Low dust charging rate induced weakly dissipative dust acoustic solitary waves: Role of nonthermal ions

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The effects of low dust charging rate compared to the dust oscillation frequency and nonthermal ions on small but finite amplitude nonlinear dust acoustic wave have been investigated. It is seen that because of the low dust charging rate, the nonlinear wave exhibits weakly dissipative solitary wave that is governed by a modified form of the Korteweg-de Vries equation. The solitary wave possesses both rarefactive and compressive soliton solution depending on the values of ion nonthermality parameter a. An analytical solution reveals that because of the simultaneous effects of low dust charging rate and nonthermal ions, the wave amplitude may grow exponentially with time if the ion nonthermality parameter (a) exceeds a critical value provided the ion-electron temperature ratio (σ_i) is less than 0.11. © *2007 American Institute of Physics*.
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I. INTRODUCTION

It is well known that the presence of highly charged heavy mass dust grains modifies the collective behavior of plasmas and introduces a very low frequency wave mode called the "dust acoustic wave" (DAW)[1,2] in which dust grains provide the inertia and the inertialess Boltzmann distributed thermal electrons, and ions provide the restoring force. In reality, the charge on a dust grains is not fixed: it varies with time according as the surrounding plasma potential. The electrostatic charging of dust grains in plasma is the main feature of dust plasma interactions in truly dusty plasma. The time-dependent dust charge variation, i.e., the nonsteady dust charge variation due to collective perturbations modifies the dust floating potential, which in turn self-consistently opposes the buildup of the plasma currents at the surface of the dust grains and consequently causes dissipative effects in dusty plasma. In the case of linear propagation, the time-dependent charge variation, i.e., nonsteady charge variation, causes a collisionless non-Landau damping of wave modes.[3] On the other hand, in the nonlinear regime this nonsteady charge variations cause anamolous dissipation in the plasma that leads to the formation of collisionless shock wave in a dusty plasma.[4,5]

In collisionless plasmas, the velocity distribution is commonly considered as Maxwellian. However, observations on space plasmas indicate that the presence of electron and ion populations is far from thermodynamic equilibrium. Thus, the occurrence of nonthermal ions and electrons is a common feature in such an environment. Nonthermal ions from the earth's bow-shock have been observed by the *Vela* satellite,[6] as well as in and around the Earth's foreshock.[7] The ASPERA experiment on the *Phobos* satellite has detected nonthermal ion fluxes from the upper ionosphere of Mars.[8] Closer to Earth, fast nonthermal ions have been observed recently by the *Nozomi* satellite in the vicinity of the Moon.[9] It appears from the observations that the nonthermal ions have a ring structure in the velocity phase space. On the basis of this observation, a possible three-dimensional nonthermal ion velocity distribution function has been suggested which satisfies collisionless Vlasov equation.[10] Moreover, the presence of nonthermal ions drastically modifies the collective behavior of electrostatic modes.[11]

Both the linear and nonlinear (solitary waves, double layers, etc.) collective behavior of DAWs have been investigated in a dusty plasma consisting of negatively fixed charged dust grains, Maxwellian electrons, and nonthermal ions.[12–14] Recently, Ghosh *et al.*[10] have investigated the effects of both *adiabatic*[15] and *nonadiabatic*[4,5] dust charge variations in presence of nonthermal ions on linear propagation characteristics of dust acoustic waves. It has been shown that instead of usual damping, the DAWs become unstable due to *nonadiabatic* dust charge variation in presence of nonthermal ions.

In this paper, the effects of low dust charging rate compared to dust oscillation frequency and nonthermal ions on DA solitary wave are investigated. It is seen that a modified form of KdV equation governs the DA solitary wave. An approximate time evolution solution of the solitary wave shows that the wave amplitude decays with time, but the product of amplitude and square of the soliton width remains constant through out the propagation, which implies the DA solitary wave is dissipative in nature. However, due to the presence of nonthermal ions, instead of decay, it grows with time, and the DA solitary wave possesses both rarefactive and compressive solitons.

The paper is organized in the following manner. Mathematical formulation of the problem is given in Sec. II. The

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nonlinear evolution equations are derived in Sec. III. An approximate analytical solution of the modified form of the KdV equation and its nature is discussed in Sec. IV. Numerical analyses and their graphical representations are given in Sec. V. Finally, the results of the present investigation are summarized in Sec. VI.

II. FORMULATION OF THE PROBLEM

A one-dimensional, unbounded, unmagnetized, collisionless dusty plasma whose constituents are Boltzmann distributed electrons, nonthermal distributed ions, and charge fluctuating negatively charged dust grains is considered Hence, in this case the number densities of electrons and ions are given by[10,11]

$$n_e = n_{e0}\exp(\Phi); \quad n_i = n_{i0}\left[1+\frac{4a}{1+3a}\left(\frac{\Phi}{\sigma_i}+\frac{\Phi^2}{\sigma_i^2}\right)\right] \times\exp\left(-\frac{\Phi}{\sigma_i}\right), \tag{1}$$

where a is the ion nonthermality parameter, which determines the number of fast (energetic) ions; i.e., σ_i $(=T_i/T_e)$ is ion-electron temperature ratio and Φ $(=e\phi/T_e)$ is the normalized plasma potential.

The nonlinear dynamics of low phase velocity dust acoustic wave are governed by the following normalized dust dynamical equations:

$$\frac{\partial N_d}{\partial T}+\frac{\partial(N_dV_d)}{\partial X}=0, \tag{2}$$

$$\frac{\partial V_d}{\partial T}+V_d\frac{\partial V_d}{\partial X}=-(\Delta Q-1)\frac{\partial\Phi}{\partial X}-\gamma_d\sigma_dN_d^{\gamma_d-2}\frac{\partial N_d}{\partial X}; \quad \sigma_d=\frac{T_d}{z_dT_e}, \tag{3}$$

$$\frac{\partial^2\Phi}{\partial X^2}=\frac{\delta}{1-\delta}N_e-\frac{1}{1-\delta}N_i-N_d(\Delta Q-1); \quad \delta=\frac{n_{e0}}{n_{i0}}, \quad 1-\delta=\frac{z_dn_{d0}}{n_{i0}}. \tag{4}$$

The dust velocity (V_d), time (T), and space (X) scales are normalized by dust acoustic speed $(=\sqrt{z_dT_e/m_d})$, dust plasma frequency ω_{pd}, and dust Debye length $(=\sqrt{\varepsilon_0T_e/z_dn_{d0}e^2})$, respectively. In deriving Eq. (4), we use the charge neutrality condition $n_{e0}+z_dn_{d0}=n_{i0}$, where n_{e0} (n_{d0},n_{i0}) is the electron (dust, ion) equilibrium number density and $-z_de$ is the charge residing on the dust grains. In addition, we denote $N_j=n_j/n_{j0}$ $(j=e,i,d)$, $(\Delta Q-1)=Q_d/z_de$, Q_d is the dust charge, and ΔQ is the fluctuating part of the dust charge, and γ_d is the adiabatic index.

The normalized dust charging equation is

$$\frac{\partial\Delta Q}{\partial T}+V_d\frac{\partial\Delta Q}{\partial X}=\left(\frac{\nu_d}{\omega_{pd}}\right)\frac{(I_e+I_i)}{\nu_dz_de}. \tag{5}$$

The normalized expressions for the electron current (I_e) and ion current (I_i) for spherical dust grain of radius r_0 are as follows:[10]

$$I_e=-n_{e0}J_e\exp[\Phi+z(\Delta Q-1)]; \quad z=\frac{z_de^2}{4\pi\varepsilon_0r_0T_e}, \tag{6}$$

$$I_i=\frac{n_{i0}J_i}{1+3a}\left\{\left[\left(1+\frac{24a}{5}\right)+\frac{16a}{3\sigma_i}\Phi+\frac{4a}{\sigma_i^2}\Phi^2\right] -\frac{z(\Delta Q-1)}{\sigma_i}\left[\left(1+\frac{8a}{5}\right)+\frac{8a}{3\sigma_i}\Phi +\frac{4a}{\sigma_i^2}\Phi^2\right]\right\}\exp\left(\frac{-\Phi}{\sigma_i}\right), \tag{7}$$

where $J_\alpha=\pi r_0^2e\sqrt{8T_\alpha/\pi m_\alpha}$ $(\alpha=e,i)$ and the dust charging frequency ν_d is given by[10]

$$\nu_d=\frac{r_0}{\sqrt{2\pi}}\frac{\omega_{pi}^2}{V_{ti}}\left(\frac{5+8a}{5+15a}\right)\left[1+z+\left(\frac{5+24a}{5+8a}\right)\sigma_i\right]. \tag{8}$$

III. NONLINEAR EVOLUTION EQUATIONS

To study the small but finite amplitude nonlinear DAW, we introduce the standard reductive perturbation technique and the independent variables can be stretched as

$$\xi=\sqrt{\varepsilon}(X-\lambda T); \quad \tau=\varepsilon^{3/2}T, \tag{9}$$

where λ is the normalized phase velocity of the linear DAW and ε is a small parameter characterizing the strength of the nonlinearity. The dynamical variables N_d, V_d, ΔQ, and Φ are expanded in powers of ε as follows:

$$f=f^{(0)}+\sum_{i=1}^{\infty}\varepsilon^if^{(i)}; \quad f=N_d,V_d,\Delta Q,\Phi, \tag{10}$$

where $f^{(0)}=1$ for $f=N_d$, and $=0$ for $f=V_d$, ΔQ, Φ. Substituting Eqs. (9) and (10) into Eqs. (1)–(4) and equating the coefficient of the terms in the lowest powers of ε, we obtain the following relations:

$$N_d^{(1)}=-\frac{\Phi^{(1)}}{\lambda^2-\gamma_d\sigma_d}; \quad V_d^{(1)}=-\frac{\lambda\Phi^{(1)}}{\lambda^2-\gamma_d\sigma_d}, \tag{11}$$

$$\left(\delta-\frac{a-1}{\sigma_i(1+3a)}\right)\Phi^{(1)}=(1-\delta)(\Delta Q^{(1)}-N_d^{(1)}). \tag{12}$$

To the next higher order in ε, we obtain

$$\frac{\partial N_d^{(1)}}{\partial\tau}+\frac{\partial(N_d^{(1)}V_d^{(1)})}{\partial\xi}=\frac{\partial(\lambda N_d^{(2)}-V_d^{(2)})}{\partial\xi}, \tag{13}$$

$$\frac{\partial V_d^{(1)}}{\partial\tau}+\frac{1}{2}\frac{\partial(V_d^{(1)^2}+\gamma_d(\gamma_d-2)\sigma_dN_d^{(1)^2})}{\partial\xi}+\Delta Q^{(1)}\frac{\partial\Phi^{(1)}}{\partial\xi} =\frac{\partial(\Phi^{(2)}+\lambda V_d^{(2)}-\gamma_d\sigma_dN_d^{(2)})}{\partial\xi}, \tag{14}$$

$$\frac{\partial^2\Phi^{(1)}}{\partial\xi^2} = \left(\frac{\delta}{1-\delta} - \frac{a-1}{\sigma_i(1-\delta)(1+3a)}\right)\Phi^{(2)} + \frac{1}{2}\left(\frac{\delta}{1-\delta} - \frac{1}{\sigma_i^2(1-\delta)}\right)\Phi^{(1)^2} - (\Delta Q^{(2)} - N_d^{(2)} + N_d^{(1)}\Delta Q^{(1)}). \quad (15)$$

A system can be termed as weakly dissipative system if the system dissipation rate ($\nu_{\rm diss}$) is sufficiently low compared to the characteristic oscillation frequency of the system. In the present model, the dissipation rate and oscillation frequency of the dynamical system are determined by the dust charging rate ν_d and dust plasma frequency ω_{pd}. Thus, for low dust charging rate compared to ω_{pd}, i.e., for $\nu_d \ll \omega_{pd}$, the present model is a weakly dissipative system. Thus, for consistent perturbation expansion, we can consider the following scaling:

$$\frac{\nu_d}{\omega_{pd}} = \nu\varepsilon^{3/2}; \quad \nu \approx O(1). \quad (16)$$

In this case, due to the nonzero value of ν_d/ω_{pd}, the dust charge Q_d does not instantaneously reach its equilibrium value, but instead plays a dissipative role. Using the above scaling in Eq. (5) together with Eqs. (6) and (7) and then equating the terms in lowest order and next to the lowest order of ε, we obtain

$$\frac{\partial\Delta Q^{(1)}}{\partial\xi} = 0 \Rightarrow \Delta Q^{(1)} = \Delta Q^{(1)}(\tau) = 0, \quad (17)$$

$$\lambda\frac{\partial\Delta Q^{(2)}}{\partial\xi} = \nu\beta_{\rm ch}\Phi^{(1)}; \quad \beta_{\rm ch} = \frac{\left[z + \left(\frac{5+24a}{5+8a}\right)\sigma_i\right]}{z\left[1 + z + \left(\frac{5+24a}{5+8a}\right)\sigma_i\right]}, \quad (18)$$

by virtue of the boundary condition that all perturbations vanish at $X=-\infty(\xi=-\infty)$ for all time scales, slow or fast. Equating all the terms of lowest order of ε and using Eq. (17) in Eqs. (11) and (12), we obtain

$$\lambda = \sqrt{\gamma_d\sigma_d + \frac{1-\delta}{\delta - \frac{a-1}{\sigma_i(1+3a)}}}. \quad (19)$$

Using Eqs. (11), (12), (17), and (18) and eliminating all the second-order quantities from Eqs. (13)–(15), we obtain the following modified form of KdV equation with a linear damping or growth term

$$\frac{\partial\Phi^{(1)}}{\partial\tau} - \alpha\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial\xi} + \beta\frac{\partial^3\Phi^{(1)}}{\partial\xi^3} - \gamma\Phi^{(1)} = 0. \quad (20)$$

The coefficient of nonlinearity α, coefficient of dispersion β and damping (growth term) γ are as follows:

$$\alpha = \frac{(\lambda^2 - \gamma_d\sigma_d)^4}{2\lambda}\left[\frac{3\lambda^2 + \gamma_d(\gamma_d - 2)\sigma_d}{(\lambda^2 - \gamma_d\sigma_d)^3} + \frac{\delta\sigma_i^2 - 1}{(1-\delta)\sigma_i^2}\right], \quad (21)$$

$$\beta = \frac{(\lambda^2 - \gamma_d\sigma_d)^2}{2\lambda}, \quad (22)$$

$$\gamma = \nu\beta_{\rm ch}\frac{\beta}{\lambda^2}\left[\frac{\frac{8a - 15 - (15+72a)\sigma_i}{15+24a} + \frac{z}{\sigma_i}\left(\frac{16a - 15 - (15+24a)\sigma_i}{15+24a}\right)}{z + \left(\frac{5+24a}{5+8a}\right)\sigma_i}\right]. \quad (23)$$

This expression shows that γ disappears if we set $\nu=0$, this should be so because $\nu=0$ implies that the right-hand side of Eq. (5) =0 ($d_T\Delta Q=0$); i.e., there is no dust charge variation [by Eqs. (17) and (18); $\Delta Q^{(1)}=\Delta Q^{(2)}=0 \Rightarrow$ fixed charge on the dust grain surface]. Hence, the low charging rate of dust grains compared to the dust oscillation frequency, i.e., $\nu_d/\omega_{pd} \ll 1$, is responsible for the term $\gamma=\gamma(z,\sigma_i,a)$ in the KdV equation (20).

IV. APPROXIMATE ANALYTICAL SOLUTION AND DISCUSSIONS

Karpman and Maslov's method and the boundary condition that all perturbations vanish at $|\xi|=\infty$, yields the following time evolution solution of modified form of KdV equation (20):

$$\Phi^{(1)}(\xi,\tau) = -\Phi^{(1)}(\tau)\mathrm{sech}^2\left(\frac{\xi - V(\tau)\tau}{W(\tau)}\right); \quad \Phi^{(1)}(\tau) = \Phi^{(1)}(0)e^{4\gamma\tau/3}, \quad (24)$$

$$\Phi^{(1)}(0) = \Phi^{(1)}(\tau=0) = \frac{3V}{\alpha}, \quad (25)$$

where

$$V(\tau) = \left(\alpha\frac{\Phi^{(1)}(0)}{3}\right)e^{4\gamma\tau/3}; \quad W(\tau) = \sqrt{\frac{12\beta e^{-4\gamma\tau/3}}{\alpha\Phi^{(1)}(0)}} \quad (26)$$

are the velocity and width of the solitary wave, respectively.

The expressions (24)–(26), for $\gamma=\gamma(z,\sigma_i,a)\neq 0$, the soliton amplitude $[=\Phi^{(1)}(\tau)]$, and velocity $[=V(\tau)]$ decay ($\gamma<0$) or grow ($\gamma>0$) exponentially with time τ at decay or growth rate γ and the spatial width $[=W(\tau)]$ of the soliton acts conversely. However, the quantity

$$L=\Phi^{(1)}(\tau)W^2(\tau)=\frac{12\beta}{\alpha} \tag{27}$$

is independent of time (τ).

It is interesting to note that in absence of nonthermal ions, i.e., for Boltzmann distributed ions ($a=0$), the expression for γ becomes

$$\gamma(z,\sigma_i,a=0)=-\nu\frac{(1+\sigma_i)(z+\sigma_i)}{z\sigma_i(1+z+\sigma_i)}\frac{\beta}{\lambda^2(a=0)}<0. \tag{28}$$

This negative value of γ reveals that the nonlinear wave amplitude decays exponentially with time according to Eq. (24). Thus, dust charge variations for low dust charging rate compared to dust oscillation frequency, i.e., $\nu_d/\omega_{pd}\ll 1$, produces a damping effect.

On the other hand, the expression (23) for $\gamma(z,\sigma_i,a)$ shows that presence of nonthermal ions ($a\neq 0$) may change the algebraic sign of $\gamma(z,\sigma_i,a)$; i.e., $\gamma(z,\sigma_i,a)>0$ *or* <0. The condition for damping ($\gamma<0$) is

$$\gamma(z,\sigma_i,a)<0\Rightarrow a<\frac{15(1+\sigma_i)}{16-24\sigma_i}=a_1. \tag{29}$$

This condition shows that for $\sigma_i>2/3$, $a_1<0$, and in this case the wave amplitude becomes damped for all nonnegative values of ion nonthermality parameter a. However, the condition for growth ($\gamma>0$) is

$$\gamma(z,\sigma_i,a)>0\Rightarrow a>\frac{15(1+\sigma_i)}{8-72\sigma_i}=a_2;\quad \sigma_i<\frac{1}{9} \tag{30}$$

as the ion nonthermality parameter $a>0$ and subject to the condition that $\lambda^2>0$ [Eq. (19)]; otherwise, there will be no DAW mode.[10] However, if $a_1<a<a_2$, then there will be growth or damping depending on the values of z/σ_i. Hence, dust charge variations for low dust charging rate compared to dust oscillation frequency produces a damping effect, whereas the growth is possible only due to the presence of nonthermal ions provided ion-electron temperature ratio $\sigma_i<1/9\approx 0.11$. The growth is possible because of the fact that for $a>a_2\sim 2(\sigma_i\ll 1)$, i.e., the ion nonthermality parameter a, which determines the number of fast (energetic) ions present in the system is greater than unity and thereby acts as source of free energy.

In addition, in the case of damping ($\gamma<0$) as the solitary wave propagates, the product of the soliton amplitude and square of the soliton width, i.e., the quantity L [Eq. (27)], remains constant and hence in this case the single soliton obtained from the solutions (24) can be regarded as a weakly dissipative soliton as γ arises due to $\nu_d/\omega_{pd}\ll 1$.

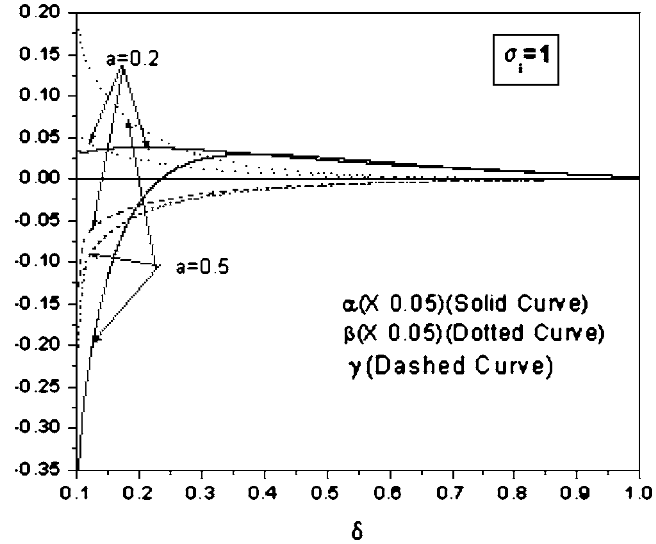

FIG. 1. Variations of coefficient on nonlinear term α [Eq. (21)], dispersive term β [Eq. (22)], and damping (growth) term γ [Eq. (23)] with electron-ion number density ratio $\delta(=n_{e0}/n_{i0})$ and ion nonthermality parameter a.

V. NUMERICAL ANALYSIS

For numerical computations, we choose the following physical parameters of cometary dusty plasma:[9,16,17] $n_{i0}\sim 10^7$ m^{-3}, $T_e\sim 100$ eV, $m_i=16m_p\sim 2.68\times 10^{-26}$ kg, $r_0\sim 1$ μm, $n_{d0}\sim 10^{-4}$ m^{-3}, $\gamma_d\sim 1$, and the dust mass density $\rho_d\sim 10^3$ kg m^{-3}. On the basis of the above-mentioned plasma parameters, for $a\sim 0.5$, $\sigma_i\sim 1$, and $z\sim 2.5$, the value of the ratio $\nu_d/\omega_{pd}\sim 4.6\times 10^{-2}\ll 1$, thus justifying the scaling equation (16).

The variations of coefficient of nonlinearity α [Eq. (21)], the coefficient of dispersion β [Eq. (22)], and damping (growth) term γ [Eq. (23)] as a function of electron-ion number density ratio δ are drawn in Fig. 1 with ion-electron temperature ratio $\sigma_i=1$ for $a=0.2$ and $a=0.5$, respectively. Figure 1 shows that for lower value of a (=0.2), $\alpha>0$, i.e., DAW possesses negative potentials as $\alpha>0\Rightarrow\Phi^{(1)}(\xi,\tau)<0$ by Eq. (24). On the other hand, for higher value of a (=0.5), $\alpha<0$; i.e., DAW possesses positive potentials as $\alpha<0\Rightarrow\Phi^{(1)}(\xi,\tau)>0$ by Eq. (24). Dotted and dashed curves in Fig. 1 show that both the coefficient of dispersion and damping rate is larger for higher value of ion nonthermality parameter a.

The variations of α, β, and γ against δ for Boltzmann distributed ions ($a=0$) and $\sigma_i=0.01$ are plotted in Fig. 2. The solid curve in Fig. 2 shows that in absence of ion nonthermality parameter a, i.e., for Boltzmann ion, DAW possesses only negative potentials as $\alpha>0\Rightarrow\Phi^{(1)}(\xi,\tau)<0$ by Eq. (24). The nature of the other two curves (dotted and dashed) are qualitatively same as that of Fig. 1.

For damping $a<a_1$ [Eq. (27)], we take $a=a_1-0.1$ and the variations of α, β, and γ (<0) against δ are plotted in Fig. 3 for $\sigma_i=0.01$. The solid curve in Fig. 3 shows that for this value of ion nonthermality parameter a ($<a_1$), the DAW possesses only positive potentials as $\alpha<0\Rightarrow\Phi^{(1)}(\xi,\tau)>0$ by Eq. (24). The nature of the other two curves (dotted and dashed) is qualitatively same as that of Fig. 1 (or Fig. 2).

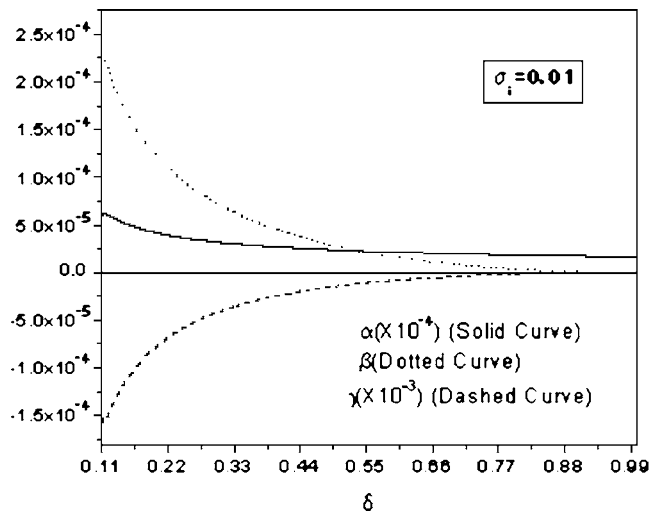

FIG. 2. Variations of coefficient on nonlinear term α [Eq. (21)], dispersive term β [Eq. (22)], and damping (growth) term γ [Eq. (28)] with electron-ion number density ratio $\delta\,(=n_{e0}/n_{i0})$ for Boltzmann distributed ions; i.e., for $a=0$.

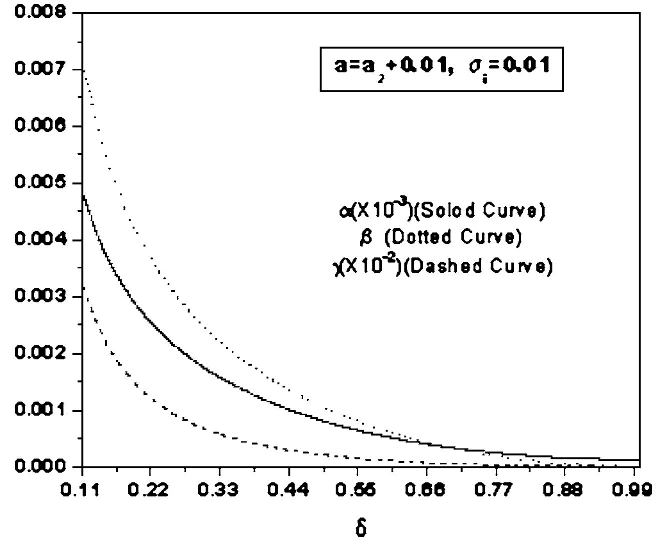

FIG. 4. Variations of coefficient on nonlinear term α [Eq. (21)], dispersive term β [Eq. (22)], and damping (growth) term γ [Eq. (23)] with electron-ion number density ratio $\delta\,(=n_{e0}/n_{i0})$ for $a=a_2+0.01$ $(>a_2)$ [Eq. (30)].

Note that in all of the above cases, the amplitude of the DA solitary wave decays with time τ, with decay rate $\frac{4}{3}|\gamma|$ as $\gamma<0$ [by Eq. (24)].

For growth $a>a_2$ [Eq. (28)], we take $a=a_1+0.01$ and the variations of α, β, and $\gamma\,(>0)$ against δ are plotted in Fig. 4 for $\sigma_i=0.01$. The solid curve in Fig. 4 shows that for this value of ion nonthermality parameter a $(>a_2)$, the DAW possesses only negative potentials as $\alpha>0\Rightarrow\Phi^{(1)}(\xi,\tau)<0$ by Eq. (24). The nature of the dotted curve in this figure is qualitatively same as that of other curves for the coefficient of dispersion β. However, the dashed curve in this figure shows that $\gamma\,(>0)$ possesses only positive values, unlike the other figures. Thus, in this case the amplitude of the dust acoustic solitary wave grows with time τ with growth rate $\frac{4}{3}|\gamma|$ as $\gamma>0$ [by Eq. (24)].

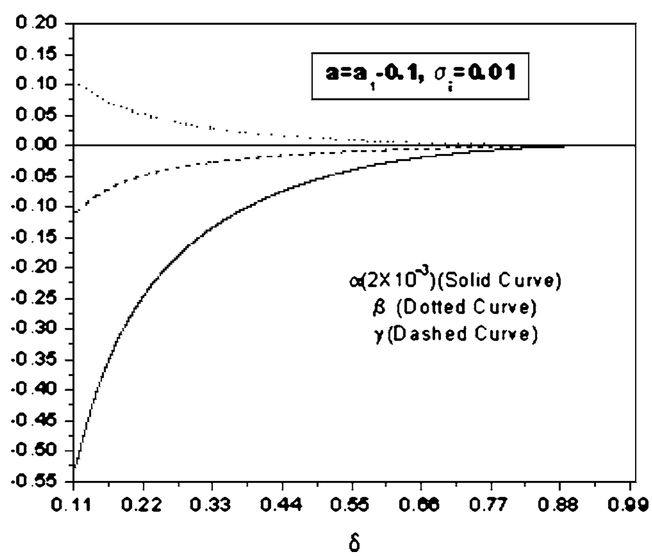

FIG. 3. Variations of coefficient on nonlinear term α [Eq. (21)], dispersive term β [Eq. (22)], and damping (growth) term γ [Eq. (23)] with electron-ion number density ratio $\delta\,(=n_{e0}/n_{i0})$ for $a=a_1-0.1$ $(<a_1)$ [Eq. (29)].

All the figures also reveal that the magnitude of coefficient of nonlinearity, coefficient of dispersion and damping or growth term decreases with the increase of electron-ion number density ratio (δ) and all are tending to 0 for $\delta\rightarrow1$. This is to be expected as the latter implies decrease in dust density.

VI. SUMMARY

The results of this paper can be summarized as follows:

(a) It is seen that the dust charge variations due to the assumption that dust charging rate is low but comparable to dust oscillation frequency $(\nu_d/\omega_{pd}\ll1,\neq0)$ [Eq. (16)], produces a dissipative effect whereby the dust acoustic solitary wave is governed by KdV equation with a linear damping term [Eq. (20)]. However, the presence of nonthermal ions makes the possibility of growth of the nonlinear wave under certain conditions.

(b) The approximate analytical time evolution solution of KdV equation with a linear damping (growth) term shows that the nonlinear wave amplitude decays (grows) and consequently the spatial width of the solitary wave grows (decays) with time τ [Eqs. (24) and (26)].

(c) The coefficient of nonlinearity α and coefficient of dispersion β decreases with the increase of electron-ion number density ratio δ (solid and dotted curves in Figs. 1–4). Thus, in all cases the soliton amplitude, which is proportional to $1/\alpha$, increases, and the spatial width of the solitary wave, which is proportional to $\sqrt{\beta}$, decreases; i.e., the dust acoustic solitons sharpen as the electron-ion density ratio increases.

(d) For ion nonthermality parameter $a<a_1$, the dust acoustic wave has positive potential and thus is associated with dust density depletion, while for $a>a_2$, dust acoustic wave has negative potential and the density is enhanced as $\alpha > < 0 \Rightarrow \Phi^{(1)}(\xi,\tau) < > 0 \Rightarrow N_d^{(1)} > < 0$ [Eq. (11), solid curves in Figs. 1, 3, and 4]. In case of Boltzmann distributed ions, i.e., for $a=0$, the dust acoustic wave has only negative potential and thus possesses only compressive soliton (solid curve in Fig. 2). Therefore, due to presence of nonthermal ions, the dust acoustic solitary wave possesses both rarefactive and compressive soliton.

(e) The damping (growth) γ [Eq. (23)] decreases with the increase of electron-ion number density ratio δ and $\gamma \rightarrow 0$ for $\delta \rightarrow 1$ (dashed curves in Figs. 1–4). This is to be expected as the latter implies decrease in dust density and hence decrease in dust charge fluctuation induced damping. In addition, due to the presence of the ion nonthermality parameter, instead of damping the wave amplitude grows with time τ as for $a > a_2$, γ is positive (Fig. 4)].

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