structure of Eq. (35) in case of fast mode against ζ for the plasma parameter $M = 0.94$, $\theta = 45°$ $\nu = 0.1$, and $U = 1.0$, around the fixed point $(2U/\alpha, 0)$.

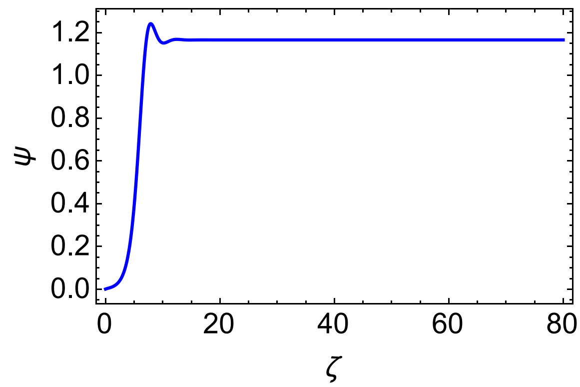

FIG. 2. Monotonic shock structure of Eq. (35) in case of fast mode against ζ for the plasma parameter $M = 0.94$, $\theta = 45°$ $\nu = 1.0$, and $U = 1.0$, around the fixed point $(2U/\alpha, 0)$.

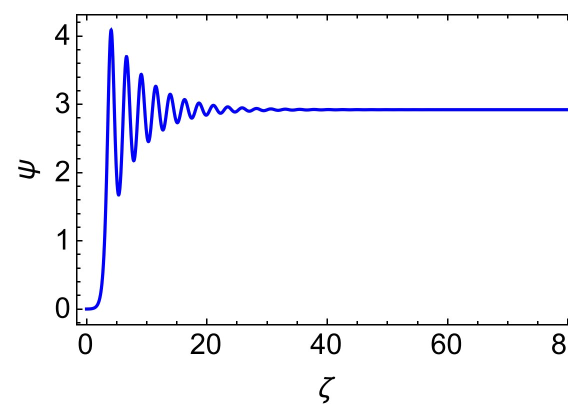

FIG. 3. Oscillatory shock structure of Eq. (35) in case of slow mode against ζ. The plasma parameters are same as in Fig. 1.

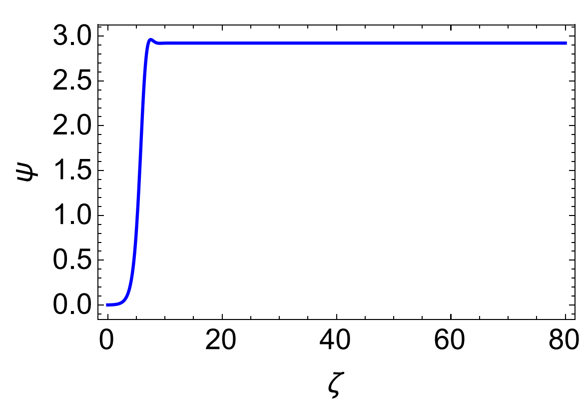

FIG. 4. Monotonic shock structure of Eq. (35) in case of slow mode against ζ. The plasma parameter are same as in Fig. 2.

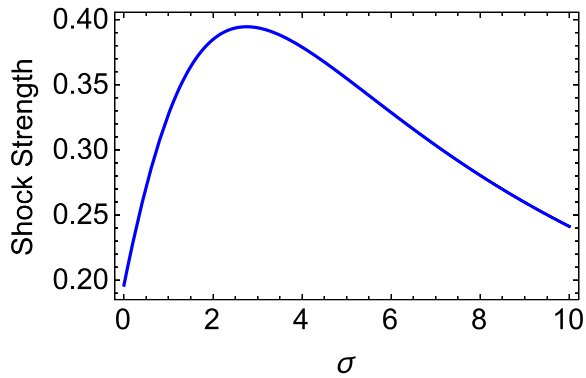

FIG. 5. Shock strength of magnetosonic shock wave in case of fast mode for different values of temperature ratio σ. The value of other plasma parameters are $M = 0.94$, $\theta = 45°$, and $U = 1.0$.

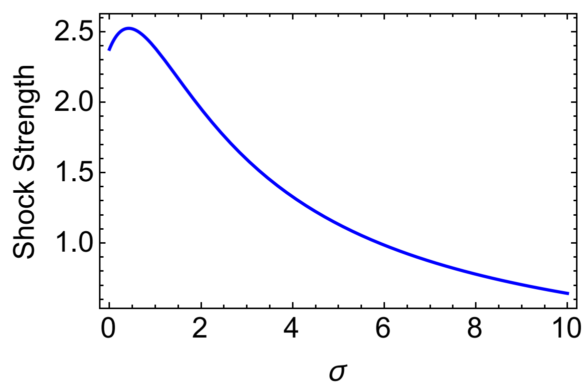

FIG. 6. Shock strength of magnetosonic shock wave in case of slow mode for different values of temperature ratio σ. The other plasma parameters are same as in Fig. 5.

pair-ion plasma. This interplay between the phase velocity and thermal velocity plays crucial role for the formation of shock wave even in electron-ion plasma.[35] As a consequence, in case of fast mode owing high phase velocity, the wave-particle interaction mechanism acts into play for relatively large value of σ that yields the maximum shock strength and after that the shock strength decreases as observed in Fig. 5. Qualitatively similar behavior is observed for slow mode as shown in Fig. 6. However, in this case, the wave-particle interaction acts into play for relatively low value of σ due to the low phase velocity of the wave. Also note that instead of the above expressions, the representations of T and M in terms of T_+ and M_+ will not change our result as we have considered the (mass) symmetric plasma.

V. CONCLUSIONS

In this paper, we have investigated the nonlinear propagation of magnetosonic shock wave in collisional pair-ion plasma consisting of positive and negative ions under the action of uniform magnetic field. The external magnetic field lies in $x - z$ plane making an angle θ with x-axis and as a result the propagation vector lies in $x - y$ plane. As per the experimental observation, the masses of both ions are taken to be equal and the temperatures are different. We have taken the two-fluid model to describe the dynamics of magnetosonic wave. Investigation shows that the magnetosonic wave has two modes: one is fast mode and another is slow mode. The propagation of nonlinear magnetosonic wave is governed by the KdVB equation. The Burgers' term represents the existence of dissipation in the system. The dissipation in this dynamical model arises through the collision between positive (negative) ions and negative (positive) ions. This indicates that ion-ion collision in the system is responsible for the Burgers' term in KdVB equation. The Burgers' term in KdVB equation possesses the physics of magnetosonic shock wave. The computational results have shown the existence of both oscillatory and monotonic shock wave depending on the strength of the dissipation in both fast and slow mode. The amplitude of the shock wave is positive as the nonlinear coefficient (α) of KdVB equation is always positive, though our investigation is valid in $(0, \pi/2)$ [range of θ]. At $\theta = 0$, the nonlinear coefficient α becomes infinitely large for both modes (fast and slow). On the other hand, magnetosonic wave disappears for the slow mode at $\theta = \pi/2$. As a result, RPT is not applicable to study the nonlinear magnetosonic shock wave when the magnetic field is directed along x-axis or along z-axis in the pair-ion plasma. Finally, we can conclude that the present investigation may be applicable to study the shock wave generation in astrophysical environment, supernova explosions, etc., and in laboratory experiments where equal mass pair-ion plasma exists.

[1] I. Kourakis, A. Esfandyari-Kalejahi, M. Mehdipoor, and P. K. Shukla, Phys. Plasmas **13**, 052117 (2006).
[2] U. A. Mofiz, Phys. Rev. A **40**, 2203 (1989).
[3] G. P. Zank and R. G. Greaves, Phys. Rev. E **51**, 6079 (1995).
[4] D. H. E. Dubin, Phys. Rev. Lett. **92**, 195002 (2004).
[5] N. Iwamoto, Phys. Rev. E **47**, 604 (1993).
[6] P. K. Shukla, B. Eliasson, and L. Stenflo, Phys. Rev. E **84**, 037401 (2011).
[7] C. M. Surko, M. Leventhal, and A. Passner, Phys. Rev. Lett. **62**, 901 (1989).
[8] R. G. Greaves, M. D. Tinkle, and C. M. Surko, Phys. Plasmas **1**, 1439 (1994).
[9] H. Boehmer, M. Adams, and N. Rynn, Phys. Plasmas **2**, 4369 (1995).
[10] M. Amoretti, C. Amsler, G. Bonomi, A. Bouchta, P. D. Bowe, C. Carraro, C. L. Cesar, M. Charlton, M. Doser, V. Filippini, A. Fontana *et al.*, Phys. Rev. Lett. **91**, 055001 (2003).
[11] F. C. Michel, *Theory of Neutron Star Magnetospheres* (Chicago University Press, Chicago, 1991).
[12] P. Goldreich and W. H. Julian, Astrophys. J. **157**, 869 (1969).
[13] M. J. Rees, in *The Early Universe*, edited by G. W. Gibbons, S. W. Hawking, and S. Siklas (Cambridge University Press, Cambridge, 1983).
[14] H. R. Miller and P. J. Witta, *Active Galactic Nuclei* (Springer-Verlag, Berlin 1987), p. 202.
[15] E. P. Liang, S. C. Wilks, and M. Tabak, Phys. Rev. Lett. **81**, 4887 (1998).
[16] W. Oohara and R. Hatakeyama, Phys. Rev. Lett. **91**, 205005 (2003).
[17] W. Oohara, D. Date, and R. Hatakeyama, Phys. Rev. Lett. **95**, 175003 (2005).
[18] W. Oohara, Y. Kuwabara, and R. Hatakeyama, Phys. Rev. E **75**, 056403 (2007).
[19] W. Oohara and R. Hatakeyama, Phys. Plasmas **14**, 055704 (2007); R. Hatakeyama and W. Oohara, Phys. Scr. **T116**, 101 (2005).
[20] P. K. Shukla and L. Stenflo, Phys. Plasmas **12**, 044503 (2005).
[21] W. Masood and H. Rizvi, Phys. Plasmas **19**, 012119 (2012).
[22] S. Mahmood and H. Ur-Rehman, Phys. Plasmas **17**, 072305 (2010).
[23] W. Masood, N. Jehan, and A. M. Mirza, Phys. Plasmas **17**, 032314 (2010).
[24] I. Kourakis, F. Verheest, and N. F. Cramer, Phys. Plasmas **14**, 022306 (2007).

[25]A. Adak, S. Ghosh, and N. Chakrabarti, Phys. Plasmas **21**, 092120 (2014).
[26]S. Ghosh, A. Adak, and M. Khan, Phys. Plasmas **21**, 012303 (2014).
[27]W. M. Moslem and P. K. Shukla, Phys. Lett. A **362**, 463–467 (2007).
[28]S. Ghosh and R. Bharuthram, Astrophys. Space. Sci. **331**, 163 (2011).
[29]A. Esfandyari-Kalejahi, I. Kourakis, and P. K. Shukla, Phys. Plasmas **13**, 122310 (2006).
[30]W. Oohara, H. Iwata, D. Date, and R. Hatakeyama, Thin Solid Films **475**, 49 (2005); W. Oohara and R. Hatakeyama, *ibid.* **435**, 280 (2003).
[31]W. Oohara, H. Iwata, and R. Hatakeyama, Surf. Coat. Technol. **201**, 5446 (2007).
[32]A. Adak, S. Ghosh, and N. Chakrabarti, Phys. Plasmas **22**, 102307 (2015).
[33]M. S. Janaki, B. Dasgupta, M. R. Gupta, and B. K. Som, Phys. Scr. **44**, 203 (1991).
[34]V. I. Karpman, *Nonlinear Waves in Dispersive Media* (Pergamon, London, 1975).
[35]H. K. Andersen, N. D'Angelo, P. Michelsen, and P. Nielsen, Phys. Rev. Lett. **19**, 149 (1967); Phys. Fluids **11**, 606 (1968).