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Magnetization and other characteristics of exact relativistic dispersion of circularly polarized radiation in a magnetized plasma

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A relativistic determination has been made of the zero frequency magnetic moment field [the inverse Faraday effect (IFE)], synchrotron radiation, and other consequences of the exact dispersion relation of a circularly polarized wave propagating in a two-component, cold, magnetized plasma. The relativistically correct Faraday angle of rotation of the plane of polarization of the electromagnetic (EM) wave due to the variation of the dispersion rates of the right- and left-circularly polarized components of the wave has been obtained. For Alfvén wave frequencies (wave frequency less than the ion gyrofrequency), a relativistically correct analysis of the interesting new features which appear due to the predominance of the wave-induced ion dynamics over electron motion is also presented. © *1996 American Institute of Physics.* [S1070-664X(96)01301-3]

I. INTRODUCTION

A circularly polarized intense laser radiation propagating in a field-free electron plasma drives the electrons in circular orbits. The consequent magnetic moment field, the inverse Faraday effect (IFE), is either parallel or antiparallel to the laser beam depending on the sign of the charged species. Including the relativistic variation of mass for electron motion, it was first studied by Steiger and Woods[1] in unmagnetized plasma. A nonrelativistic treatment of IFE by a microwave radiation was given by Pomeau and Quemada[2] in a collisionless electron plasma. Deschamps *et al.*[3] have observed this magnetic field experimentally. In some problems of wave–plasma interaction, and wave–wave interaction in plasma, the zero frequency nonrelativistic part of this field has been evaluated.[4–7]

The response of a plasma to a high intensity radiation ($\approx 10^{17}$ W/cm^2) field is nonlinear, and produces a substantial diamagnetic field as well as relativistic electrons. The circular polarization is the only mode in which electromagnetic energy can propagate as pure transverse waves, and even including the relativistic variation of mass, it does not couple parametrically with other waves. The charged constituents have magnetic moment due to their wave-induced circular velocity. Depending on the sign of the charge, it is antiparallel or parallel to the direction of wave propagation.[1] Moreover, the organized generation of local waves of circular polarization and circular currents of charges as well as masses, all circles having a common local axis, is a strong cause of whirls of turbulent processes.

For finding the relativistic characteristics of waves of Alfvén wave frequencies (wave frequencies smaller than the ion gyration frequency) in a two-component plasma, an external magnetic field is assumed to exist along the direction of wave propagation. Inclusion of ion dynamics is important because, in the low frequency regime, the massive ions respond dynamically more effectively than the mobile electrons; both species experience the gyratory motion with different radius and sense of gyration. This circular motion of charges is mixed up with the circular motion under the influence of the incident circularly polarized electromagnetic (EM) radiation. The orbit radii of the electrons and ions depend on their respective gyration frequency, radiation intensity, and frequency of the pump field. The induced zero frequency IFE field of magnetic moment lying along the direction of wave propagation modifies the background magnetic field of the plasma and the gyration of charges, and changes the wave-induced orbital motion due to a feedback mechanism. Since its magnitude is large, it effectively freezes the charged particles along the lines of forces and enhances synchrotron radiation from gyrating charges. This radiation, emitted by an accelerated (rotating) relativistic particle, is equivalent to that emitted by a particle moving instantaneously at constant speed on an appropriate circular path. For periodic circular motion the frequency spectrum of the synchrotron radiation is a discrete integral multiple of the fundamental frequency.[8]

For high frequencies, synchrotron radiation dominates over the bremsstrahlung radiation loss. For low frequencies, the mechanisms of dissipation of energy are different from those for high frequencies. For Alfvén wave frequencies, the loss from bremsstrahlung could be more than that from synchrotron radiation. Also, since collisional absorption of wave power by a medium is much larger than that from other sources of loss, by Kirchhoff's laws the bremsstrahlung loss should also be significant in this case. Moreover, in the absence of a strong static magnetic field the bremsstrahlung process is known to be dominant at low frequencies.[9] The semiclassical method of evolution of the bremsstrahlung loss followed by Steiger and Woods,[1] and the literature cited by these authors, are not helpful for extension of our study. So we could not resolve the uncertainty about the conclusion on bremsstrahlung loss of Alfvén waves.

When a linearly polarized wave splits up into a left-circularly polarized (LCP) and a right-circularly polarized (RCP) wave by a static magnetic field along the direction of wave propagation, the LCP and RCP components move with different phase velocities, and so the plane of polarization of the linearly polarized wave rotates through the angle of Faraday rotation. The relativistically correct Faraday rotation angle in the two-component plasma, including that at Alfvén wave frequencies, has been calculated.

The IFE field of zero frequency in the Alfvén wave limit was earlier obtained by a nonrelativistic treatment.[4] In that paper, we investigated the interaction of both propagating and standing Alfvén waves with a two-component cold magnetized plasma. The feature obtained here is that the cutoff value of wave frequency in the low frequency Alfvén wave limit disappears which is significant for high frequency cases, since beyond this cutoff limit high frequency transverse waves cannot propagate. Moreover, the importance of wave-induced ion dynamics over electron dynamics is easily understandable from our model, which is not realizable from single fluid magnetohydrodynamic (MHD) theory.

In Sec. II, we have obtained the relativistically correct exact dispersion relation in a two-component plasma of a wave of circular polarization propagating along the direction of the ambient magnetic field. Its phase velocity, cutoff frequency, and Poynting flux have been calculated in Sec. III. The relativistic IFE magnetization of the zero frequency and the consequent synchrotron radiation have been evaluated in Sec. IV; and in Sec. V, the relativistically correct angle of Faraday rotation has been calculated.

There are several circular motions in a multicomponent plasma in the presence of circularly polarized waves propagating along the direction of the ambient field. The radii of gyration due to the ambient magnetic field depend on the thermal motion. The radii of the wave-induced circular motion depend on the wave field amplitude. Both gyration and circular motion are in opposite senses for charges of opposite sign. Also, since mass of an electron is much less than that of an ion, mass circulation is dominant in both circular and gyratory motion of ions, and charge circulation is dominant in those motions of the electrons. So, waves of Alfvén wave frequencies are the result of interaction between circular kinetic energy and circular current of charges in opposite senses.

II. THE BASIC EQUATIONS AND THE EXACT DISPERSION RELATION

The basic equations are

$$\dot{\mathbf{p}}_\alpha = q_\alpha \mathbf{E} + \frac{q_\alpha}{c}(\mathbf{v}_\alpha \times \mathbf{H}), \tag{1}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \tag{2}$$

$$\nabla \times \mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j}, \tag{3}$$

$$\nabla \cdot \mathbf{E} = 4\pi\rho, \tag{4}$$

$$\nabla \cdot \mathbf{H} = 0, \tag{5}$$

where $\rho = \Sigma_{\alpha=e,i} N_\alpha q_\alpha$ and $\mathbf{j} = \Sigma_{\alpha=e,i} N_\alpha q_\alpha \mathbf{v}_\alpha$ are the charge density and the current density. The electrons and ions have charge $q_e(=-e)$ and $q_i(=e)$ per particle, number densities N_e, N_i and velocities $\mathbf{v}_e$ and $\mathbf{v}_i$, respectively.

The circularly polarized EM wave, propagating along the z axis, of a rectangular Cartesian coordinate system, has the electric field vector

$$\mathbf{E} = E(\cos\theta, \lambda \sin\theta, 0), \tag{6}$$

where $\theta = kz - \omega t$, k and ω are wave number and wave frequency, λ (helicity) is $+1$ for left-circular polarization and -1 for right-circular polarization. The magnetic vector of this field and the current $\mathbf{j}$, following from (2) and (3), are

$$\mathbf{H} = nE(-\lambda \sin\theta, \cos\theta, 0), \tag{7}$$

$$\mathbf{j} = \frac{E\omega}{4\pi}(n^2-1)(\sin\theta, -\lambda\cos\theta, 0), \tag{8}$$

where $n(=kc/\omega)$ is the refractive index of the plasma medium. Since the current density $\mathbf{j}(=\Sigma_{\alpha=e,i} N_\alpha q_\alpha \mathbf{v}_\alpha)$ is generated by the wave-field-induced motion of electrons and ions, the wave drives the charges into circular orbits; so that we can write

$$\mathbf{v}_\alpha = v_\alpha^0(\sin\theta, -\lambda\cos\theta, 0), \tag{9}$$

where v_α^0 is the amplitude of the velocity vector $\mathbf{v}_\alpha$ of the αth species of charged particles.

In presence of the ambient magnetic field,

$$\mathbf{H}_0 = (0,0,H_0), \tag{10}$$

the equations of motion for electrons and ions are

$$\dot{\mathbf{p}}_\alpha = q_\alpha \mathbf{E} + q_\alpha/c(\mathbf{v}_\alpha \times \mathbf{H}) + q_\alpha/c(\mathbf{v}_\alpha \times \mathbf{H}_0). \tag{11}$$

For circular polarization (7) and (9) show that

$$\mathbf{v}_\alpha \times \mathbf{H} = 0, \tag{12}$$

Substituting (6), (9), and (10) in (11) and then integrating with respect to time, we obtain the expression for the momentum of the αth species of the charged particles

$$\mathbf{p}_\alpha = -q_\alpha/\omega(E - v_\alpha^0 H_0/c)(\sin\theta, -\lambda\cos\theta, 0), \tag{13}$$

$$= m_\alpha \gamma_\alpha \mathbf{v}_\alpha, \tag{14}$$

where $\gamma_\alpha[=1/(1-v_\alpha^2/c^2)^{1/2}]$ is the relativistic factor of the particles of the species α. Since, $\mathbf{v}_\alpha$ is circularly polarized with amplitude v_α^0, we have

$$v_\alpha^2 = \mathbf{v}_\alpha \cdot \mathbf{v}_\alpha = v_\alpha^{0^2}, \tag{15}$$

$$\gamma_\alpha = \gamma_\alpha^0 = \frac{1}{(1 - v_\alpha^{0^2}/c^2)^{1/2}}, \tag{16}$$

so that γ_α is a constant in this case.

Substituting (9) and (16) in (14) and equating it with (13) for $\beta_\alpha^0(=v_\alpha^0/c)$, we obtain the biquadratic equation

$$\beta_\alpha^{0^4} + b\beta_\alpha^{0^3} + c\beta_\alpha^{0^2} + d\beta_\alpha^0 + e = 0, \tag{17a}$$

where $b = -2\lambda\delta$, $c = \delta^2 - 1 + 1/y_\alpha^2$, $d = 2\lambda\delta$, $e = -\delta^2$,

$$\delta = \frac{E}{H_0}, \quad y_\alpha = \frac{Q_\alpha}{\omega}, \quad \Omega_\alpha = \frac{q_\alpha H_0}{m_\alpha c}, \quad \beta_\alpha^0 = \frac{v_\alpha^0}{c}. \tag{17b}$$

Ferrari's method of solving this biquadratic equation gives

$$\beta^0_{\alpha 1,2}=\frac{1}{2}\left[\lambda\,\delta+\left(1+\xi_\alpha-\frac{1}{Y_\alpha^2}\right)^{1/2}\right]\pm\frac{1}{2}\left[\delta^2+\left(1+\xi_\alpha-\frac{1}{Y_\alpha^2}\right)\right.$$
$$\left.+2\lambda\,\delta\left(1+\xi_\alpha-\frac{1}{Y_\alpha^2}\right)^{1/2}-2\xi_\alpha+2\xi_\alpha\left(1+\frac{4\delta^2}{Y_\alpha^2}\right)^{1/2}\right]^{1/2}, \tag{18a}$$

$$\beta^0_{\alpha_{3,4}}=\frac{1}{2}\left[\lambda\,\delta-\left(1+\xi_\alpha-\frac{1}{Y_\alpha^2}\right)^{1/2}\right]\pm\frac{1}{2}\left[\delta^2+\left(1+\xi_\alpha-\frac{1}{Y_\alpha^2}\right)\right.$$
$$\left.-2\lambda\,\delta\left(1+\xi_\alpha-\frac{1}{Y_\alpha^2}\right)^{1/2}-2\xi_\alpha-2\xi_\alpha\left(1+\frac{4\delta^2}{Y_\alpha^2}\right)^{1/2}\right]^{1/2}, \tag{18b}$$

where

$$\xi_\alpha=\frac{1}{3}\left(\delta^2-1+\frac{1}{Y_\alpha^2}\right)\left[1+\left(1+\frac{27\delta^2/Y_\alpha^2}{[\delta^2-1+(1/Y_\alpha^2)^3]}\right)^{1/3}\right.$$
$$\times\left(1+\frac{2\delta/Y_\alpha}{\{\frac{1}{27}[\delta^2-1+(1/Y_\alpha^2)]^3+(\delta^2/Y_\alpha^2)\}^{1/2}}\right)^{1/3}$$
$$+\left(1+\frac{27\delta^2/Y_\alpha^2}{[\delta^2-1+(1/Y_\alpha^2)]^3}\right)^{1/3}$$
$$\left.\times\left(1-\frac{2\delta/Y_\alpha}{\{\frac{1}{27}[\delta^2-1+(1/Y_\alpha^2)]^3+(\delta^2/Y_\alpha^2)\}^{1/2}}\right)^{1/3}\right]. \tag{18c}$$

The exact dispersion relation, which depends on the wave intensity, external magnetic field, wave frequency, and gyration frequency of charged particles, is therefore given by

$$k^2c^2-\omega^2=\frac{4\pi\omega c}{E}\,(N_e q_e\beta_e^0+N_i q_i\beta_i^0). \tag{19}$$

III. SOME PROPAGATION CHARACTERISTICS

The exact dispersion relation (19) can be expressed in the form

$$k^2c^2=\omega^2\left(1-\sum_{\alpha=e,i}\frac{\omega_{p_\alpha}^2}{\omega^2}\left|\frac{r_\alpha^0}{a_\alpha}\right|\right), \tag{20}$$

where

$$\omega_{p_\alpha}=\left(\frac{4\pi N_\alpha q_\alpha^2}{m_\alpha}\right)^{1/2}\quad\text{and}\quad a_\alpha=\frac{q_\alpha E}{m_\alpha\omega^2} \tag{21}$$

are the plasma frequency and the field-induced displacement of the αth species of the charged particles, $r_\alpha^0(=v_\alpha^0/\omega)$ is the amplitude (radius) of the wave-induced circular motion, and

$$\mathbf{r}_\alpha=r_\alpha^0(\cos\,\theta,\lambda\,\sin\,\theta,0). \tag{22}$$

The phase velocity and group velocity are

$$v_p=\frac{\omega}{k}=\frac{c}{[1-\Sigma_{\alpha=e,i}(\omega_{p_\alpha}^2/\omega^2)|r_\alpha^0/a_\alpha|]^{1/2}}, \tag{23a}$$

$$v_g=\frac{d\omega}{dk}=c\left(1-\sum_{\alpha=e,i}\frac{\omega_{p_\alpha}^2}{\omega^2}\left|\frac{r_\alpha^0}{a_\alpha}\right|\right)^{1/2}. \tag{23b}$$

Evidently, $v_p>c$, $v_g<c$. The wave reflects back from the region of ionization where

$$\omega=\left(\sum_{\alpha=e,i}\omega_{p_\alpha}^2\left|\frac{r_\alpha^0}{a_\alpha}\right|\right)^{1/2}. \tag{24}$$

This critical cutoff frequency depends on the intensity of the wave, ambient magnetic field, and the gyration frequency of charges. The intensity of the laser radiation (the magnitude of the Poynting vector) is

$$I=|\mathbf{s}|=\frac{c}{4\pi}\,|\mathbf{E}\times\mathbf{H}|=\frac{c}{4\pi}\left(1-\sum_{\alpha=e,i}\frac{\omega_{p_\alpha}^2}{\omega^2}\left|\frac{r_\alpha^0}{a_\alpha}\right|\right)^{1/2}E^2, \tag{25}$$

where the orbit radius r_α^0 is obtained by making use of (6) and (7) in (18a)–(18c) and (9).

IV. IFE MAGNETIZATION AND SYNCHROTRON RADIATION

A. IFE magnetization

The expressions for the induced magnetic moment and field (the inverse Faraday effect) are

$$\boldsymbol{\mu}_\alpha=\frac{1}{2c}\sum_{\alpha=e,i}(\mathbf{r}_\alpha\times\mathbf{j}_\alpha), \tag{26}$$

$$\mathbf{H}_{\text{in}}=4\pi\sum_{\alpha=e,i}N_\alpha\boldsymbol{\mu}_\alpha. \tag{27}$$

Substituting (26) in (27) we obtain the static (zero frequency) field,

$$H_{\text{in}}=2\pi e(N_i r_i^0\beta_i^0-N_e r_e^0\beta_e^0) \tag{28}$$

parallel to the direction of wave propagation. It modifies the equation of motion of charges and a feedback process is developed which freezes all the charges along their lines of force.

B. Synchrotron radiation

Energy loss due to synchrotron radiation[1] is

$$P_{\text{syn}}=\frac{2e^2\omega^4}{3c^3}\,(\gamma_i^{0^4}r_i^{0^2}+\gamma_e^{0^4}r_e^{0^2}), \tag{29}$$

where r_α^0, γ_α^0 $(\alpha=e,i)$ have been defined in Secs. II and III. It is a process of attenuation of the incident EM wave. It is beamed in a narrow cone in the direction of the velocity vector, and is seen by the observer as a short pulse of radiation as the search light beam sweeps across the observation point. Such radiations have been observed in Crab Nebula, sunspots, and from the particle radiation belts of Jupiter. For the Crab Nebula the radiation spectrum extends from radio frequencies into the extreme ultraviolet region, and shows very strong polarization.

V. RELATIVISTICALLY CORRECT FARADAY ROTATION

The difference of dispersion rates between left- and right-circular components created by a static magnetic field determines the Faraday rotation angle

$$\psi=\tfrac{1}{2}(k_+-k_-)z, \tag{30}$$

where k_+ and k_- are the wave numbers of the LCP and RCP components of the wave. The relativistically correct Faraday rotation angle, obtained from the exact dispersion relation of the circularly polarized wave,

$$k^2c^2=\omega^2-\sum_\alpha \omega_{p_\alpha}^2\left|\frac{r_\alpha^0}{a_\alpha}\right| \tag{31}$$

is

$$\psi\approx-\frac{z}{4}\sum_\alpha \frac{x_\alpha}{a_\alpha}(|\beta_\alpha^0|_{\lambda=+1}-|\beta_\alpha^0|_{\lambda=-1}), \tag{32}$$

where

$$k_+=\frac{\omega}{c}\left(1-\sum_\alpha x_\alpha\left|\frac{r_\alpha^0}{a_\alpha}\right|\right)^{1/2}_{\lambda=+1}, \tag{33a}$$

$$k_-=\frac{\omega}{c}\left(1-\sum_\alpha x_\alpha\left|\frac{r_\alpha^0}{a_\alpha}\right|\right)^{1/2}_{\lambda=-1}, \tag{33b}$$

$x_\alpha = \omega_{p_\alpha}^2/\omega^2$ and $r_\alpha^0[=(c/\omega)\beta_\alpha^0]$ is obtained from (18a)–(18c). For the LCP $\lambda=+1$ and $\lambda=-1$ for the RCP components of the wave. Substituting (18a) and (18b) in (33) we get

$$\begin{aligned}\psi\simeq&-\frac{z}{8}\sum_\alpha \frac{x_\alpha}{a_\alpha}\Bigg\{2\delta\pm\Bigg[\delta^2+\left(1+\xi_\alpha-\frac{1}{Y_\alpha^2}\right)\\&+2\delta\left(1+\xi_\alpha-\frac{1}{Y_\alpha^2}\right)^{1/2}-2\xi_\alpha+2\xi_\alpha\left(1+\frac{4\delta^2}{Y_\alpha^2}\right)^{1/2}\Bigg]^{1/2}\\&\mp\Bigg[\delta^2+\left(1+\xi_\alpha-\frac{1}{Y_\alpha^2}\right)-2\delta\left(1+\xi_\alpha-\frac{1}{Y_\alpha^2}\right)^{1/2}-2\xi_\alpha\\&+2\xi_\alpha\left(1+\frac{4\delta^2}{Y_\alpha^2}\right)^{1/2}\Bigg]^{1/2}\Bigg\}.\end{aligned} \tag{34}$$

Due to Faraday rotation from splitting of dispersion of the LCP and RCP parts of the wave, these polarization components are destroyed quickly and this solution is no longer valid.

VI. ALFVÉN WAVE FREQUENCIES

Alfvén waves are the low frequency limit (wave frequency smaller than the ion gyrofrequency) of transverse waves propagating parallel to an ambient field $\mathbf{H}_0$ in a two-component plasma. The gyration of the charges of both signs in the presence of the static magnetic field, and the ordered gyratory motion under the influence of the RCP ($\lambda=-1$) and LCP ($\lambda=+1$) components, couple with both these parts of the wave for the waves of Alfvén wave frequencies.

Since the EM wave features depend on the dispersive magnetic field which is basically relativistic in nature,[10] the relativistic nonlinear aspects of Alfvén waves in plasmas should not be ignored in the computations of the nonlinear effects. But the relativistic mass effects are completely lost when Alfvén waves, including their nonlinear modifications, are evaluated with the help of the field equations of the magnetohydrodynamics continuum approximation, for which the working relativistic generalization is not available. Also, then, other consequences like the generation of the induced magnetization from Alfvén waves do not follow directly. So, the Alfvén wave characteristics should be evaluated from characteristics of transverse waves in a two-component magnetized cold plasma model where the particle dynamical treatment is possible in the low frequency limit $\omega\ll(\Omega_e,\Omega_i)$ and $E\ll H_0$. Then the terms involving the third and higher powers of $\delta(=E/H_0)$ are neglected. Moreover, in the wave characteristics evaluated in Secs. III–V, we put $r_\alpha^0=(c/\omega)\beta_\alpha^0$. The exact dispersion relation being

$$k^2c^2-\omega^2=\frac{ec\omega}{E}(4\pi N_i\beta_i^0-4\pi N_e\beta_e^0), \tag{35}$$

the roots of the biquadratic equation (17a) with (17b) in the Alfvén wave frequency limit are given by

$$\begin{aligned}\beta_{\alpha_{1,2}}^0=&\frac{q_\alpha a_\alpha\omega^2}{2C_{\mathrm{A}}^2\Sigma_{\alpha=e,i}m_\alpha E\omega_{p_\alpha}^2}[\lambda H_0E+H_0^2(1+\xi_\alpha)^{1/2}]\\&\pm\frac{q_\alpha a_\alpha H_0\omega^2}{2C_{\mathrm{A}}^2\Sigma_{\alpha=e,i}m_\alpha E\omega_{p_\alpha}^2}[E^2+H_0^2+H_0^2\xi_\alpha\\&+2\lambda EH_0(1+\xi_\alpha)^{1/2}-2\xi_\alpha H_0^2+2H_0(4E^2\\&+H_0^2\xi_\alpha^2)^{1/2}]^{1/2},\end{aligned} \tag{36a}$$

$$\begin{aligned}\beta_{\alpha_{3,4}}^0=&\frac{q_\alpha a_\alpha\omega^2}{2C_{\mathrm{A}}^2\Sigma_{\alpha=e,i}m_\alpha E\omega_{p_\alpha}^2}[\lambda H_0E-H_0^2(1+\xi_\alpha)^{1/2}]\\&\pm\frac{q_\alpha a_\alpha H_0\omega^2}{2C_{\mathrm{A}}^2\Sigma_{\alpha=e,i}m_\alpha E\omega_{p_\alpha}^2}[E^2+H_0^2+H_0^2\xi_\alpha\\&-2\lambda EH_0(1+\xi_\alpha)^{1/2}-2\xi_\alpha H_0^2-2H_0\\&\times(4E^2+H_0^2\xi_\alpha^2)^{1/2}]^{1/2},\end{aligned} \tag{36b}$$

where

$$\begin{aligned}\xi_\alpha=&\frac{1}{3H_0^2}(E^2-H_0^2)+\frac{1}{3H_0^{2/3}}(3E^2-H_0^2)^{1/3}\\&\times\left[\left(1+\frac{6E(1/Y_\alpha)}{[E^2-(H_0^2/3)]^{1/2}}\right)^{1/3}\right.\\&\left.+\left(1-\frac{6E(1/Y_\alpha)}{[E^2-(H_0^2/3)]^{1/2}}\right)^{1/3}\right],\end{aligned}$$

$$C_{\mathrm{A}}^2=\frac{H_0^2}{4\pi\rho},\quad \rho=\sum_{\alpha=e,i}N_\alpha m_\alpha. \tag{36c}$$

Expressions for the phase velocity, group velocity, the cutoff frequency, the induced magnetization, and synchrotron radiation, and also its Poynting flux follow easily from the relations (23), (24), (28), (29), and (25), where the values of β_i^0

and β_e^0 obtained in (36a) should be used only for the Alfvén wave frequency limit. The relativistically correct angle of Faraday rotation is

$$\Psi \approx \frac{eH_0}{4C_A^2 \Sigma_\alpha m_\alpha} \sum_\alpha \left(1 \pm \frac{1-\frac{1}{4}\xi_\alpha^2}{1-\xi_\alpha} \right). \tag{37}$$

It is interesting to note that Ψ is inversely proportional to the ambient magnetic field $|\mathbf{H}_0|$.

VII. NUMERICAL ESTIMATION

In sunspots the magnetic pressure is very high compared to the radiation pressure because of the presence of high ambient magnetic field, approximately 2000 G. Its material density being 10^{-7} g/cc, electron and ion gyration frequencies are of the order of 10^2 and 10^{-1} Hz, respectively. So waves of the Alfvén wave frequency range, propagating there should have frequencies almost of the order of 10^{-2} Hz. The relativistic quantity $\beta^0 = v^0/c$ calculated from (18a)–(18c) has the value 0.5. Thus the charged particles are moving with relativistic velocity $\sim 10^{10}$ cm/s. there. The induced magnetic field calculated from (28) for an Alfvén wave frequency is approximately thousand megagauss.

VIII. CONCLUSIONS

Relativistically correct motion of plasma constituents induced by a circularly polarized high-intensity electromagnetic wave field in the presence of an external magnetic field remains circular with gyration radii depending on the radiation intensity, wave frequency, and the ambient magnetic field. This wave-induced circular motion of charged constituents induces a zero frequency magnetic moment field along the direction of wave propagation that enhances the synchrotron radiation by relativistically moving charged particles. A relativistically correct derivation of the angle of Faraday rotation, which occurs because its LCP and RCP components move with different phase velocities parallel to the static magnetic field, have been obtained. The relativistically correct formalism for evaluating such effects in unmagnetized electron plasma developed by Steiger and Woods[1] (1972) has been extended here for the two-component magnetized plasma, including the case of waves of Alfvén wave frequencies.

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