LOW ENERGY CONSTRAINTS ON $N = 2$ SUPERSYMMETRIC MODELS

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We derive constraints on the masses of mirror particles in $N = 2$ supersymmetric theories. We consider the K_L-K_S mass difference, the $\pi \to e\nu/\pi \to \mu\nu$ branching ratio and the anomalous magnetic moment of the muon. The K_L-K_S mass difference gives a lower bound of 15 TeV on the mirror gauge scalar and mirror gauge fermion masses, under suitable assumptions.

There has been recent interest in $N = 2$ supersymmetric (SUSY) theories [1] because they soften the divergences of quantum field theory and also relate various coupling constants. In this letter we address the question of constraints put on such theories by the low energy phenomenology.

These theories contain new fermions and scalar particles ("mirror particles"). We show that these give rise to extra contributions to the flavour violating neutral current processes, the $\pi \to e\nu/\pi \to \mu\nu$ branching ratio and the anomalous magnetic moment of the muon. These contributions are in addition to the standard model and $N = 1$ supersymmetric model contributions. We discuss constraints on the masses of new mirror particles from these processes. The couplings which we discuss are not present in $N = 1$ supersymmetric models.

We first summarize the features of $N = 2$ supersymmetric theories relevant for our discussion, essentially following ref. [1]. The $N = 2$ supersymmetric transformations are generated by the spinorial charges Q_1 and Q_2. In the language of $N = 1$ supersymmetry generated by Q_1, the $N = 2$ supermultiplet of scalars and fermions contains $N = 1$ chiral multiplets $X = (\psi_X, \tilde{\psi}_X)$ and $Y = (\psi_Y, \tilde{\psi}_Y)$ transforming as R and $\bar{R}$ respectively, under the internal symmetry group. The $N = 2$ vector multiplet contains the $N = 1$ vector multiplet $V_a = (V_a^\mu, \lambda_a)$ and a chiral multiplet $\Phi_a = (\psi_a, \phi_a)$ in the adjoint representation of the group (the a are group generator indices). Thus, $N = 2$ SUSY introduces mirror gauge scalars ϕ_a related to the usual vector gauge bosons. Fig. 1 shows a schematic representation of the transformations of the fields under the two spinorial charges.

The lagrangian for these theories [1] is given by

$$\mathcal{L} = \tfrac{1}{8} g^{-2} [WW]_F + [\sqrt{2}\, ig\, Y\Phi X]_F + \text{h.c.}$$
$$+ [2\, \text{tr}\, \Phi^\dagger e^{2gV} \Phi e^{-2gV} + X^\dagger e^{2gV} X + Y^\dagger e^{-2gV^T} Y]_D, \qquad (1)$$

where $\Phi = \Phi_a T^a$, $V = V_a T^a$ with T^a as the group generators, and W is the $N = 2$ supersymmetric analogue of the $F^{\mu\nu}$ tensor. The term of particular importance to us is $\sqrt{2}\, ig [Y\Phi X]_F$. The origin of this term is in a global SU(2) symmetry acting on the doublet Q_i of

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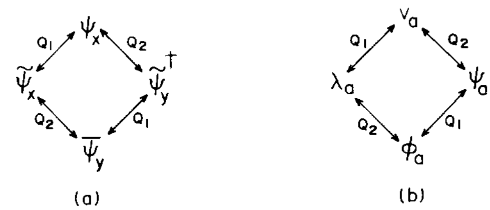

Fig. 1. Action of the two supersymmetry charges Q_1 and Q_2 on the fields in the matter multiplet and vector multiplet.

the generators. (ψ_ϕ, λ) and $(\widetilde{\psi}_{x_i}, \widetilde{\psi}^\dagger_{y_i})$ are doublets under this symmetry. The action of this global SU(2) rotates the kinetic terms of $N = 1$ supersymmetric theories into $\sqrt{2}\, ig[\mathrm{Y}\Phi\mathrm{X}]$. Thus the strength of this interaction is fixed by the gauge coupling. In component form this interaction gives rise to terms $\sqrt{2}\, ig\ \psi_x \phi \psi_y$, $\sqrt{2}\, ig \psi_x \psi_\phi \widetilde{\psi}_y$ and $\sqrt{2}\, ig \widetilde{\psi}_x \psi_\phi \psi_y$. In the following we discuss implications of these extra couplings in two models.

In the first model (the 3–2–1 model), the gauge group is $SU(3)_C \times SU(2)_L \times U(1)_Y$. The ordinary fermions are put in the usual representation R of the gauge group and are in the X superfield. The Y superfields contain mirror fermions which transform as the conjugate representation $\bar{R}$. We now consider the contributions to the flavour violating processes of the extra mirror particles in this model.

Among these processes the most stringent constraint comes from the $K_L - K_S$ mass difference. The diagrams contributing to this process are shown in fig. 2. There is a GIM cancellation between different diagrams. The diagrams of fig. 2a give an effective $\Delta S = 2$ interaction,

$$\mathcal{L}_{\text{eff}} = G \cos^2\theta \sin^2\theta (\bar{s}_L \gamma_\mu d_L)(\bar{s}_L \gamma_\mu d_L) , \qquad (2)$$

where $x_1 = m^2_{\tilde{u}'}$, $x_2 = m^2_{\tilde{c}'}$, $x_3 = m^2_{\psi W}$ and

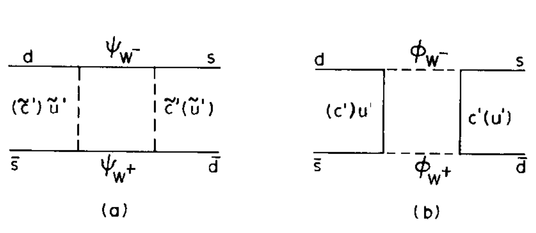

Fig. 2. Contribution of the mirror particles to the $K_L - K_S$ mass difference.

$$G = \frac{ig^4}{16\pi^2}\left(\frac{2x_1 \ln(x_1/x_3)(x_1^2 - x_2 x_3)}{(x_1 - x_3)^2 (x_1 - x_2)} - \frac{2x_1(x_3 - x_2)^2 + x_3(x_1 - x_2)^2}{2(x_3 - x_2)^2 (x_1 - x_3)^2} + (x_1 \leftrightarrow x_2)\right) \qquad (3)$$

and θ is the mixing angle. G is zero when $x_1 = x_2$. This reflects the GIM cancellation. A priori there is no reason to expect cancellation between diagrams of fig. 2 and other contributions [2,3]. Therefore we require that the diagrams of figs. 2a and 2b separately make contributions no greater than the experimental value. The contribution of eq. (2) to the $(K_L - K_S)$ mass difference is given by

$$2G \cos^2\theta \sin^2\theta \langle \bar{K}^0 | (\bar{s}_L \gamma_\mu d_L)(\bar{s}_L \gamma^\mu d_L) | K^0 \rangle .$$

We evaluate the $K^0 - \bar{K}^0$ matrix element in the vacuum insertion method [3] and demand

$$2G \sin^2\theta \cos^2\theta \times 1.82 \times 10^{-2}\ \text{GeV}^3 \lesssim \Delta m_K (\text{exp.}). \qquad (4)$$

In general the angles θ could be complex. Thus a stringent bound will come from $\mathrm{Im}[\Delta m_K]$, for which the experimental value is 3.5×10^{-18} GeV. The limit given in eq. (4) can be translated into useful constraints on the theory provided one has some knowledge of mass ratios. Since this information is lacking at present, we have to make some assumptions.

For illustrative purposes we assume $m_{\tilde{u}'} = M + \delta_1$, $m_{\tilde{c}'} = M + \delta_2$ and $m_{\psi W} = M$, where $\delta_{1,2} \ll M$. Then, to leading order in $\delta_{1,2}$ we get,

$$G \cdot \mathrm{Im}[\sin^2\theta \cos^2\theta] = (g^4/480\pi^2)[(\delta_1 - \delta_2)^2/M^4]\ \mathrm{Im}[\cos^2\theta \sin^2\theta] \lesssim 10^{-16}\ \text{GeV}^{-2} . \qquad (5)$$

Assuming for convenience that $\delta_{1,2} \simeq O(0.1\ M)$, $\mathrm{Im}(\sin^2\theta \cos^2\theta) \simeq 0.06$ and using the known value of g, eq. (5) gives us $M > 13$ TeV. Alternatively, taking $m_{\tilde{u}'}, m_{\tilde{c}'} \ll m_{\psi W}$ e.g. $m_{\tilde{u}'} = 90$ GeV and $m_{\tilde{c}'} = 100$ GeV, we get $m_{\psi W} > 15$ TeV.

The diagrams of fig. 2b again lead us to eqs. (2) and (3) with $x_1 = m^2_{u'}$, $x_2 = m^2_{c'}$ and $x_3 = m^2_{\phi W}$. Since u′ and c′ occur in nontrivial representation of $SU(2)_L$ and are related to u and c by a mirror symmetry, it is difficult to make them heavier than $O(M_W)$. Thus we

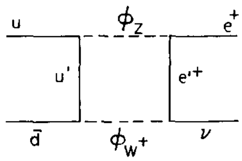

Fig. 3. Contribution to $\pi \to e\nu$ in the 3–2–1 model.

expect $x_1, x_2 \ll x_3$. In this limit we find G to be

$$(x_1 - x_2)^2 g^4/192\pi^2 x_1 x_3^2 .$$

This gives $M_{\phi W} \gtrsim 15$ TeV, if for illustrative purposes we assume $m_{u'} \simeq 90$ GeV, $m_{c'} \simeq 100$ GeV and $\mathrm{Im}(\sin^2\theta \cos^2\theta) \simeq 0.06$.

The new interactions also contribute to various flavour changing processes absent in the standard model. The constraints coming from such processes will be much weaker than from the $K_L - K_S$ mass difference. Hence we will not discuss all of them here. However, there is no GIM cancellation in the case of $\pi \ell_2$ decay, which receives contributions from fig. 3. Hence it is potentially capable of putting stringent bounds and is discussed below. The diagram in fig. 3 leads to an effective interaction

$$\frac{g^4}{4\pi^2} m_{u'} m_{c'} \left[\sum_{i=1}^{4} \left(x_i \ln(x_i) \Big/ \prod_{j=1, j\neq i}^{4} (x_i - x_j) \right)\right]$$

$$\times\, (\bar{\nu}_L e_R)(\bar{d}_L u_R) , \qquad (6)$$

where $x_1 = m_{u'}^2, x_2 = m_{c'}^2, x_3 = m_{\phi Z}^2$ and $x_4 = m_{\phi W}^2$. This interaction involves a pseudoscalar current and hence is severely constrained by the $\pi \to e\nu/\pi \to \mu\nu$ branching ratio, B_π. As discussed earlier, $m_{u'}, m_{c'} \lesssim O(M_W)$. Assuming therefore, for illustration, $m_{u'} = m_{c'} = 100$ GeV, $m_{\phi Z} = m_{\phi W} = M$ and using the calculation from ref. [4], we get $M > 30\, M_W$. The diagram with particles inside the loop replaced by their supersymmetric partners undergoes a GIM type cancellation and hence does not put stringent bounds.

Next we consider the constraints coming from the anomalous magnetic moment of the muon, a_μ. It has long been known [5] that a_μ vanishes in an exactly supersymmetric theory. In a broken theory however, nonzero contributions do arise. In $N = 2$ supersymmetric theories there are additional contributions coming from diagrams shown in fig. 4. Fig. 4a and its supersymmetric analogue contribute terms proportional to m_μ^2 and hence do not severely constrain any

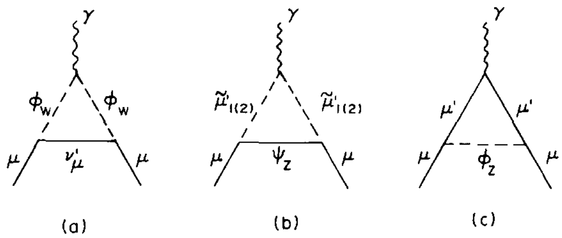

Fig. 4. Diagrams contributing to the anomalous magnetic moment of the muon.

masses. Dominant contributions, proportional to $m_\mu m_{\psi Z}$ and $m_\mu m_{\mu'}$ come from figs. 4b, 4c respectively. For the mirror scalar contribution (fig. 4b) a GIM type cancellation occurs. This is similar to the cancellation observed [6] for gauge fermion contributions in $N = 1$ supersymmetric theories. We find:

$$\Delta a_\mu = -(g_A^2/8\pi^2)(m_\mu/m_{\psi Z}) \sin\theta \cos\theta$$

$$\times \{[1/(1-\eta_1) - 2\eta_1/(1-\eta_1)^2$$

$$- 2\eta_1^2 \ln(\eta_1)/(1-\eta_1)^3] - (\eta_1 \to \eta_2)\} , \qquad (7)$$

for the $\tilde{\mu}'_{1(2)}$ exchange diagrams, and

$$\Delta a_\mu = -(g_A^2/8\pi^2)(m_\mu m_{\mu'}/m_{\phi Z}^2)$$

$$\times [2\ln\eta/(1-\eta)^3 + 1/(1-\eta) + 2/(1-\eta)^2] , \qquad (8)$$

for the μ' exchange contribution, where

$$g_A = g/4\cos\theta_W ,$$

$$\eta = m_{\mu'}^2/m_{\phi Z}^2 , \quad \eta_i = m_{\tilde{\mu}'_i}^2/m_{\psi Z}^2 ,$$

and θ is the mixing angle between $\tilde{\mu}'_1, \tilde{\mu}'_2$.

Now possible mass constraints can be obtained by comparing contributions of eqs. (7) and (8) with the experimental constraint [7], $-2.0 \times 10^{-8} < \Delta a_\mu < 2.6 \times 10^{-8}$. Since the two constrain different masses, we consider them separately. For the contribution of eq. (7), we consider the same limits as used in the $K_L - K_S$ calculations, i.e. $m_{\tilde{\mu}'_1} = M + \delta_1, m_{\tilde{\mu}'_2} = M + \delta_2$ and $m_{\psi Z} = M$ with $\delta_1 - \delta_2 \sim (0.1\, M)$. In this limit eq. (7) gives

$$-2.0 \times 10^{-8} < (-g^2/96\pi^2) m_\mu \sin\theta \cos\theta$$

$$\times [(m_{\tilde{\mu}'_1}^2 - m_{\tilde{\mu}'_2}^2)/M^3] F(\eta_1, \eta_2) < 2.6 \times 10^{-8} ,$$

where $F(\eta_1, \eta_2)$ is a factor of O(1). Using this we find $M > 100$ GeV. The mirror fermion masses can at the most be $O(M_W)$, as discussed earlier. Using this, eq. (8) implies $M_{\phi Z} > 20\, M_W$. Both these constraints are weaker than the one obtained from the $K_L - K_S$ mass difference.

The constraints derived so far are applicable in any $N = 2$ supersymmetric model which reduces to the standard (3–2–1) model at low energies. Additional constraints are present in some classes of models, e.g. the 3–4–1 model of ref. [1]. This model seems to be the simplest one which can accommodate realistic mirror fermion masses without upsetting the stability of vacuum. In this model the quarks and leptons are put into the $SU(3)_C \times SU(4) \times U(1)_Y$ representations:

$$X_L = \begin{pmatrix} L \\ \bar{L}' \end{pmatrix} \sim (1, 4, -1/2) , \quad Y_L = \begin{pmatrix} L' \\ \bar{L} \end{pmatrix} \sim (1, \bar{4}, 1/2)$$

$$X_Q = \begin{pmatrix} Q \\ \bar{Q}' \end{pmatrix} \sim (3, 4, 1/6) , \quad Y_Q = \begin{pmatrix} Q' \\ \bar{Q} \end{pmatrix} \sim (\bar{3}, \bar{4}, -1/6) . \tag{9}$$

In the above Q, L are quark and lepton $SU(2)_L$ doublets $\bar{Q}$, $\bar{L}$ are antiquark and antilepton $SU(2)_R$ doublets with primes denoting the corresponding superfields.

An interesting aspect of this model is the presence of new gauge bosons connecting fermions with antifermions. The mirror scalars ϕ associated with these gauge bosons couple left handed quarks (leptons) to right handed quarks (leptons). These give rise to interesting new currents even at the tree level. Fig. 5 shows a scalar boson mediating the $\pi \to e\nu$ process. This generates a tree-level pseudoscalar current,

$$(g^2/M_\phi^2)[\bar{d}_R u_L][\bar{\nu}_L e_R] . \tag{10}$$

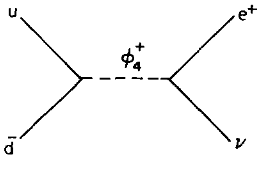

Fig. 5. Tree level contribution to $\pi \to e\nu$ in the 3–4–1 model.

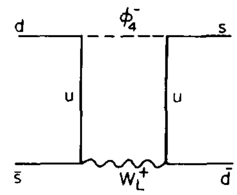

Fig. 6. Contribution to the $K_L - K_S$ mass difference of the mirror gauge scalars in the 3–4–1 model.

Thus M_ϕ is strongly constrained by the data on B_π. Using the calculations of ref. [4], we find

$$2g^2/M_\phi^2 < 2.5 \times 10^{-7}\, G_F , \tag{11}$$

i.e., $M_\phi > 8000\, M_W$.

The above, indeed, is a large scale for supersymmetry breaking. An even more stringent bound comes from the virtual contributions to the $K_L - K_S$ mass difference. One such contribution is shown in fig. 6. The mixing angles occurring at the W and ϕ vertex will in general be different. Hence there will be no GIM cancellation for these diagrams [‡1]. As a result these diagrams lead to an effective interaction,

$$G[4(\bar{s}_L d_R)(\bar{s}_L d_R) - (\bar{s}_L \sigma_{\mu\nu} d_R)(\bar{s}_L \sigma^{\mu\nu} d_R)] \tag{12}$$

where

$$G = (16\pi^2 m_\phi^2)^{-1}\, i g^4 \sin\theta_1 \cos\theta_1 \sin\theta_2 \cos\theta_2 \times \ln(m_u^2/m_\phi^2) .$$

The $K^0 - \bar{K}^0$ matrix elements required to get an experimental bound from eq. (12) can be taken from the vacuum insertion calculation [8]. We will consider only the $(\bar{s}_L d_R)(\bar{s}_L d_R)$ term in eq. (12), since we are interested in an order of magnitude estimate. The experimental limit on $\mathrm{Im}[\Delta m_K]$ then gives $G \lesssim 1.6 \times 10^{-17}\ \mathrm{GeV}^{-2}$. Assuming θ_1, θ_2 to be of the order of $\theta_{\mathrm{Cabibbo}}$ and log factor ~ 15, we find $M_\phi > 700$ TeV [i.e. 90 000 M_W]. This is a very stringent bound for the supersymmetry breaking parameter. The

‡1 In the L–R symmetric theories also there are $W_R - W_L$ interference diagrams similar to fig. 6, and there is no GIM cancellation for these. However, these are suppressed due to an extra factor of $O(m_F^2/m_W^2)$. This comes from the internal fermion propagators. In our diagram such a suppression is absent due to the scalar nature of ϕ.

bound from other flavour violating interactions will be much weaker. The contribution to a_μ, due to these additional interactions is proportional to m_μ^2 and hence does not give very severe constraints.

In conclusion, we have studied the constraints from the virtual effects of new particles (and interactions) in $N = 2$ supersymmetric models. The strongest bound comes from the imaginary part of the $K_L - K_S$ mass difference. This bound depends on the mass differences, due to GIM cancellations between different diagrams. We assume the mass differences of mirror fermions and their scalar superpartners to be roughly 10% of their average values; we then find that the masses of the additional gauge scalars and gauge fermions of $N = 2$ supersymmetric models have to be greater than 15 TeV (i.e. 200 M_W). The $\pi \to e\nu/\pi \to \mu\nu$ branching ratio needs $M_{\phi Z}, M_{\phi W} > 30\, M_W$, and the anomalous magnetic moment of the muon, a_μ gives the weaker limit of 20 M_W for the mass of the gauge scalars. In models like the 3–4–1 model of ref. [1], one gets additional constraints. The $\pi \to e\nu/\pi \to \mu\nu$ branching ratio gives us $M_\phi > 8000\, M_W$. Here ϕ is the gauge scalar corresponding to additional generators of SU(4). The $K_L - K_S$ mass difference gives a still stronger bound, $M_\phi > 10^5\, M_W$. The masses constrained by the above processes are of the order of the supersymmetry breaking scale. Hence the above discussion tells us how low such a scale can be without coming into conflict with experiments.

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