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Large amplitude dust acoustic solitary waves and double layers in positively charged warm dusty plasma with nonthermal electrons

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Large amplitude dust acoustic solitary waves and double layers in a nonthermal plasma consisting of positively charged dust grains, nonthermal electrons, and isothermal ions including the effect of dust temperature have been studied using the Sagdeev potential technique by a computational scheme. The effect of different parameters on the nature of existence of solitary waves and double layers has been investigated to delimit their compositional parameter space. The physics corresponding to the computational result has been pointed out. © *2010 American Institute of Physics*. [doi:10.1063/1.3291060]

In the past two decades much emphasis has been given on different types of wave propagation in dusty plasma due to their involvement in the study of astrophysical and space environments.[1–5] Rao *et al.*[6] theoretically predicted the existence of extremely low phase velocity dust acoustic (DA) waves in an unmagnetized collisionless negatively charged dusty plasma. The existence of the DA waves has been supported by many experimental works.[7–9] Rao *et al.*[6] also considered the dust acoustic solitary (DAS) waves in the same dusty plasma system using the reductive perturbation method, whereas Mamun[10] investigated the same problem by Sagdeev potential technique. However lighter dust grains may be positively charged due to photoionization or secondary electron emission.[4,5] The presence of positively charged dust grains has been observed in different regions of space and in laboratory plasmas.[11]

Verheest and Pillay[12] have studied the large amplitude DAS waves and double layers (DLs) in dusty plasmas with positive cold dust, Boltzmann distributed ions and nonthermally distributed electrons as prescribed by Cairns *et al.*[13] In this paper, they have reported that there is no change in the qualitative behavior of the DAS waves and DLs if one consider nonthermally distributed ions instead of Boltzmann distributed ions, i.e., the nonthermal ions are unable to change the qualitative behavior of the DAS waves and DLs. Verheest[14] has reported the coexistence of both positive and negative potential solitary waves as well as the existence of negative potential DLs (NPDLs) in a dusty plasma consisting of both positively and negatively charged cold dust grains along with the nonthermally distributed ions and electrons at the same temperature. Recently, Djebli and Marif[15] have reported the existence of large amplitude NPDLs in a plasma composed of positively charged warm dust particles, cold electrons, and hot electrons. In this brief communication, we reconsider the problem of Verheest and Pillay[12] to include the effect of dust temperature and to elaborate their investigations on delimitation of compositional parameter space. A computational scheme has been developed to study large amplitude DAS waves and DLs in a plasma consisting of positively charged dust grains, Cairns distributed nonthermal electrons, and isothermal ions including the effect of dust temperature. The Sagdeev potential approach, which is valid to study the large amplitude solitary waves and DLs, has been employed.

In this brief communication, we proceed in the same way as in our earlier paper[16] on DAS waves, by suitably modifying for the presence of positive dust instead of negative dust. However here we normalize with respect to temperature of Cairns species by T_e instead of by T_i, so that $c_d=\sqrt{(Z_d K_B T_e)/m_d}$, $\lambda_{Dd}=\sqrt{(K_B T_e)/(4\pi Z_d n_{d0} e^2)}$, and ϕ is normalized by $K_B T_e/e$, where Z_d is the number of positive unit charges residing on the dust grains and other notations have their usual meanings. The nonthermal electron and isothermal ion number densities of the present system can be obtained from our earlier paper by simply replacing ϕ by $-\phi$, α by $1/\alpha$, and making the interchange between the subscripts e and i. The parameters σ_d, α, and μ are defined by $\sigma_d=T_d/(Z_d T_e)$, $\alpha=T_i/T_e$, and $\mu=n_{i0}/n_{e0}$, where n_{i0} and n_{e0} are the unperturbed number densities of ions and electrons, respectively. μ_i and μ_e can also be obtained by making an interchange between e and i in the subscripts in our previous paper.[16] Making these changes in the basic equations and following the same algebra in our earlier paper[16] with corresponding adaptation to the length scale, we get

$$n_d^2=\frac{1}{6\sigma_d}(\sqrt{\Psi_M-\phi}-\sqrt{\Phi_M-\phi})^2, \tag{1}$$

where

$$\Psi_M=\tfrac{1}{2}(M-\sqrt{3\sigma_d})^2, \quad \Phi_M=\tfrac{1}{2}(M+\sqrt{3\sigma_d})^2. \tag{2}$$

Obviously n_d^2 is defined for $\phi \leq \Psi_M$. However for computational scheme, we have expressed Eq. (1) in the form of Eq. (19) of Das *et al.*[16] with appropriate changes for positive dust. Now integrating Poisson equation using the boundary condition (15) of Das *et al.*,[16] we arrive at the following energy integral with $V(\phi)$ as Sagdeev pseudopotential

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + V(\phi) = 0, \tag{3}$$

where

$$V(\phi) = \mu_e V_e - \mu_i V_i - V_d, \tag{4}$$

$$V_d = -(M^2 + \sigma_d) + n_d(M^2 + 3\sigma_d - 2\varphi - 2\sigma_d n_d^2), \tag{5}$$

$$V_e = (1 + 3\beta_1) - (1 + 3\beta_1 - 3\beta_1\phi + \beta_1\phi^2)e^{\phi}. \tag{6}$$

$$V_i = -\alpha(1 - e^{-\phi/\alpha}), \tag{7}$$

where β_1 is the nonthermal parameter restricted to lie within $0 \leq \beta_1 < 4/3$. The physical interpretation of the energy integral and different conditions applied on pseudopotential $V(\phi)$ for the existence of different solitary waves and DL solutions have been discussed explicitly in our earlier paper.[16]

For the present problem, from the condition $V''(0) < 0$, we have $M > M_c$ where

$$M_c^2 = 3\sigma_d + \frac{\alpha(1-\mu)}{\mu + \alpha(1-\beta_1)}. \tag{8}$$

From Eq. (8) we find that β_1 is restricted to lie within $0 \leq \beta_1 < \beta_{1T} = \min\{1 + \mu/\alpha, 4/3\}$.

To find the upper limit of M ($M_{\max}$), up to which positive potential solitary wave (PPSW) solution can exist, we use the same treatment as made for negative potential in our earlier paper.[16] However for the present dusty plasma system, depending on the behavior of $V(\Psi_M)$ with respect to β_1, we have the following three cases: (a) $V(\Psi_{M_c}) > 0$ and $V(\Psi_M)$ is an increasing function of M for $M_c < M < M^*$ and a decreasing function of M for $M^* \leq M \leq M_{\max}$, i.e., $V(\Psi_M)$ attains its global maximum at $M = M^*$. Now as Ψ_M and $V(\Psi_M)$ both are increasing functions of M for $M_c < M < M^*$, it is a simple to check that $V(\Psi_M)$ is an increasing function of Ψ_M for $M_c < M < M^*$, and consequently, $M_c < M < M^* \Rightarrow \Psi_{M_c} < \Psi_M < \Psi_{M^*} \Rightarrow V(\Psi_{M_c}) < V(\Psi_M) < V(\Psi_{M^*}) \Rightarrow V(\Psi_M) > V(\Psi_{M_c}) > 0$, i.e., $V(\Psi_M) > 0$ for all $M_c < M < M^*$. Again as Ψ_M is an increasing function of M and $V(\Psi_M)$ is a decreasing function of M for $M^* \leq M \leq M_{\max}$, it is easy to verify that $V(\Psi_M) \geq 0$ for all $M_c < M \leq M_{\max}$. (b) $V(\Psi_{M_c}) > 0$ and $V(\Psi_M)$ is a strictly decreasing function of M for $M_c < M \leq M_{\max}$. Pursuing similar arguments as in (a) we have $V(\Psi_M) \geq 0$ for all $M_c < M \leq M_{\max}$. (c) $V(\Psi_{M_c}) \leq 0$ and $V(\Psi_M)$ is a strictly decreasing function of M for $M > M_c$. Following the same arguments as earlier, we have found $V(\Psi_M) < 0$ for $M > M_c$. Thus there is no possibility that $V(\Psi_M) = 0$, i.e., no existence of $M_{\max}$, i.e., no existence of PPSW. Therefore, for the existence of PPSWs, the Mach number M is restricted by $M_c < M \leq M_{\max}$, where $M_{\max}$ is the largest positive root of the equation $V(\Psi_M) = 0$ subject to the condition $V(\Psi_M) \geq 0$ for all $M \leq M_{\max}$. Again to find an upper limit or upper bound of M, up to which negative potential solitary wave (NPSW) solution can exist, we shall first of all find a value M_D of M for which NPDL solution exists. It can be easily checked numerically that NPDL solution is the ultimate solution of the energy integral

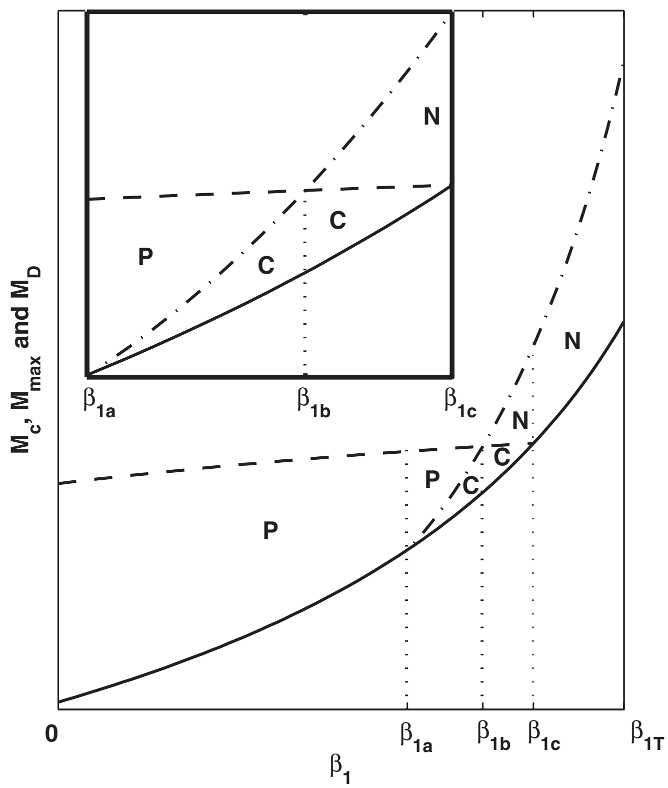

FIG. 1. M_c (—), $M_{\max}$ (– – –), and M_D (–·–) have been plotted against β_1. By "P," "C," and "N" we mean, respectively, the regions for existence of PPSWs, coexistence of both PPSWs and NPSWs, and existence of NPSWs. At any point on the curve $M = M_D$, one can always get a NPDL solution. The region between β_{1a} and β_{1c} is shown in larger scale in the inset.

(3) that the present system can encounter, i.e., increasing M beyond M_D, no NPSW solution can be obtained. From this consideration, it is also clear that NPSW solution of the energy integral (3) exists when $M_c < M < M_D$ provided that energy integral (3) gives a NPDL solution at $M = M_D$.

With the above theoretical development we investigate the present system using a computational scheme. We focus mainly on delimitation of different parameters involved in the system with respect to different types of solitary wave and DL solutions. First of all we concentrate on the parameter β_1. We have found three points β_{1a}, β_{1b}, and β_{1c} lying in between 0 and β_{1T} so that entire range of β_1 can be split up to four subintervals I, II, III, and IV defined as I: $0 \leq \beta_1 \leq \beta_{1a}$, II: $\beta_{1a} < \beta_1 \leq \beta_{1b}$, III: $\beta_{1b} < \beta_1 \leq \beta_{1c}$, and IV: $\beta_{1c} < \beta_1 < \beta_{1T}$. The variation in the nature of solitary wave and DL solution has been represented graphically in Fig. 1. From this figure we see that the solitary wave solution starts just above the lower curve $M = M_c$ and ends up with the uppermost curve. For each of M in between these curves there is either solitary wave solution or DL solution. The nature of the solitary wave is determined by the corresponding upper curve(s). For clarity, if we pick a β_1 in $0 \leq \beta_1 \leq \beta_{1a}$ and goes with M, all intermediate M, bounded by the curves $M = M_c$ and $M = M_{\max}$, generate PPSW. Again for a β_1 in $\beta_{1a} < \beta_1 \leq \beta_{1b}$ we always have coexistence of both PPSW and NPSW for all values of M lies between the curves $M = M_c$ and $M = M_D$, whereas all intermediate M bounded by $M = M_D$ and $M = M_{\max}$ generate only PPSW. At any point on the curve

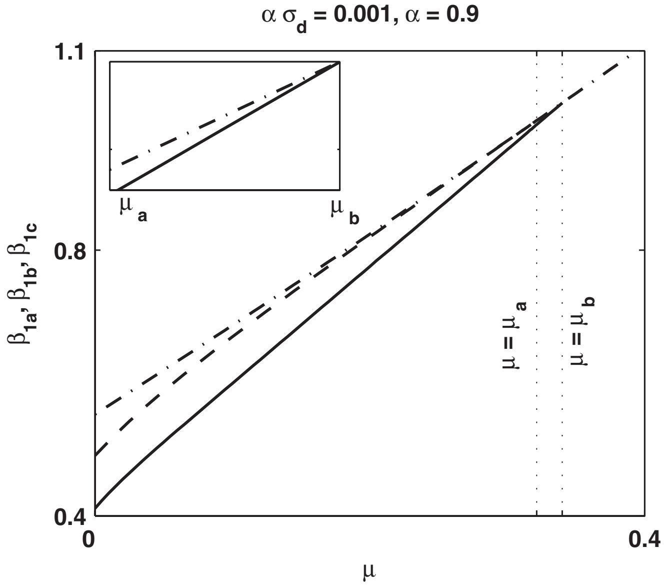

FIG. 2. β_{1a} (—), β_{1b} (– – –), and β_{1c} (–·–) are plotted against μ. The region between μ_a and μ_b is shown in larger scale in the inset.

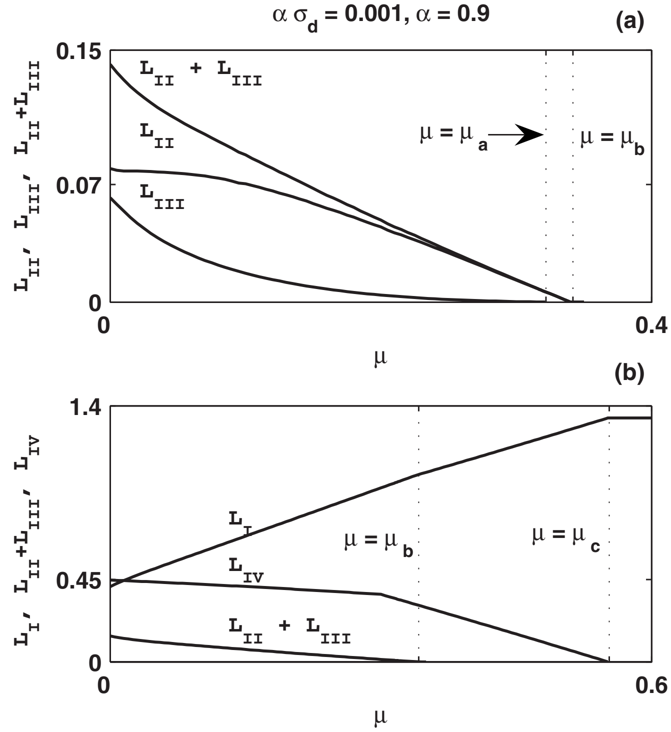

FIG. 3. L_{II}, L_{III}, and $L_{\mathrm{II}}+L_{\mathrm{III}}$ are plotted against μ in (a) and in (b) L_{I}, $L_{\mathrm{II}}+L_{\mathrm{III}}$, and L_{IV} are plotted against μ.

$M=M_D$, one can always get a NPDL solution. Similar interpretation can be made for other two subintervals. We have observed that in general β_{1a}, β_{1b}, and β_{1c} are increasing function of σ_d.

However for fixed σ_d and α the above analysis is not valid for the entire range of μ. This phenomena has been shown in Fig. 2. It is clear from this figure that β_{1b} meets β_{1c} at a point $\mu=\mu_a$ (say), whereas β_{1a} meets β_{1c} at a point $\mu=\mu_b$ (say), i.e., at $\mu=\mu_a$, L_{III} is equal to zero and at $\mu=\mu_b$, $L_{\mathrm{II}}+L_{\mathrm{III}}$ is equal to zero and consequently, L_{II} is equal to zero at $\mu=\mu_b$ because L_{III} is equal to zero for $\mu\geq\mu_a$, where L_k is the length of the kth subinterval. Again we find that β_{1c} meets β_{1T} at point $\mu=\mu_c$. To understand the situation clearly it is helpful to draw the length of the different subintervals of β_1 against μ. In Fig. 3(a), L_{II}, L_{III}, and $L_{\mathrm{II}}+L_{\mathrm{III}}$ are plotted against μ. We see from this figure that all the curves decrease with increasing μ. It is also clear from this figure that L_{III} vanishes at $\mu=\mu_a$, whereas L_{II} and $L_{\mathrm{II}}+L_{\mathrm{III}}$ vanish at $\mu=\mu_b$, and consequently all facts as pointed out from Fig. 2 are true. In Fig. 3(b), L_{I}, $L_{\mathrm{II}}+L_{\mathrm{III}}$, and L_{IV} are plotted against μ. We see from this figure that $L_{\mathrm{II}}+L_{\mathrm{III}}$ vanishes at $\mu=\mu_b$. From this figure it is also clear that L_{IV} vanishes at $\mu=\mu_c$, and consequently for $\mu\geq\mu_c$ subinterval I is actually the entire admissible range of β_1. Now as L_{III} vanishes at $\mu=\mu_a$, all the four subintervals of β_1 exist for $0\leq\mu<\mu_a$. In $\mu_a\leq\mu<\mu_b$, all the subintervals of β_1 exist except subinterval III. Again there is no existence of subintervals II and III in $\mu_b\leq\mu<\mu_c$, and consequently, for $\mu_b\leq\mu<\mu_c$, only PPSW is possible if $0\leq\beta_1\leq\beta_{1c}$ and only NPSWs and NPDLs are possible if $\beta_{1c}<\beta_1<\beta_{1T}$. For $\mu\geq\mu_c$, only subinterval I exists, i.e., only PPSWs are possible if $0\leq\beta_1<\beta_{1T}$. Hence, if μ crosses μ_b, it is impossible to find coexistence of both PPSW and NPSW and for $\mu\geq\mu_c$ only positive potential solitary structures are possible, i.e., it is impossible to find a NPSW as well as NPDL for $\mu\geq\mu_c$. Here, μ_c acts as a switching value of μ with the property that if μ crosses μ_c, only PPSW exists for any value of β_1, whereas for $(\mu_b\leq)\mu<\mu_c$, only PPSW is possible if $0\leq\beta_1\leq\beta_{1c}$ and only NPSWs and NPDLs are possible if $\beta_{1c}<\beta_1<\beta_{1T}$, i.e., if $\mu\geq\mu_c$, nonthermal parameter β_1 is unable to change the solitary structures. Therefore, we find that all four subintervals of β_1 are found to exist if $0\leq\mu<\mu_a$. For $\sigma_d=0.001$ and $\alpha=0.9$, we find $\mu_a\approx0.32$, $\mu_b\approx0.34$, and $\mu_c\approx0.55$. Thus we have delimited the range of the parameters with respect to the nature of solitary wave and DL. One can easily verify the delimitation simply by plotting pseudopotential against ϕ as we did in our earlier paper.[16]

The physics corresponding to the numerical results can be interpreted as follows: when the value of β_1 lies in a small neighborhood of the point $\beta_1=0$, the nonthermal distribution of electrons deviates little from Boltzmann distribution and thus the nonthermal electrons are unable to acquire enough strength to make a NPSW. If β_1 crosses a critical value β_{1a}, i.e., if sufficient number of energetic electrons are present in the system the nonthermal distribution is capable of making a root of $V(\phi)=0$ on the negative potential side and thus we have a NPSW. Again NPDLs occur when $\beta_1>\beta_{1a}$. Therefore, for the existence of NPSWs and NPDLs it is required to have a sufficient number of energetic electrons and positively charged dust grains. In this context one might interpret that the presence of ions is not really required for the existence of NPSWs and NPDLs. However if μ increases, i.e., inclusion of more ions in the system sustain only PPSWs and delay the occurrence of NPSWs and NPDLs. Once $\beta_1>\beta_{1c}$, i.e., the nonthermal distribution includes too many energetic electrons, the positively charged particles are unable to keep the PPSWs if μ is restricted to lie within $0\leq\mu<\mu_c$. However if

μ increases then the system can have PPSW for the same β_1 and thus a little more ions are needed for that. When there are sufficient number of ions in the system so that $\mu \geq \mu_c$, the nonthermal electrons cannot acquire enough strength compared with positively charged particles to produce a negative potential solitary waves. Now as β_1 increases the number of energetic electrons also increases in the system, and consequently NPSWs are expected. Therefore, for restricted values of β_1, coexistence of both NPSWs and PPSWs are expected. In this communication, for existence of both NPSWs and PPSWs, the values of β_1 is restricted by the inequality: $\beta_{1a} < \beta_1 < \beta_{1c}$. Again for constant average temperature of both dust grains and electrons, we find that $Z_d \propto \sigma_d^{-1}$. Therefore, as σ_d decreases the number of positive charges (i.e., ions) residing on the dust grain surface increases. However to keep the value of μ intact, the system must include more energetic electrons which actually strengthen the nonthermal distribution. These excess electrons are responsible for the occurrence of NPSW faster than earlier. Thus NPSWs are expected for smaller value of β_1 and consequently β_{1a} decreases with respect to σ_d. For the same reason β_{1b} and β_{1c} also decrease. In a nutshell, inclusion of ions try to sustain only PPSWs and delay the occurrence of NPSWs or NPDLs, whereas increasing value of the nonthermal parameter β_1, actually, make the existence of both NPSWs and NPDLs faster.

In summary, the properties of large amplitude DAS waves and DLs in nonthermal plasmas consisting of positively charged warm dust grains, isothermal ions and nonthermal electrons, are systematically and explicitly investigated with the help of Sagdeev potential method. We have found that for any fixed values of α and σ_d, there exists a value μ_a of μ such that for $0 \leq \mu < \mu_a$, the entire interval of β_1 can be broken up into four disjoint subintervals, depending on the nature of existence of solitary waves and DLs. From $\mu = \mu_a$ onwards the subintervals successively disappear and for $\mu \geq \mu_c$, only PPSWs can exist for the entire range of β_1, i.e., if μ crosses the value μ_c, nonthermal parameter is unable to change the qualitative behavior of the solitary structures. The results of the present investigation should be useful for understanding the nonlinear structures of DA waves in a number of heliospheric environments where positively charged warm dust particles and nonthermally distributed electrons are the major plasma species such as in the upper Earth's atmosphere.

For $\sigma_d = 0$, $\alpha = 1$, and $\mu = 0$, if we apply our computational scheme, we get $\beta_{1a} = 0.411\,61$, $\beta_{1b} = 0.529\,64$, and $\beta_{1c} = 0.614\,55$. The values of β_{1a} and β_{1c} are exactly same as those of Verheest and Pillay.[12]

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