

# Laser Diode to Single-mode Fiber Excitation via Hemispherical Lens on the Fiber Tip: Efficiency Computation by ABCD Matrix with Consideration for Allowable Aperture

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## Summary

Employing ABCD matrix for hemispherical lens under paraxial approximation and Gaussian field distributions for both the source and the fiber, we report a simple theoretical investigation of the coupling efficiency of a laser diode to a single-mode fiber via hemispherical lens on the fiber tip. The analysis also takes care of the allowable aperture of the hemispherical lens. Our model simplifies calculations which are executable by a pocket calculator. Our results agree excellently with the experimental observations.

## 1 Introduction

Very recently tremendous interests have been generated in the fabrication of microlenses on the fiber tip in order to maximize the source to single-mode fiber coupling efficiency [1, 2]. These microlenses which are formed either in the hemispherical or conical shape have the common advantage of being self-centred. However, in designing the optimum launch optics to achieve maximum light coupling efficiency involving such lenses, theoretical techniques [1–3] which are already presented require lengthy numerical integrations.

In this communication, we take into consideration the truncating aperture [1] of the hemispherical lens within which coupling is permitted and also use the ABCD matrix for refraction by a hemispherical lens [4] to study the source to single-mode fiber coupling via hemispherical lens on the fiber tip. We also employ Gaussian approximation for field distributions of both the source and the fiber. The results may be useful in designing suitable hemispherical microlenses on the fiber tip to achieve maximum coupling efficiency.

## 2 Theory

The basic coupling scheme to be studied is shown in Fig. 1. The hemispherical lens grown on the fiber tip is essentially expected to match the modes of semiconductor laser diode and the circular core single-mode fiber. The light beams from practical laser diodes possess elliptical intensity profile. The intensity profiles are approximated by Gaussian functions of spot sizes  $W_{1x}$

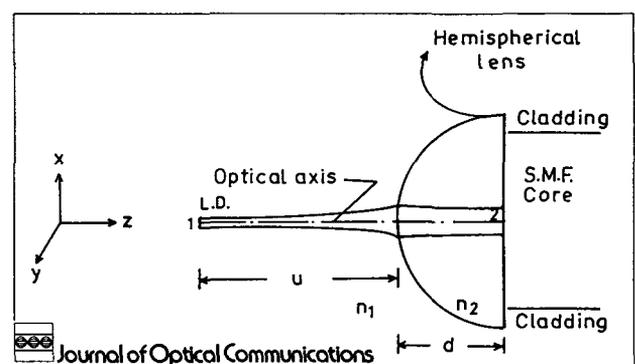


Fig. 1: Geometry of an optical beam emitted from the input plane 1 of a laser facet and refracted through a hemispherical lens onto plane 2 which is the end face of a single-mode fiber

and  $W_{1y}$  along the two mutually perpendicular directions, one perpendicular and another parallel to the junction planes respectively. The field  $\psi_u$  at the output of the laser at a distance “u” from the lens surface can be given by [5]

$$\psi_u = \exp\left[-\left(x^2/w_{1x}^2 + y^2/w_{1y}^2\right)\right] \exp\left[-jk_1(x^2 + y^2)/2R_1\right] \quad (1)$$

where  $R_1$  is the radius of curvature of the wavefront from the laser diode and  $k_1$  is the wave number in the incident medium. We also use the Gaussian approximation for the fundamental mode in the single-mode circular core fiber to express it as [5]

$$\psi_f = \exp\left[-(x^2 + y^2)/w_f^2\right] \quad (2)$$

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where the spot size  $w_f$  can be approximated by [6] as follows

$$w_f = a \left[ 0.65 + 1.619/v^{1.5} + 2.879/v^6 \right] \quad (3)$$

where "a" is the core radius and "v" is the normalised frequency given by  $k_0 a (n_{co}^2 - n_{cl}^2)^{1/2}$  with  $n_{co}$  and  $n_{cl}$  being refractive indices of the core and the cladding, respectively, and  $k_0$  being free space wave number.

The lens transformed laser field  $\psi_v$  on the fiber plane 2 can be given by

$$\psi_v = \exp \left[ - \left( x^2 / w_{2x}^2 + y^2 / w_{2y}^2 \right) \right] \cdot \exp \left[ -jk_2 \left( x^2 / R_{2x} + y^2 / R_{2y} \right) / 2 \right] \quad (4)$$

where  $k_2$  is the wave number in the lens and  $w_{2x}$  and  $w_{2y}$  are transformed spot sizes with  $R_{2x}$  and  $R_{2y}$  being radii of curvature of the refracted wavefront in the x and y directions respectively. In the appendix, we have shown the determination of  $w_{2x,2y}$  and  $R_{2x,2y}$  in terms of  $w_{1x,1y}$  and  $R_1$  with the help of the ABCD matrix corresponding to hemispherical lens on the tip of the fiber. Further, the coupling efficiency for source to single-mode fiber via hemispherical lens on the fiber tip is given by the overlap integral [7]

$$\eta_0 = \frac{\left| \iint \psi_v \psi_f^* dx dy \right|^2}{\iint |\psi_v|^2 dx dy \iint |\psi_f|^2 dx dy} \quad (5)$$

Now using (2) and (4) in (5), we obtain

$$\eta_0 = 4w_{2x}w_{2y}w_f^2 / \left[ \left( w_f^2 + w_{2x}^2 \right)^2 + \left( k_2^2 w_f^4 w_{2x}^4 \right) / \left( 4R_{2x}^2 \right) \right]^{0.5} \cdot \left[ \left( w_f^2 + w_{2y}^2 \right)^2 + \left( k_2^2 w_f^4 w_{2y}^4 \right) / \left( 4R_{2y}^2 \right) \right]^{0.5} \quad (6)$$

### 2.1 Coupling due to allowable aperture

The performance of a hemispherical lens is seriously impaired due to its limited aperture within which transmission is permitted. The radius  $\rho_c$  beyond which transmission is not allowed corresponds to the grazing angle of incidence and is shown in [1] with proper explanations as

$$\rho_c = (n_1 R_L) / n_2 \quad (7)$$

where  $n_1$  and  $n_2$  are refractive indices of the incident medium and hemispherical lens, respectively.

Accordingly the corrected efficiency  $\eta$  will be the product of  $\eta_0$  and lens transmittivity factor T which is given by [1]

$$T = \frac{\int_0^{\rho_c} |\psi_f|^2 r dr}{\int_0^{\infty} |\psi_f|^2 r dr} \quad (8)$$

Here the transmission coefficient (t) of the lens under paraxial approximation is given by

$$t = \frac{2(n_1 n_2)^{1/2}}{n_1 + n_2} \quad (9)$$

Using (2) and (7) in (8), we get

$$T = t^2 \left[ 1 - \exp \left( -2\rho_c^2 / w_f^2 \right) \right] \quad (10)$$

### 3 Results and discussions

Our method based on the ABCD matrix under paraxial approximation with consideration for allowable aperture of a hemispherical lens on the fiber tip aims at simplifying the coupling efficiency calculations. The calculations can be executed by a pocket calculator. However for comparison of our analysis, we refer to the earlier work [3] where the theoretical investigations take care of the phase shift on the lens surface in both the cases of planar and spherical wave models for the incident wavefront. Parameters similar to those taken by [3] are used, namely a laser diode of wavelength  $1.5 \mu\text{m}$  with  $w_{1x} = 0.843 \mu\text{m}$ ,  $w_{1y} = 0.857 \mu\text{m}$  and a single-mode fiber of core diameter  $7.3 \mu\text{m}$  with  $w_f = 4.794 \mu\text{m}$ . We also take [3] the refractive index of the microlens and t as 1.55 and 1, respectively. It is also relevant to mention here that the value of t as found from (9) is 0.9765, very close to unity. In Fig. 2 the solid curve and the dash-dotted curve represent the variation of maximum coupling efficiency calculated by our formalism against the radius ( $R_L$ ) of the hemispherical lens for the cases of spherical and planar wave models respectively for the incident wavefront. These curves are compared with the two theoretical curves [3], one represented by dots in the planar wave model for incident wavefront and the other represented by dashes in the spherical wave model for the incident wavefront. The experimental points [3] are represented by crosses in Fig. 2. Our curves agree

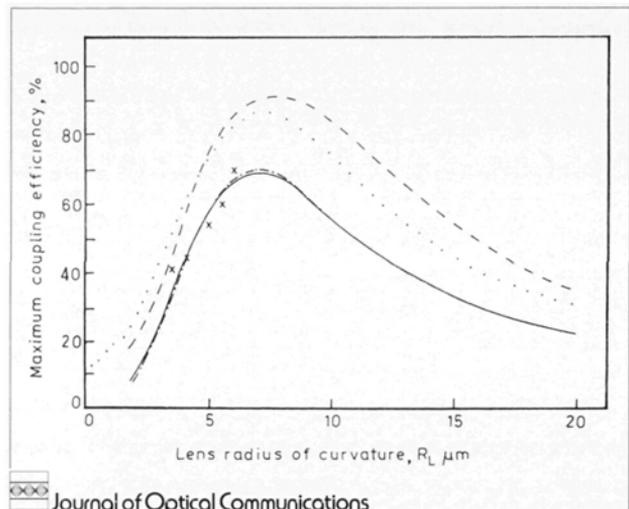


Fig. 2: Variation of maximum coupling efficiency with radius of curvature of the hemispherical lens on the fiber tip: Comparison of our theoretical results with the results of [3]; x experiment [3]; — our theory based on ABCD matrix using the spherical wave model; — · — · our theory based on ABCD matrix using the planar wave model; — · — · theory using the spherical wave model [3]; · · · · theory using the planar wave model [3]

exceedingly well with the experimental findings and follow nicely the nature of variations of the two theoretical curves [3]. The latter two curves seem to overestimate the experimental values whereas our formalism estimates the practical situation properly. The earlier investigations [2] point out that the maximum achievable coupling efficiency can be at best 55 %. The more recent experimental observations [3] give the coupling efficiency nearly 70 % for  $R_L = 6 \mu\text{m}$ . Our formalism predicts values of coupling efficiency 69.36 % and 68.98 % in planar and spherical wave models respectively for incident wavefront when  $R_L = 7 \mu\text{m}$ . It is also seen that the value of  $R_L$  which gives optimum coupling efficiency in our formalism agrees excellently with that predicted by the theoretical curve [3] in planar wave model for incident wavefront. Thus our formalism can safely be recommended for computation in coupling optics involving hemispherical lenses.

## 4 Conclusions

Using the ABCD matrix for refraction by a hemispherical lens on the fiber tip, we present a simple and realistic method to calculate the coupling efficiency of a source to a single-mode fiber excitation via hemispherical lens on the fiber tip. It is realistic in the sense that it takes care of the allowable aperture of such lenses. The excellent agreement of our results with the available experimental results justifies our simple theoretical model. The technique developed should be useful in designing suitable hemispherical microlenses in coupling optics.

## 5 Acknowledgement

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## 6 Appendix

Input and output parameters ( $q_1, q_2$ ) of the light beam are given by

$$q_2 = (Aq_1 + B)/(Cq_1 + D) \quad (A1)$$

where

$$1/q_{1,2} = 1/R_{1,2} - (j\lambda_0)/(\pi w_{1,2}^2 n_{1,2}) \quad (A2)$$

The ray matrix (M) for the hemispherical lens on the tip of the fiber is given by [4]

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ (1-n)/(nR_L) & 1/n \end{pmatrix} \begin{pmatrix} 1 & u \\ 0 & 1 \end{pmatrix} \quad (A3)$$

whereby

$$\begin{aligned} A &= 1 + [d(1-n)]/(nR_L) \\ B &= u + [(1-n)ud]/(nR_L) + d/n \\ C &= (1-n)/(nR_L) \\ D &= 1/n + [(1-n)u]/(nR_L). \end{aligned} \quad (A4)$$

Here the maximum depth (d) of the lens is equal to its radius of curvature ( $R_L$ ) and u is the distance of the laser diode from the hemispherical lens. The refractive index of the material of the lens with respect to the incident medium is represented by n where  $n = n_2/n_1$ . The transformed beam spot sizes and radii of curvature in the x and y directions are found by using (A4) in (A1) and (A2) and expressed as

$$\begin{aligned} w_{2x,2y}^2 &= \frac{A_1^2 W_{1x,1y}^2 + (\lambda_1^2 B^2)/(\pi^2 w_{1x,1y}^2)}{n(A_1 D - B C_1)} \\ \frac{1}{R_{2x,2y}} &= \frac{A_1 C_1 w_{1x,1y}^2 + (\lambda_1^2 B D)/(\pi^2 w_{1x,1y}^2)}{A_1^2 w_{1x,1y}^2 + (\lambda_1^2 B^2)/(\pi^2 w_{1x,1y}^2)} \end{aligned} \quad (A5)$$

where  $\lambda_1 = \lambda_0/n_1$ ;  $A_1 = A + B/R_1$ ;  $C_1 = C + D/R_1$ .

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